Sensitivity to Doppler Shift and Carrier Frequency Errors in OFDM Systems -- The Consequences and Solutions

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***Abstract** : Due to the intercarrier interference (ICI) in Orthogonal Frequency Division Multiplexing (OFDM) mobile radio communication systems, the Bit Error Rate (BER) of the received signals are extremely sensitive to Doppler frequency shifts and carrier synchronization errors. This paper studies the mechanism of the ICI signals, and then proposes a new method to overcome such problems without any equalization procedures. The results show that by using the self ICI cancellation scheme proposed here, the ICI can be reduced significantly. Therefore this OFDM system suffers much less from performance degradation caused by frequency errors than the normal OFDM systems.*

I. INTRODUCTION

Along with the increase of bit rates in digital mobile radio communication systems, the intersymbol interference (ISI) and channel deep fading become big problems in conventional single carrier systems. A proposal to use Orthogonal Frequency Division Multiplexing (OFDM) in radio mobile environment has been analyzed[1]. Since a long symbol interval is used, the system has strong ability to combat channel deep fading, and the ISI can be mitigated by means of guard intervals.

In an OFDM system, the whole available bandwidth is divided into N small parts, and a block of N data symbols are modulated on N corresponding subcarriers which are orthogonal to each other. The spectra of the subcarriers are overlapping, therefore precise frequency recovering is needed. However, in the mobile radio environment, the relative movement between transmitter and receiver causes Doppler frequency shifts, in addition, the carriers can never be perfectly synchronized. These random frequency errors in OFDM system distort orthogonality between subcarriers, as a result, intercarrier interference (ICI) occurs. Literature show that in such systems, the bit error rate (BER) increases rapidly with increasing frequency offsets [2,3,5]. This is a main restriction in OFDM systems.

The main intention of this paper is to present a solution to this problem. To explore the ICI mechanism in OFDM systems, a new concept of ***subcarrier frequency offset response*** **(SFO response)** is established. By using this basic concept, the theoretical expression of the Carrier to Interference Ratio (CIR) is given. Finally, the ***self ICI cancellation scheme*** is introduced to compress ICI influence. Simulations have been done to show the performance improvement when using the self ICI cancellation scheme.

II. PERFORMANCE ANALYSIS OF ICI

A. Impulse response of mobile radio channels

In mobile radio environment, the time variant impulse response model of the multipath channel is defined as[4]

$$h(t)=\sum_{i=0}^{M-1} h_i e^{j(2\pi f_{Di}(t)t+\theta_i)}\delta(t-\tau_i), \qquad (1)$$

where,
M = the total number of propagation paths;
$f_{Di}(t)$ = the Doppler frequency at time t of ith path;
θ_i = the initial angle of the ith path, here it is assumed to be 0 without loosing generality.
τ_i = the delay time of the ith path.

Normally, the changes of the $f_{Di}(t)$ are not very fast, therefore a constant value f_{Di} is assumed within each data block in OFDM systems. Furthermore, the f_{Di} is normalized by the subcarrier frequency separation Δf to define a new parameter ε_i:

$$\varepsilon_i=\frac{f_{Di}}{\Delta f}. \qquad (2)$$

The ε_i is called the normalized frequency offset of ith path. It is a more efficient parameter when analyzing frequency offset impact in OFDM systems.

Using discrete time domain index n instead of t, the channel impulse response model for one data block is expressed as:

$$h(n)=\sum_{i=0}^{M-1} h_i e^{j\frac{2\pi}{N}\varepsilon_i(n-n_i)}, \qquad (3)$$

where
N = the number of subcarriers;
n_i = the delay chip number of the ith path.

For each path, the amplitude of h_i is Rayleigh distributed.

B. Subcarrier frequency offset response (SFO response)

Since the spectral behavior of OFDM is more important than that the temporal behavior of the time domain, the analysis of the frequency domain representation of the channel response is preferred. Referring to a conventional single carrier communication system, a time domain impulse response (or transfer function) is used to express the channel distortion. In OFDM, a basic response function is needed to describe the spectrum distortion due to frequency offsets. It can be obtained by using one out of *N* subcarriers to transmit an unit signal into the channel. Then the frequency domain response through all of the subcarriers is defined as this function.

Assume a block of data takes value

$$X(k)=\delta(k)=\begin{cases}1, & k=0;\\ 0, & k\neq 0;\end{cases} \qquad (4)$$

where *k* represents the subcarrier *k*. When *X(k)* is linearly modulated on *N* subcarriers, only the *0*th subcarrier is used and the rest of the subcarriers are *0*. Then this block is transmitted over a time variant multipath propagation channel. Consider one typical path with amplitude of h_i, delay chip number n_i and the normalized frequency offset ε_i, the channel impulse response of the *i*th path can be expressed by

$$h_i(n)=h_i e^{j\frac{2\pi}{N}\varepsilon_i(n-n_i)}. \qquad (5)$$

Therefore, the corresponding frequency domain response can be obtained by FFT, which gives[5]

$$\begin{aligned}H_i(k)&=\frac{1}{N}\sum_{n=0}^{N-1}h_i e^{j\frac{2\pi}{N}\varepsilon_i(n-n_i)}e^{-j\frac{2\pi}{N}nk}\\ &=h_i e^{-j\frac{2\pi}{N}\varepsilon_i n_i}\left(\frac{\sin(\pi(k-\varepsilon_i))}{N\sin\left(\frac{\pi}{N}(k-\varepsilon_i)\right)}\right)e^{-j\left(1-\frac{1}{N}\right)\pi(k-\varepsilon_i)}.\end{aligned} \qquad (6)$$

Equation (6) is a function of n_i, ε_i and k for a fixed *N*. The first exponential component $e^{-j\frac{2\pi}{N}\varepsilon_i n_i}$ is independent of *k*, which means that the received signals on any subcarriers are rotated by the same phase angle of $-\frac{2\pi}{N}\varepsilon_i n_i$. To give a clear idea of Eq.(6), the n_i will be assumed to be *0* in the following steps.

In the case of $\varepsilon_i=0$, Eq. (6) reduces to

$$H_i(k)=h_i\delta(k). \qquad (7)$$

The $H_i(k)$ appears as an ideal channel response function. Thus there is no intercarrier interferences. Notice that the Eq. (7) is derived by using the fact $\sin\left(\frac{\pi\varepsilon_i}{N}\right)\approx\frac{\pi\varepsilon_i}{N}$ for the values of $N>>\pi\varepsilon_i$.

To examine the case when frequency offset $\varepsilon_i\neq 0$, change the form of Eq. (6) into (assume $n_i=0$)[5]

$$H_i(k)=h_i\frac{\sin(-\pi\varepsilon_i)}{Ne^{-j\pi\varepsilon_i}}\left(ctg\left(\frac{\pi}{N}(k-\varepsilon_i)\right)+j\right); \qquad (8)$$

where $j=\sqrt{-1}$. The $H_i(k)$ values for $k\neq 0$ are not *0* anymore, it becomes a function of frequency offset ε_i, and reduces as a cotangent function when k increases. If the transmitted data block is $X(k)=\delta(k-k_0)$, where $0\le k_0<N$, the k in the Eq.(8) will be replaced by $k-k_0$. The example 1 will give more detail explanation about it.

Example 1.

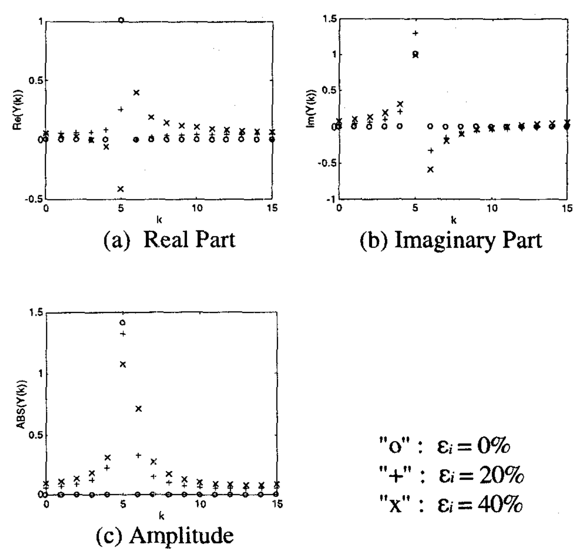
Consider a 4PSK modulation OFDM system. Let $k_0=5$ and assume the transmitted signal is

$$X(k)=(1+j)\delta(k-5); \qquad (9)$$

Then the received frequency domain signal $Y_i(k)$ (over path *i*) is

$$\begin{aligned}Y_i(k)&=X(k)H_i(k)\\ &=(1+j)h_i\frac{\sin(-\pi\varepsilon_i)}{Ne^{-j\pi\varepsilon_i}}\left(ctg\left(\frac{\pi}{N}(k-5-\varepsilon_i)\right)+j\right).\end{aligned} \qquad (10)$$

Usually *N* is rather large. However to graphically show the results, here is assumed *N=16* and $h_i=1$. Fig.1 shows the real part, imaginary part and the amplitude of $Y_i(k)$ for $\varepsilon_i=0$, $\varepsilon_i=0.2$ and $\varepsilon_i=0.4$ respectively.

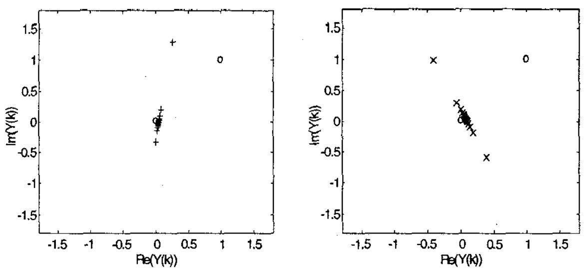


(a) Real Part (b) Imaginary Part (c) Amplitude

Fig.1 The frequency domain response of path i.

Due to the frequency offset, the signal energy in the k_0th subcarrier is reduced. The energy lost is transferred to the other subcarriers within the same DFT block (see Fig.1 (c)). The energy gain by the kth subcarrier (for $k \neq k_0$) is the ICI energy of the single path i. For multipath radio channels, the ICI signal is the sum of the ICI signals caused by each path. This is the mechanism explanation of ICI signals.

The ICI energy distorts the signal constellation, therefore the receiver detection error rate increases. Fig.2 shows the received signal constellation given by Eq. (10). In the ideal case, the received signals should be just 2 points in both figures (small circles): *(1,1)*,the response of the subcarrier 5, and *(0,0)*,the response of the rest of subcarriers. In this case, the ICI values are *0*. When $\varepsilon_i \neq 0$, the phase rotation and signal dispersion around point *(0,0)* can be clearly seen (the "+" and "x" marks).



(a). "o" : $\varepsilon_i = 0\%$; "+": $\varepsilon_i = 20\%$

(b). "o" : $\varepsilon_i = 0\%$ "x" : $\varepsilon_i = 40\%$

Fig.2 Received signal constellation of path i.

If all paths are considered, the channel frequency domain response *H(k)* becomes

$$H(k) = \sum_{i=0}^{M-1} h_i e^{-j\frac{2\pi}{N}\varepsilon_i n_i} \frac{\sin(-\pi\varepsilon_i)}{Ne^{-j\pi\varepsilon_i}} \left(ctg\left(\frac{\pi}{N}(k-\varepsilon_i)\right) + j \right), \quad (11)$$

and the ICI signals appear in more complicated forms.

Equation (11) is called ***subcarrier frequency offset response*** (**SFO response**). This function expresses the ICI property with respect to the system frequency offset in the time variant multipath radio channel. It is a basic function for analyzing frequency domain performance of OFDM systems.

For a signal modulated on subcarrier k_0, the shifted SFO response is

$$H(k) = \sum_{i=0}^{M-1} h_i e^{-j\frac{2\pi}{N}\varepsilon_i n_i} \frac{\sin(-\pi\varepsilon_i)}{Ne^{-j\pi\varepsilon_i}} \left(ctg\left(\frac{\pi}{N}(k-k_0-\varepsilon_i)\right) + j \right). \quad (12)$$

C. Analysis of ICI signals in OFDM system

For the MPSK modulations, the phase rotation of the received signals (see Fig.2) can be simply removed by using differential coding. The signal dispersion around *(0,0)* is know as intercarrier interference. Whenever a frequency offset exists, the signal transmission on one subcarrier will produce ICI signals to the rest of the subcarriers. In practice, when all subcarriers are concerned, the interference signal in each individual subcarrier is the sum of the interference signals caused by the rest of subcarriers within the same DFT block. Thus for the k_0th subcarrier, the ICI signal is

$$I(k_0) = \sum_{\substack{k=0 \\ k \neq k_0}}^{N-1} X(k)H(k) \quad (13)$$

where *H(k)* is expressed by Eq. (12). Theoretical analysis[5] shows that for a MPSK modulation OFDM mobile radio communication system, the Carrier to ICI ratio is

$$CIR = \frac{1}{2\varepsilon_B^2}, \quad (14)$$

where ε_B is the normalized Doppler bandwidth which is

$$\varepsilon_B = \frac{f_c V}{c\Delta f}, \quad (15)$$

where,

f_c = carrier frequency;

V = speed of the vehicle;

c = speed of light in the air.

To keep CIR value larger than 20dB, the ε_B has to be less than 7%. This restriction limits the application of OFDM mobile communication system, since Doppler shift and carrier synchronization error always need to be considered.

In the next section, a new method is proposed to prevent frequency offset influence by using the property of the SFO response.

III. SELF ICI CANCELLATION SCHEME

The SFO response for each individual subcarrier is already shown by Eq.(11) and Eq.(12). Now let's consider a pair of adjacent subcarriers k_0 and k_0+1 and consider another example.

Example 2.

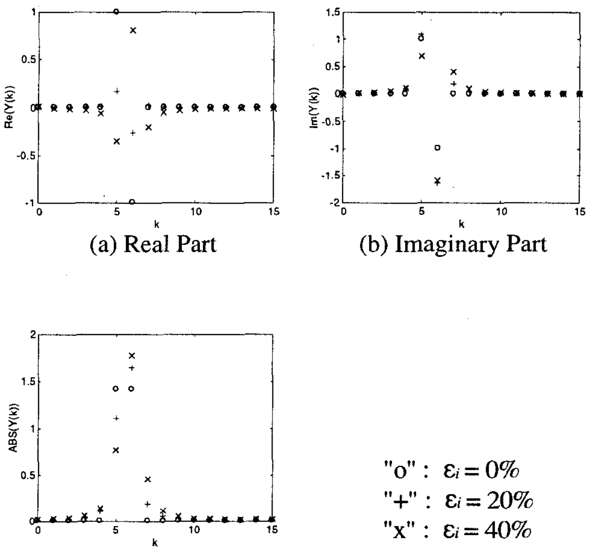
In the special case where $X(k_0) = a + jb$ and $X(k_0+1) = -(a+jb)$ respectively (i.e., *(a+jb)=1+j*), the received signal over the ith path will be

$$Y_i(k_0) + Y_i(k_0+1) = (1+j)H_i(k_0, k_0+1), \quad (16)$$

and,

$$H_i(k_0,k_0+1) = H_i(k_0) - H_i(k_0+1)$$
$$= h_i \frac{\sin(-\pi\varepsilon_i)}{Ne^{-j\pi\varepsilon_i}} \left(ctg\left(\frac{\pi}{N}(k-k_0-\varepsilon_i)\right) - ctg\left(\frac{\pi}{N}(k-k_0-1-\varepsilon_i)\right) \right). \quad (17)$$

The received signal points of the ith path for k_0=5 is plotted in Figure 3. The other parameters are the same as in example 1. Compare this results with the Fig.1, it can be seen that the signals for $k_0 \neq 5$ and $k_0 \neq 6$ in the Fig.3 is much smaller than the corresponding signals in the Fig.1. It means that the ICI signals caused by this frequency pair is much less than that caused by each individual one.

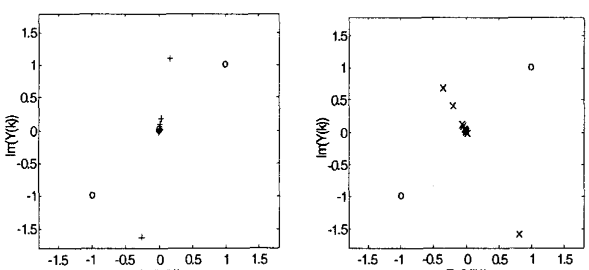


(a) Real Part (b) Imaginary Part

(c) Amplitude

Fig.3 Frequency domain response of the frequency pair.

The signal constellations of the frequency pair is shown in Fig.4. Obviously, when $\varepsilon_i = 0$, the received signals have 3 point (small circles): *(1,j)* for *k=5*, *(-1,-j)* for *k=6* and *(0,0)* for the others. Compare Fig.4 with Fig.2, it shows that the dispersion around the *(0,0)* in the Fig.4 is much smaller than that in the Fig.2. This is because the ICI value caused by subcarrier k_0 and k_0+1 has canceled out each others.



(a). "o" : $\varepsilon_i = 0\%$; "+": $\varepsilon_i = 20\%$

(b). "o" : $\varepsilon_i = 0\%$ "x" : $\varepsilon_i = 40\%$

Fig.4 Received signal constellation of the frequency pair

When the cancellation effect is considered, if the number of the subcarriers is *N*, the data block length is *N/2*, and each signal is modulated on one pair of adjacent subcarriers by using the original signal and signal multiplied by *-1* respectively, the ICI value within the data block can be reduced significantly. This is called ***self ICI cancellation scheme***.

In the multipath propagation environment, the Eq.(17) will become a sum through the M paths. The carrier frequency errors can be seen as a special case of system frequency offset.

By using this scheme, the CIR value of OFDM system can be derived as[5]

$$CIR = \frac{1}{0.2\varepsilon_B^2}. \tag{18}$$

The *CIR* has increased 10dB compared with the result from Eq.(14). To keep *CIR* larger than 20dB, the allowed ε_B can increase to 22%.

IV. SIMULATIONS

The simulations of an OFDM system have been done using a 2.16 MHz bandwidth and a carrier frequency of 2 GHz. In the simulations, the length of the DFT block is 1024, therefore, the subcarrier separation is about 2.1 kHz. The radio channel is a standard 6 path Rayleigh fading channel with a maximum delay time of 5μs (urban areas).

There are 3 different OFDM systems will be compared here, which are:

System 1 (S1): Normal BPSK subcarrier modulation system;

System 2 (S2): Normal 4PSK subcarrier modulation system;

System 3 (S3): 4PSK subcarrier modulation with self ICI cancellation.

The S3 is proposed system in this paper, its simulation block diagram is shown in Fig.5. The purpose of the "ICI cancellation coding" block is that modulate one 4PSK signal on a pair of adjacent subcarriers using original signal and the signal multiplied by "-1". It can also be seen as a type of repeat coding, therefore, the group decoding is used.

The S1 and S2 are normal OFDM systems, which have no special procedure to against frequency offset. Their block diagrams can be got by removing "ICI cancellation coding" and "ICI cancellation decoding" blocks from Fig.5.

To combat the phase rotation of the received signals, frequency domain differential coding and decoding are used in all the systems.

A common value of E_b/N_0 (the signal energy per bit-to-noise power density ratio) has been used to examine BER performance, thus the signal energy in S3 is half of which in S2 since two subcarriers are used to transmit one signal.

The simulation results are shown in Fig.6. For the walking speed (V = 3km/h, f_D=6.5Hz, ε_B=3%), the BER performances of S2 and S3 are nearly the same. For a certain BER value, the needed E_b/N_0 in S1 is 1.5dB lower than that in both S2 and S3. However, when the vehicle speed is high (V = 195km/h, f_D=421Hz, ε_B=20%), the S3 (dashed line) shows much better BER performance than that of S1 and S2 (solid line and dash-dotted line).

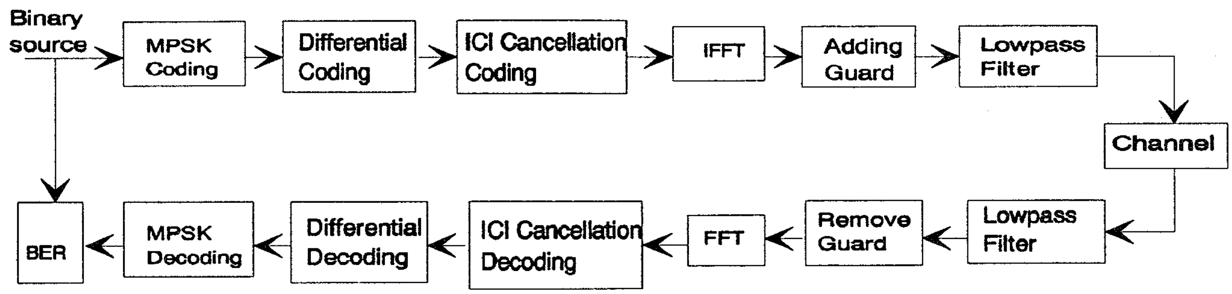


Fig. 5 Simulation Block diagram of OFDM system

It is more fair to compare two systems under the same conditions. In simulations, the spectrum efficiency and the E_b/N_0 are same for S1 and S3. Fig.6 shows that for the bigger E_b/N_0 values, the S3 have stronger ability to against system frequency shift than S1 does. When E_b/N_0 is small, since the BER performance are dominated by additive white Gaussian noise, the ICI cancellation results can't clearly be seen.

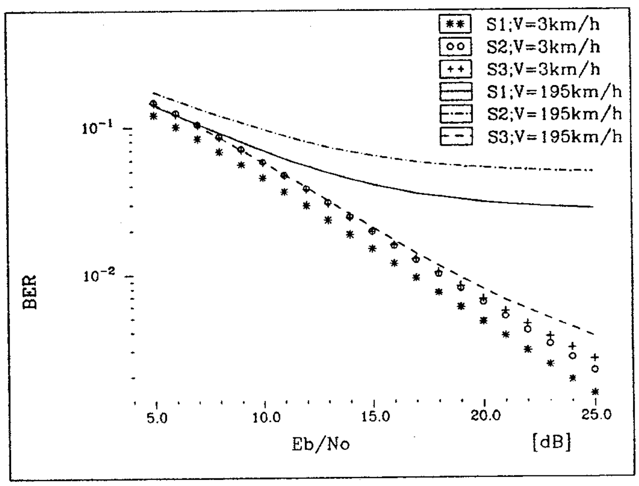


Fig.6 Comparison of the BER performance

When E_b/N_0 = 20dB for the all systems, Fig.7 shows that the BER value of the proposed system (S3, "+" mark) has slight increase when ε_B becomes large.

V. CONCLUSIONS

By analyzing the new concept of SFO response, a self ICI cancellation scheme has been proposed in this paper. The CIR value can be improved for 10 dB by using this strategy, therefore the BER of OFDM communication systems can be relatively insensitive to frequency errors. Since no equalization procedure is needed, the system can be significantly simplified.

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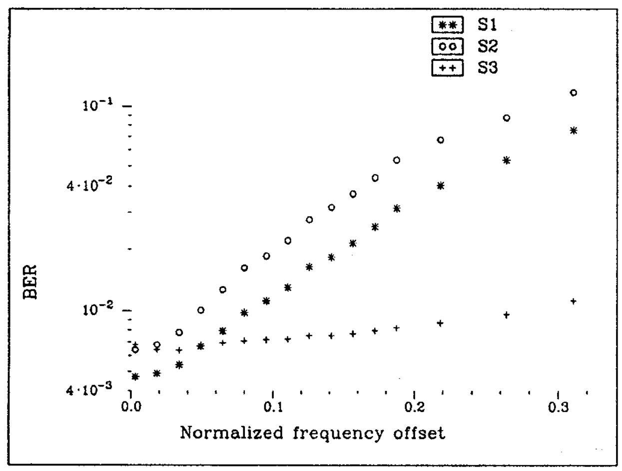

Fig.7 BER versus ε_B

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