Wideband Smart Antenna Theory Using Rectangular Array Structures

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***Abstract*—Smart antenna techniques at the base station can dramatically improve the performance of the mobile radio system by employing spatial filtering. The design of a fully spatial signal processor using rectangular array configuration is proposed in this paper. Two-dimensional (2-D) spatial filters that can be implemented by microstrip technology are capable of filtering the received signal in the angular domain as well as the frequency domain. Furthermore, it has wideband properties and, hence, eliminates the requirement of different antenna spacing for applications including various carrier frequencies. The desired frequency selectivity of the smart antenna can be combined with compensation of the undesired frequency performance of a single antenna element, and the result is quite satisfactory for practical implementation. In addition, if the elements of the array are not perfectly omnidirectional or frequency independent, we can compensate for these deficiencies in the design algorithm. Two different algorithms for calculating the real-valued weights of the antenna elements are proposed. The first algorithm is more complex but leads to sharper beams and controlled performance. The second method is simpler but has wider beam and lower fractional bandwidth. Some computer simulation results demonstrating the directional beam patterns of the designed beamformers are presented at the end of this paper.**

***Index Terms*—Formation, rectangular arrays, spatial filtering, wideband beam.**

I. INTRODUCTION

ONE OF the major problems of future mobile communication systems is the rapid increase in the demand for different broadband services and applications. Since the available spectrum for providing high data rate communication to new subscribers is limited, there is no doubt that smart antennas are the best solution for increasing the system capacity and performance. These antennas not only increase the gain but also reduce the interference and the delay spread by means of spatial filtering.

Smart antennas employ arrays of antenna elements and can improve reliability and capacity in wireless systems in three ways [1]. First, diversity-combining techniques combine the signals from multiple antennas in a way that mitigates multipath fading. In most scattering environments, antenna diversity is a practical, effective, and, hence, widely applied technique. The classical approach is to use multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of the received signal. Space–time coding using multiple antennas has been studied in recent research [2]–[4], and space–time codes have been introduced in a multipath fading environment to improve mobile system performance. Second, adaptive beamforming using antenna arrays provides capacity improvement through interference reduction and mitigates multipath fading. Adaptive arrays cancel multipath components of the desired signal and null interfering signals that have different directions of arrival (DOAs) from the desired signal. The third category of systems uses switched fixed beams to achieve coarser pattern control than adaptive arrays. Two or more of the fixed beams can be used for diversity reception. Adaptive and switched beam antenna systems are popularly referred to as smart antennas because of the dynamic system intelligence required for their operation.

Most of the smart antennas proposed in literature are narrowband beamformers [5], [6]. The antenna spacing of narrowband arrays is usually half of the wavelength of the incoming signal that is assumed to have a fractional bandwidth (FB) of less than 1%. By definition, the FB of a signal is the ratio of the bandwidth to the center frequency as follows:

$$\mathrm{FB} = \frac{f_h - f_l}{(f_h + f_l)/2} \times 100\% \tag{1}$$

where f_h and f_l are the highest and the lowest frequencies of the signal, respectively. Wideband arrays are designed for FB of up to 50% [7], [8], and ultra-wideband arrays are proposed for FB of 50 to 200% [9]. Wideband and ultra-wideband arrays use the same antenna spacing for all frequency components of the arriving signals. The interelement distance d is determined by the highest frequency of the input wave. In a uniform one-dimensional (1-D) linear array, we can write

$$d = \frac{c}{2f_h} \tag{2}$$

where c is the propagation speed. Conventional wideband antenna arrays use a combination of spatial filtering with temporal filtering. On each branch of the array, a tapped delay line (TDL) filter [10] or a recursive filter [11] allows each element to have a phase response that varies with frequency. As a result, the phase shifts due to higher and lower frequencies are equalized by temporal signal processing.

Rectangular linear arrays, when subjected to an almost fixed elevation angle, may be used for fully spatial signal processing of antenna arrays with wideband properties [12]. In addition, the sharpness of the beams is satisfactory not only in the broadside but at the endfire directions of the array as well. The beamwidth of the directional pattern can be controlled at all angles, and

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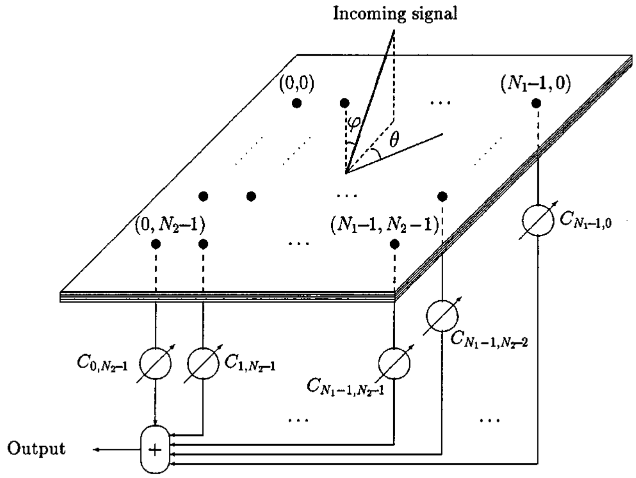

Fig. 1. Incoming signal arriving at an $N_1 \times N_2$ array with azimuth angle θ and elevation angle φ. Each element is connected to a real multiplier, although not all of them are shown.

frequency domain filtering can be achieved easily in the design procedure. This is called frequency-selective wideband beamforming (FSWB) and can compensate for the frequency dependence of the elements. Furthermore, we can selectively filter the wideband input signals in different frequency regions.

In this paper, we introduce two new algorithms that design wideband rectangular antenna arrays. The first algorithm is based on the inverse discrete Fourier transform (IDFT), and the second algorithm is a constrained beamforming technique and includes matrix inversion. In both cases, the wideband signals received by the antenna are filtered in angle and frequency domains, and interference coming from other sources with different DOAs or frequencies are suppressed efficiently by spatial signal processing. The beamforming procedure is performed at radio frequency (RF), and real multipliers are necessary for each antenna element.

The reminder of this paper is organized as follows. In Section II, the basic theory of frequency-selective rectangular wideband arrays is presented. In Section III, we consider the first design algorithm and different aspects of this method such as beamwidth control and adaptive selection of the number of antenna elements. In Section IV, we describe the second way to design a wideband rectangular array. Some simulation results are given in Section V. Finally, Section VI concludes the paper.

II. Rectangular Array Antenna in Azimuth

Fig. 1 shows the structure of a linear and uniform rectangular configuration for $N_1 \times N_2$ elements of the proposed beamforming network. Each antenna element is denoted by (n_1, n_2), where $0 \le n_1 \le N_1 - 1$ and $0 \le n_2 \le N_2 - 1$ and has a frequency-dependent gain that is the same for all elements. The direction of the arriving signal is determined by the azimuth and the elevation angles θ and φ, respectively. As in most practical cases, it is assumed that the elevation angles of the incoming signals to the base station antenna array are almost constant, and without loss of generality, we consider $\varphi \approx 90°$. The elements are placed at the distances of d_1 and d_2 in the direction of n_1 and n_2, respectively. Assuming that the phase reference point is located at $(n_1 = 0, n_2 = 0)$, the phase of the incoming wave at the element denoted by (n_1, n_2) is

$$\Phi(n_1, n_2) = \frac{2\pi f}{c}\,(d_1 n_1 \sin\theta - d_2 n_2 \cos\theta) \tag{3}$$

where f is the frequency variable. Note that if φ was constant but not necessarily near 90°, then we would have to modify d_1 and d_2 to new constant values of $(d_1 \sin\varphi)$ and $(d_2, \sin\varphi)$, respectively, which are, in fact, the effective array interelement distances in an environment with almost-fixed elevation angles. Unlike conventional wideband arrays, we assume that each antenna element is connected to only one single coefficient $C_{n_1 n_2}$. Hence, the response of the array with respect to frequency and angle can be written as

$$H(f, \theta) = G_a(f, \theta) \cdot \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} C_{n_1 n_2} e^{j(2\pi f/c)(d_1 n_1 \sin\theta - d_2 n_2 \cos\theta)} \tag{4}$$

where $G_a(f, \theta)$ represents the frequency-angle dependent gain of each antenna element. Normally, $G_a(f, \theta)$ is an even function of θ for $-90° < \theta < 90°$ and has a maximum value at $\theta = 0$, $f = f_0$, where $f_l < f_0 < f_h$. Now, two auxiliary frequencies are defined by

$$f_1 = \frac{f d_1}{c} \sin\theta \tag{5}$$

$$f_2 = \frac{f d_2}{c} \cos\theta. \tag{6}$$

These new variables are functions of the frequency f and the angle θ and are related as

$$\frac{f_1}{f_2} = \frac{d_1}{d_2} \tan\theta. \tag{7}$$

This equation shows that for any value of θ, if f is multiplied by a positive constant, then f_1 and f_2 will also be multiplied by the same factor, and their ratio will be independent of frequency. This indicates a wideband property.

Substituting (5) and (6) in (4) yields

$$H(f_1, f_2) = G_a(f_1, f_2) \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} C_{n_1 n_2} e^{j2\pi f_1 n_1} e^{-j2\pi f_2 n_2} \tag{8}$$

where $G_a(f, \theta)$ is now rewritten as a function of f_1 and f_2. The two-dimensional (2-D) frequency region $f_1 - f_2$ is limited for both f_1 and f_2 to $(-0.5, 0.5)$ because, for example, for f_1, we can write

$$|f_1| = \left|\frac{f d_1}{c} \sin\theta\right| \le \frac{f d_1}{c} \le \frac{f}{c}\,\frac{\lambda_{\min}}{2} = \frac{f}{c}\,\frac{c}{2 f_h} \le 0.5 \tag{9}$$

where $\lambda_{\min}$ is the wavelength of the highest frequency. Equation (9) is valid for f_2 as well.

Careful examination of (8) indicates that it looks like a discrete Fourier transform (DFT). Hence, the desired 2-D frequency characteristics, which are divided by $G_a(f_1, f_2)$, gives the 2-D DFT of the multipliers denoted by $C_{n_1 n_2}$. Therefore, by taking the IDFT of the values of $H(f_1, f_2)/G_a(f_1, f_2)$

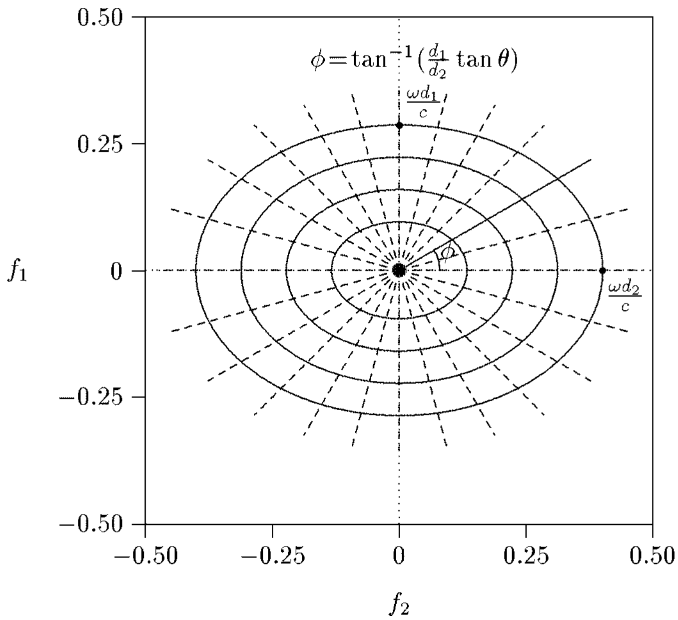

Fig. 2. Loci of constant angle θ and constant frequency f are radial and elliptical, respectively.

in the $f_1 - f_2$ plane, we may have $C_{n_1 n_2}$, and the design is complete. These are the fundamental steps of our first design method.

If ϕ is the polar angle of the $f_1 - f_2$ plane, we will have

$$\frac{f_1}{f_2} = \frac{d_1}{d_2}\tan\theta = \tan\phi \tag{10}$$

and eliminating θ from (5) and (6) yields

$$\left(\frac{f_1}{fd_1/c}\right)^2 + \left(\frac{f_2}{fd_2/c}\right)^2 = 1 \tag{11}$$

which demonstrates an ellipse with the center at $f_1 = f_2 = 0$. In the special case of $d_1 = d_2 = d$, we have circles with the equations $f_1^2 + f_2^2 = (fd/c)^2$ and radius fd/c. Equations (10) and (11) represent the loci of constant angle and constant frequency in the $f_1 - f_2$ plane, respectively. Plots of these two loci, which are given in Fig. 2, are helpful for determination of the angle and frequency characteristics of the wideband beamformer.

Assume that an array antenna system is to be designed with $\theta = \theta_0$, $f_l < f < f_h$ and a center frequency of $f = f_0$. A demonstrative plot, which shows the location of the desired points on the $f_1 - f_2$ plane, is given in Fig. 3. This location is limited by $\phi_0 = \tan^{-1}((d_1/d_2)\tan\theta_0)$ and $r_l < |r| < r_h$, where

$$r_l = \frac{f_l}{c}\bar{d}; \quad r_h = \frac{f_h}{c}\bar{d}; \quad \bar{d} = \sqrt{d_1^2\sin^2\theta_0 + d_2^2\cos^2\theta_0}. \tag{12}$$

The symmetry of the loci with respect to the origin of the $f_1 - f_2$ plane results in real values for the multipliers of each antenna element $C_{n_1 n_2}$.

III. Beamforming Using IDFT

The first proposed design method of wideband beamforming with the specifications of Fig. 3 is explained in this section. In an ideal situation, i.e., for perfect omnidirectional antenna elements with $G_a(f_1, f_2) = 1$, the amplitude of $H(f_1, f_2)$ at the selected points given in Fig. 3 must be equal to one and, at other points out of this region, must be equal to zero. This is demonstrated by

$$H_{\text{ideal}}(f_1, f_2) = \begin{cases} 1; & \phi_0 = \tan^{-1}\left(\frac{d_1}{d_2}\tan\theta_0\right), \quad r_l < |r| < r_h \\ 0; & \text{otherwise} \end{cases} \tag{13}$$

where r_l and r_h are as defined in (12). If the elements are not perfectly omnidirectional and allpass, we have to compensate the frequency and angle dependence of the elements in (13). It is easily done by replacing 1 in (13) with inverse of $G_a(f_1, f_2)$ as

$$\tilde{H}_{\text{ideal}}(f_1, f_2) = \begin{cases} G_a^{-1}(f_1, f_2); & \phi_0 = \tan^{-1}\left(\frac{d_1}{d_2}\tan\theta_0\right) \\ & r_l < |r| < r_h \\ 0; & \text{otherwise.} \end{cases} \tag{14}$$

For designing the beamformer with beamwidth control, a lowpass filter is defined by the impulse response

$$H_p(f) = \frac{\sin\pi f}{\pi f} \tag{15}$$

Then, we transform $H_p(f)$ to $H(f_1, f_2)$ as

$$H(f_1, f_2) = \tilde{H}_{\text{ideal}}(f_1, f_2) H_p\left(\alpha\left(\frac{f_1}{f_2} - \frac{d_1}{d_2}\tan\theta_0\right)\right) \tag{16}$$

where α is a constant, chosen according to the desired beamwidth of the antenna pattern, and θ_0 is the direction of the main lobe of the desired beamformer. Now, an IDFT is performed on the frequency response values of (16), and the results are $C_{n_1 n_2}$.

Now, we demonstrate that several useful parameters of the wideband beamformer can be assigned adaptively.

A. Beamwidth

Internull beamwidth $\Delta\theta$ is defined as the difference between the angles of the nearest zeros to the main lobe. To find a relation between $\Delta\theta$ and α, we first combine (15) and (16), which gives

$$H(f_1, f_2) = \tilde{H}_{\text{ideal}}(f_1, f_2)\frac{\sin\left[\alpha\pi\left(\frac{f_1}{f_2} - \frac{d_1}{d_2}\tan\theta_0\right)\right]}{\alpha\pi\left(\frac{f_1}{f_2} - \frac{d_1}{d_2}\tan\theta_0\right)}. \tag{17}$$

Now, $H(f_1, f_2) = 0$ yields

$$\alpha\left(\frac{f_1}{f_2} - \frac{d_1}{d_2}\tan\theta_0\right) = \pm 1, \pm 2, \ldots. \tag{18}$$

The first zeros correspond to the values ± 1 on the right side of (18); hence, we have

$$\alpha\left(\frac{d_1}{d_2}\tan\theta - \frac{d_1}{d_2}\tan\theta_0\right) = \pm 1 \tag{19}$$

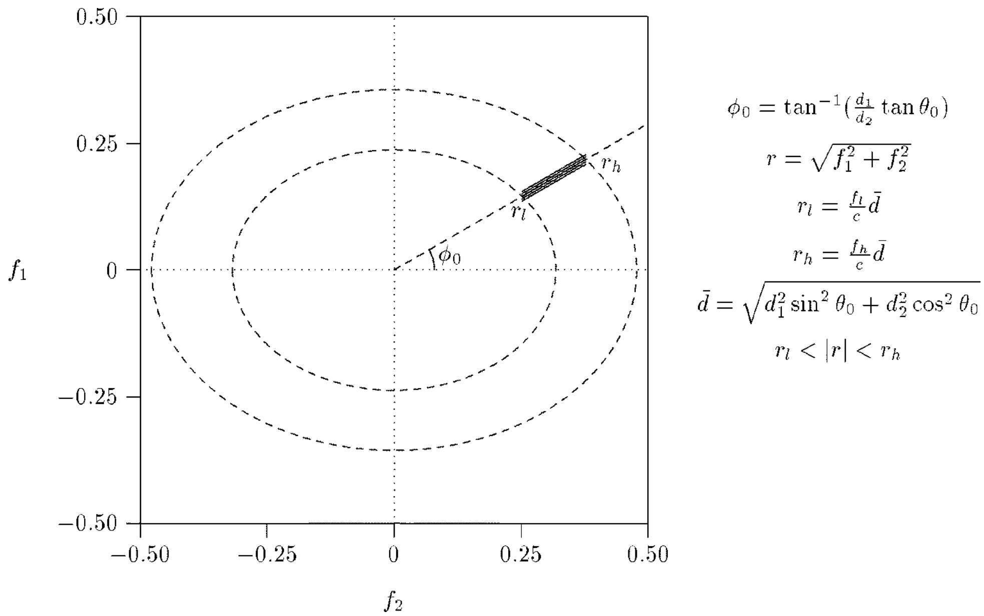

Fig. 3. Location of the desired points on the intersection of the constant angle and constant frequency loci.

which can be rewritten as

$$\theta = \tan^{-1}\left(\tan\theta_0 \pm \frac{1}{\alpha}\frac{d_2}{d_1}\right) = \theta_1,\, \theta_2. \tag{20}$$

This means that the nearest two zeros to θ_0 are θ_1 and θ_2, where $\theta_2 > \theta_1$, respectively. Therefore, $\Delta\theta = \theta_2 - \theta_1$ can be written as

$$\Delta\theta = \tan^{-1}\left(\tan\theta_0 + \frac{1}{\alpha}\frac{d_2}{d_1}\right) - \tan^{-1}\left(\tan\theta_0 - \frac{1}{\alpha}\frac{d_2}{d_1}\right). \tag{21}$$

Assuming that θ_0 and $\Delta\theta$ are known, we can derive a relation for determining α from $\Delta\theta$ as

$$\alpha = \frac{\frac{d_2}{d_1}\tan\Delta\theta}{-1 + \sqrt{1 + \tan^2\Delta\theta + \tan^2\Delta\theta\tan^2\theta_0}}. \tag{22}$$

This relation is shown in Fig. 4 for various values of θ_0 and $\Delta\theta$. As an example, for $d_1 = d_2$, $\theta_0 = 80°$, and $\Delta\theta = 20°$, we will have $\alpha = 0.275$.

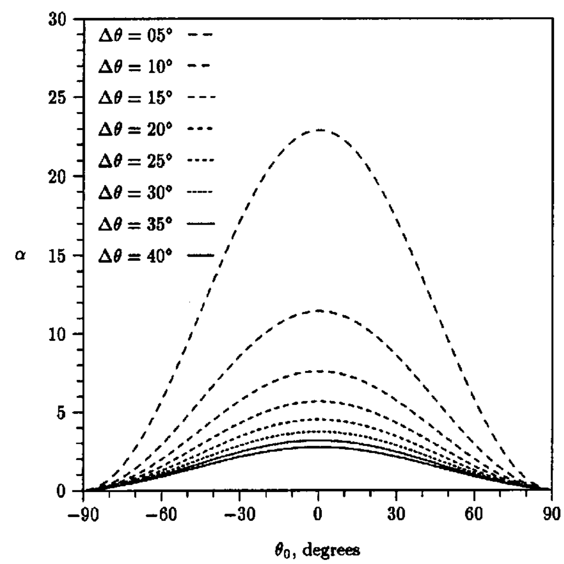

Fig. 4. Beamwidth parameter α as a function of the beam angle θ_0 and the beamwidth $\Delta\theta$.

B. Number of Antenna Elements

The dimensions of the rectangular array are determined by N_1 and N_2. The absolute value of the beamformer angle θ_0 can be related to the number of elements in two directions of n_1 and n_2 in a way that for $\theta_0 = 0°$ and $|\theta_0| = 90°$, we have a maximum number of antenna elements in the directions of n_1 and n_2 and a minimum number of elements in the directions of n_2 and n_1, respectively. Using a linear relationship, we can write

$$N_1 = \left[N_{1_{\max}} - \frac{|\theta_0|}{90°}\left(N_{1_{\max}} - N_{1_{\min}}\right)\right] \tag{23}$$

and

$$N_2 = \left[N_{2_{\max}} - \frac{(90° - |\theta_0|)}{90°}\left(N_{2_{\max}} - N_{2_{\min}}\right)\right] \tag{24}$$

where $[\cdot]$ denotes the nearest integer to the argument. As an example, for $N_{1_{\max}} = N_{2_{\max}} = 41$, $N_{1_{\min}} = N_{2_{\min}} = 11$ and $\theta_0 = 0, 45, 90°$, we will calculate $N_1 = 41, 26, 11$ and $N_2 = 11, 26, 41$, respectively. Therefore, we can save on the number of elements used considerably when designing different beamformers with various angles.

To illustrate the sharpness of the controlled beam pattern of the array at the endfire angles, we consider an example. The parameters of the array have been already mentioned in this section, except that $f_l = 0.1c/d$, and $f_h = 0.48c/d$. Fig. 5 shows the 2-D frequency response, and Fig. 6 demonstrates the beam pattern for ten frequencies between f_l and f_h. The FB is here about 131%.

IV. Beamforming Using Matrix Inversion

In the second method for the design of the real multipliers, instead of controlling all points of the $f_1 - f_2$ plane, which is very difficult to do, we consider only L points on this plane.

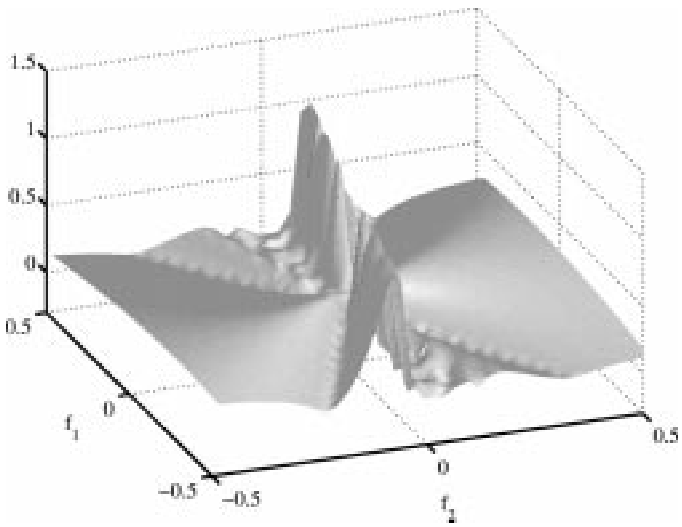

Fig. 5. Two-dimensional frequency response of the first method with $\theta_0 = 80°$.

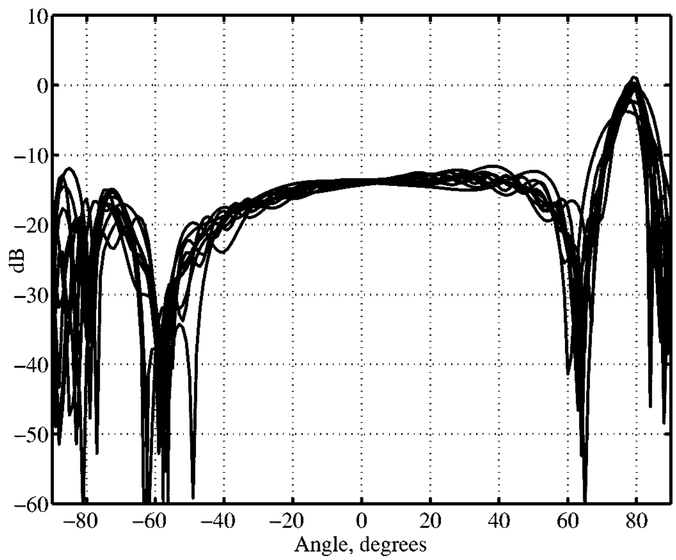

Fig. 6. Directional patterns of the beamformer of Fig. 5 for $f_l = 0.1c/d$ and $f_h = 0.48c/d$.

These L points are symmetrically distributed on the $f_1 - f_2$ plane and do not include the origin; thus, L is considered an even integer. Two vectors are defined as

$$\mathbf{b} = [b_1, b_2, \ldots, b_L]^T \tag{25}$$

$$\mathbf{H_0} = \left[H\left(f_{1_{0_1}}, f_{2_{0_1}}\right), H\left(f_{1_{0_2}}, f_{2_{0_2}}\right), \ldots H\left(f_{1_{0_L}}, f_{2_{0_L}}\right)\right]^T \tag{26}$$

where the superscript T stands for transpose. The elements of the vector $\mathbf{H_0}$ have the same values for any two pairs $(f_{1_{0_l}}, f_{2_{0_l}})$, $l = 1, 2, \ldots, L$, which are located symmetrically with respect to the origin of the $f_1 - f_2$ plane. In addition, they consider the frequency dependence of the elements in a way like (14). The vector $\mathbf{b}$ is an auxiliary vector and will be computed in the design procedure. Now, assume that $H(f_1, f_2)$ is expressed by the multiplication of two basic polynomials and, then, the summation of the weighted result as

$$H(f_1, f_2) = \sum_{l=1}^{L} b_l \left(\sum_{n_1=0}^{N_1-1} e^{j2\pi n_1 (f_1 - f_{1_{0_l}})}\right) \cdot \left(\sum_{n_2=0}^{N_2-1} e^{-j2\pi n_2 (f_2 - f_{2_{0_l}})}\right). \tag{27}$$

In fact with this form of $H(f_1, f_2)$, we have reduced the problem of direct computation of $N_1 \times N_2$ coefficients $C_{n_1 n_2}$ from a complicated system of $N_1 \times N_2$ equations to a new problem of solving only L equations because normally, we select $L \ll N_1 \times N_2$. Nevertheless, our final task will be finding $C_{n_1 n_2}$ from b_l. Now, we can obtain the relationship between b_l and $C_{n_1 n_2}$ by rearranging (27) as

$$H(f_1, f_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \left\{\sum_{l=1}^{L} b_l e^{-j2\pi n_1 f_{1_{0_l}}} e^{j2\pi n_2 f_{2_{0_l}}}\right\} \cdot e^{j2\pi n_1 f_1} e^{-j2\pi n_2 f_2}. \tag{28}$$

Comparing with (8) yields

$$C_{n_1 n_2} = \sum_{l=1}^{L} G_a^{-1}(f_{1_{0_l}}, f_{2_{0_l}}) b_l e^{-j2\pi n_1 f_{1_{0_l}}} e^{j2\pi n_2 f_{2_{0_l}}} \tag{29}$$

i.e., after calculation of $\mathbf{b}$, we can find $C_{n_1 n_2}$ from (29). The computation of $\mathbf{b}$ is not difficult from (27). With the definition of an $L \times L$ matrix $\mathbf{A}$ with the elements $\{a_{kl}\}$, $1 \leq k, l \leq L$ as

$$a_{kl} = \sum_{n_1=0}^{N_1-1} e^{j2\pi n_1 (f_{1_{0_k}} - f_{1_{0_l}})} \sum_{n_2=0}^{N_2-1} e^{-j2\pi n_2 (f_{2_{0_k}} - f_{2_{0_l}})} \tag{30}$$

if (27) is rewritten for L pairs of $(f_{1_{0_l}}, f_{2_{0_l}})$, $l = 1, 2, \ldots, L$; then, by using (25) and (26), it follows that

$$\mathbf{H_0} = \mathbf{A}\mathbf{b}. \tag{31}$$

Hence, $\mathbf{b}$ can be obtained from

$$\mathbf{b} = \mathbf{A}^{-1}\mathbf{H_0}. \tag{32}$$

It is assumed that $\mathbf{A}$ has a nonzero determinant; therefore, its inverse exists. Then, the values of $C_{n_1 n_2}$ are computed from (29), and the design is complete.

The question may arise as to how the symmetrical points $(f_{1_{0_l}}, f_{2_{0_l}})$ with the corresponding gains of $H(f_{1_{0_l}}, f_{2_{0_l}})$ should be located in the $f_1 - f_2$ plane. In the following section, we consider the design algorithm for a 4 × 4 array with $L = 4$ selected points.

V. Simulation Results

Two simulations are given to illustrate the performance of the beamformers of the first and second algorithms.

A. IDFT Beamforming Network

At the first simulation of the proposed FSWB network, we assume that a software defined radio (SDR) platform is designed to receive RF signals with frequencies 2.45 and 5.25 GHz. It

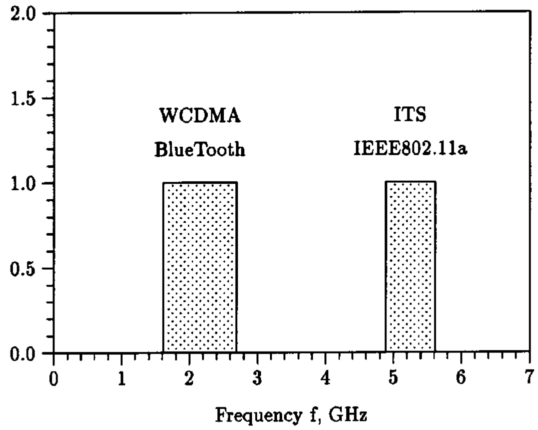

Fig. 7. Desired multibandpass behavior of the beamformer for SDR applications.

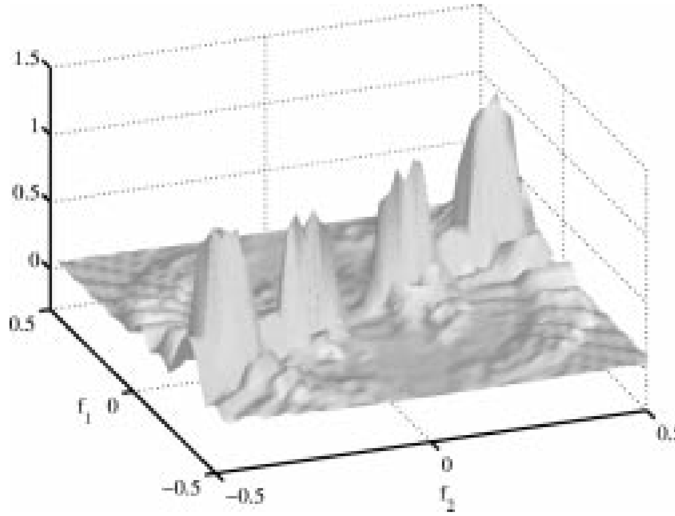

Fig. 8. Plot of $H(f_1, f_2)$ demonstrating the frequency selection and wideband properties of the proposed smart antenna.

is desired that a single antenna system can simultaneously perform beamforming and frequency selection. We assume that a microstrip rectangular array with independent gain adjustment for $N_{1_{\max}} = N_{2_{\max}} = 41$ and $N_{1_{\min}} = N_{2_{\min}} = 11$ elements is available. Beamforming is done using fully spatial analog signal processing. The desired frequency response of the beamformer is shown in Fig. 7.

Each element of the antenna array has a frequency and angle response as

$$G_a(f, \theta) = \left(-\tfrac{1}{80} f^2 + \tfrac{1}{20} f + \tfrac{19}{20}\right) \left(\tfrac{-1}{16\,200} \theta^2 + 1\right). \quad (33)$$

This approximate relation is valid for 1 GHz $< f <$ 6 GHz and $-90^\circ < \theta < 90^\circ$. The frequency f and the angle θ are measured by gigahertz and degrees, respectively. We note that (33) is a normalized equation and has a unit gain at $f = 2$ GHz and $\theta = 0^\circ$. The bandpass behavior of (33) in both frequency and angle makes it an appropriate measure of nonideal properties of each antenna element. The antenna spacing is assumed to be $d_1 = d_2 = 0.025$ m. With substitution of $c = 3 \times 10^8$ m/s for the speed of wave propagation, we will have, from (5) and (6)

$$f_1 = \tfrac{1}{12} f \sin\theta \quad (34)$$

$$f_2 = \tfrac{1}{12} f \cos\theta \quad (35)$$

where f is measured in gigahertz, and f_1 and f_2 are normalized parameters without any unit. For obtaining the desired region of the $f_1 - f_2$ plane, it is necessary to calculate (10) and (11) as

$$\frac{f_1}{f_2} = \tan\theta = \tan\phi \quad (36)$$

$$f_1^2 + f_2^2 = \left(\frac{f}{12}\right)^2. \quad (37)$$

Using these equations in (33), we will have

$$G_a(f_1, f_2) = \left(-1.8 f_1^2 - 1.8 f_2^2 + 0.6\sqrt{f_1^2 + f_2^2} + 0.95\right) \left(\frac{-1}{16\,200}\left(\tan^{-1}\frac{f_1}{f_2}\right)^2 + 1\right). \quad (38)$$

Since the maximum value of f_1 and f_2 is 0.5, the maximum operating frequency of the designed smart antenna will be 6 GHz. For each value of the beamformer angle θ_0, it is desired that the frequency response of the beamformer has bandpass properties from about 1.6 to 2.7 GHz and from about 4.9 to 5.6 GHz as well as band rejection properties between these two ranges. Hence, with the consideration of a small certainty factor, the definition of the $H(f_1, f_2)$ is done according to

$$H(f_1, f_2) = \frac{\sin\left[\alpha\pi\left(\frac{f_1}{f_2} - \tan\theta_0\right)\right]}{\alpha\pi\left(\frac{f_1}{f_2} - \tan\theta_0\right)} \cdot \begin{cases} \left[\left(-1.8 f_1^2 - 1.8 f_2^2 + 0.6\sqrt{f_1^2 + f_2^2} + 0.95\right) \left(\frac{-1}{16\,200}\left(\tan^{-1}\frac{f_1}{f_2}\right)^2 + 1\right)\right]^{-1} & \text{for } 0.38 < r < 0.49 \text{ and } 0.079 < r < 0.24 \\ \frac{1}{\sqrt{10}}, & \text{otherwise.} \end{cases} \quad (39)$$

The value of θ_0 is set to 30° for this simulation. The parameter α controls the bandwidth and is calculated to be 4.296, which corresponds to the beamwidth of 20°. Fig. 8 demonstrates the magnitude of the $H(f_1, f_2)$ according to (39). Two desired regions in lower and higher frequencies are emphasized by higher amplitudes.

For better presentation of the frequency selectivity of the beamforming network, in addition to the compensation of the imperfect antenna element specifications, the directional patterns of the system are plotted in Figs. 9–11. Fig. 9 shows the directivity of the array for frequencies from 1.61 to 2.69 GHz. Fig. 10 illustrates the patterns for the frequencies from 4.89 to 5.61 GHz. In both figures, we observe no noticeable variation in the gains of the array antenna for the specified frequency bands. In Fig. 11, the same patterns are plotted for frequencies from 3.49 to 4.21 GHz. As expected, an attenuation of 10 dB, due to the factor $\sqrt{10}$ considered in (39), is observed for all related frequencies. Therefore, the interferences at the receiver

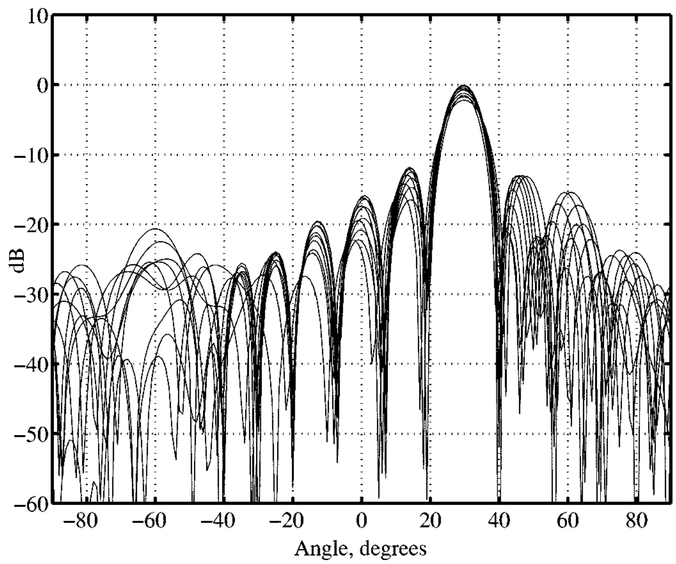

Fig. 9. Directional patterns of the beamformer plotted for frequencies from 1.61 to 2.69 GHz.

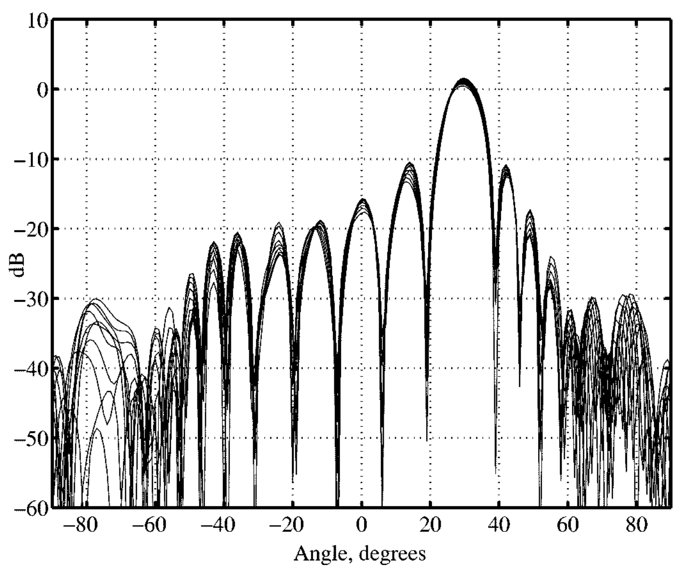

Fig. 10. Directional patterns of the beamformer plotted for frequencies from 4.89 to 5.61 GHz.

are filtered in frequency as well as angle domains. The FB of the array is at least 0.14 and 0.5 for the higher and lower regions, respectively. Hence, the overall FB is more than 64%.

B. Matrix Inversion Beamforming Network

Now, a simple and efficient 4×4 linear rectangular array is designed, and it is shown that the frequency independence of the array is quite appropriate for applications where the fractional bandwidth is supposed to be in the range of 0.2 to 0.3. The angle of the beamformer is assumed to be $\theta_0 = -40^\circ$ with the center frequency of $f_0 = 0.35c/d$, where $d = d_1 = d_2$. Because of the limitation of the number of points on the $f_1 - f_2$ plane in this example, we assume that $G_a = 1$. Initially, four pairs of critical points $(f_{1_{0_l}}, f_{2_{0_l}})$ are calculated as

$$P_1: \qquad (f_{1_{0_1}}, f_{2_{0_1}}) = (f_{1_0}, f_{2_0}) \tag{40}$$

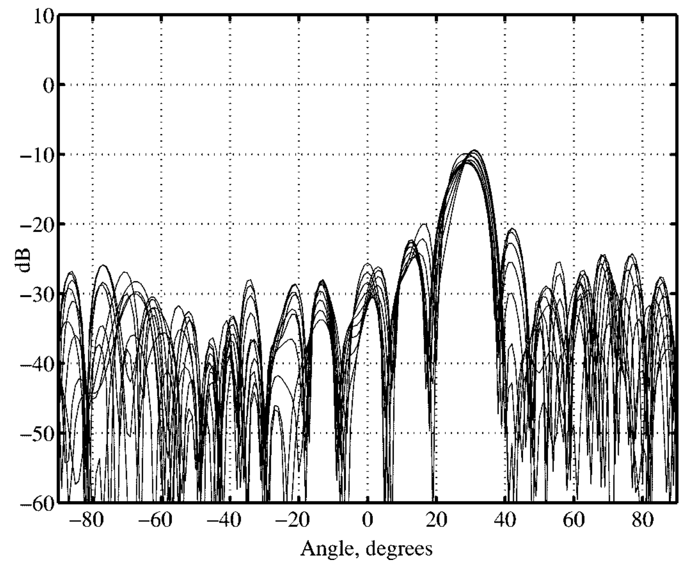

Fig. 11. Directional patterns of the beamformer plotted for frequencies from 3.49 to 4.21 GHz.

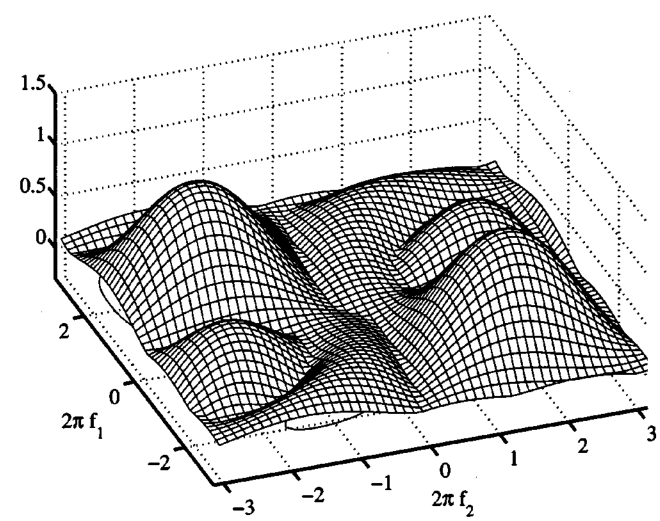

Fig. 12. Pattern of the 2-D frequency response $H(f_1, f_2)$ for the desired 4×4 rectangular array.

$$P_2: \qquad (f_{1_{0_2}}, f_{2_{0_2}}) = (-f_{1_0}, -f_{2_0}) \tag{41}$$

$$P_3: \qquad (f_{1_{0_3}}, f_{2_{0_3}}) = (-f_{2_0}, f_{1_0}) \tag{42}$$

$$P_4: \qquad (f_{1_{0_4}}, f_{2_{0_4}}) = (f_{2_0}, -f_{1_0}) \tag{43}$$

where f_{1_0} and f_{2_0} have been found from (5) and (6), respectively. Then, we form the vector $\mathbf{H}_0$ as

$$\mathbf{H}_0 = [1, 1, 0, 0]^T. \tag{44}$$

Next, $\mathbf{A}$ is constructed using (30), and $\mathbf{b}$ is calculated from (32). Finally, $C_{n_1 n_2}$, for $1 \leq n_1, n_2 \leq 4$, is computed from (29). Due to the symmetry of the selected points in the $f_1 - f_2$ plane, the values of $C_{n_1 n_2}$ are all real. This simplifies the computation in practical situations. The plot of $H(f_1, f_2)$ from (8), which is illustrated in Fig. 12, shows the actual 2-D frequency response obtained in our design algorithm. Clearly, there are two peak points at P_1 and P_2 and two zeros at P_3 and P_4. The important result of this pattern is that in a relatively large neighborhood

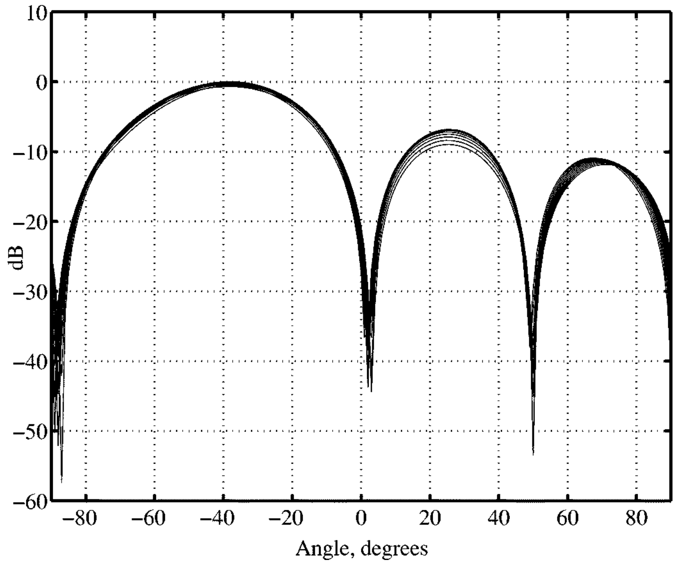

Fig. 13. Directional patterns of the 4 × 4 rectangular array for ten equally spaced frequencies between $f_l = 0.3c/d$ and $fh = 0.4c/d$.

of the point corresponding to $f = f_0$, we observe an almost constant amplitude. This observation for a 4 × 4 rectangular array makes it a wideband array when it is designed for the center frequency of the frequency band. Fig. 13 demonstrates this fact more clearly. According to this figure, the frequency response for mainlobe and sidelobe invariance of the patterns is from $f_l = 0.3c/d$ to $f_h = 0.4c/d$, i.e., a fractional bandwidth of 28.5%. Assuming a system with the carrier frequency of about 2.1 GHz, i.e., $f_0 = 2.1$ GHz, we will have

$$d = 0.35\frac{c}{f_0} = 0.05\ \mathrm{m} \tag{45}$$

and the higher and lower frequencies will be $f_h = 2.4$ and $f_l = 1.8$ GHz, respectively.

VI. Conclusions

In this paper, the following ideas regarding wideband frequency selective rectangular arrays were investigated.

- A fully spatial signal processing method for RF beamforming in smart antenna arrays was proposed. No time processing was required for beam formation, and no complex multipliers were required for phase shifting of signals.
- Filtering of the incoming wave was done in frequency and space domains without adding extra filters after beamforming.
- The wideband property of the antenna array was kept in spite of the frequency selection, and high values of FB could be obtained by different methods.
- The imperfect behavior of the elements, regarding frequency and angle, can be compensated for in the design as well. This can be extremely helpful in practical design conditions. Furthermore, reception of a wide range of frequencies is done with a single antenna spacing that is calculated for the highest frequency component of the input signal.
- Beamwidth and the number of elements selected in each direction of the array are calculated according to the analytical results. For the IDFT method, the sharp endfire beams are created with the desired beamwidth.
- As an application of the proposed smart antenna, it was simulated for a SDR platform operating frequency range, and the result was quite satisfactory because we can obtain at least a 15-dB attenuation for the interferences of the same frequency and about the same amount of suppression for the interferers that come in with the same DOA but different frequencies.
- It is predicted that using microstrip antenna array technology and analog amplifiers, this type of smart antenna can be very helpful and cost effective in future versions of an SDR platform.

Between two discussed algorithms for computation of the real weights, the first method using IDFT gave more a controlled beam pattern compared with the second method using matrix inversion. Instead, the second method, although it is for larger beamwidth, required a lower number of antenna elements.

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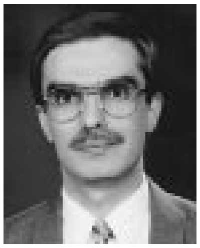

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