Reduced ICI in OFDM Systems Using the "Better Than" Raised-Cosine Pulse

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***Abstract*—A recently found "better than" raised-cosine pulse is employed in pulse-shaping orthogonal frequency-division multiplexing to reduce the inter-carrier interference due to frequency offset. The results show that new pulse outperforms the rectangular pulse and raised-cosine pulse in average inter-carrier interference power reduction.**

***Index Terms*—Inter-carrier interference (ICI), orthogonal frequency-division multiplexing (OFDM), pulse-shaping.**

I. Introduction

ORTHOGONAL frequency-division multiplexing(OFDM) is being widely used in wireless communications standards, such as IEEE 802.11, the multimedia mobile access communication (MMAC), and the HIPERLAN/2 because of its ability to effectively convert a frequency-selective fading channel into several nearly flat-fading channels. On the other hand, OFDM is sensitive to frequency offset which leads to inter-carrier interference (ICI), and hence performance degradation. Reference [1] discussed this kind of performance degradation, and [2] derived a maximum likelihood estimator for frequency offset. Recently, [3] examined the use of pulse-shaping to reduce the sensitivity of OFDM systems to frequency offset. In this regard, a novel pulse shape that is better in both intersymbol interference environments [4], [5] and co-channel interference environments [6] than the raised-cosine pulse has recently been reported. In this letter, we examine the employment of the novel pulse in OFDM systems. The results indicate that the new pulse outperforms rectangular and raised-cosine pulses in the reduction of average ICI power.

II. System Model

The complex envelope of one radio frequency (RF) N-subcarrier OFDM block with pulse-shaping is expressed as [1]

$$x(t) = e^{j2\pi f_c t} \sum_{k=0}^{N-1} a_k p(t) e^{j2\pi f_k t} \tag{1}$$

where $j = \sqrt{-1}$, f_c is the carrier frequency, f_k is the subcarrier frequency of the kth subcarrier, $p(t)$ is the time-limited pulse-shaping function, and a_k, where $k = 0, 1, \ldots, N-1$ is the data symbol transmitted on the kth subcarrier. We assume that a_k has mean zero and normalized average symbol energy. We further assume that the data symbols are uncorrelated. That is

$$E[a_k a_m^*] = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases} \tag{2}$$

where a_m^* denotes the complex conjugate of a_m. One also has that

$$f_k - f_m = \frac{k-m}{T} \tag{3}$$

to ensure subcarrier orthogonality [7]; that is,

$$\int_{-\infty}^{+\infty} p(t) e^{j2\pi(f_k - f_m)t}\, dt = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases} \tag{4}$$

where $1/T$ is the minimum subcarrier frequency spacing required. Equation (4) also indicates the important condition that the Fourier transform of the pulse $p(t)$ should have spectral nulls at the frequencies $\pm 1/T, \pm 2/T, \ldots$ to ensure subcarrier orthogonality.

We consider here three time-limited pulses. Let $p_r(t)$, $p_{rc}(t)$ and $p_{btrc}(t)$ denote the rectangular pulse, the raised-cosine pulse (in the time domain), and the "better than" raised-cosine (BTRC) pulse (in the time domain) defined as

$$p_r(t) = \begin{cases} \frac{1}{T}, & -\frac{T}{2} \leq |t| \leq \frac{T}{2} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$p_{rc}(t) = \begin{cases} \frac{1}{T}, & 0 \leq |t| \leq \frac{T(1-\alpha)}{2} \\ \frac{1}{2T}\left\{1 + \cos\left[\frac{\pi}{\alpha T}\left(|t| - \frac{T(1-\alpha)}{2}\right)\right]\right\}, & \frac{T(1-\alpha)}{2} \leq |t| \leq \frac{T(1+\alpha)}{2} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

and

$$p_{btrc}(t) = \begin{cases} \frac{1}{T}, & 0 \leq |t| \leq \frac{T(1-\alpha)}{2} \\ \frac{1}{T} e^{(-2ln2/\alpha T)[|t| - (T(1-\alpha)/2)]}, & \frac{T(1-\alpha)}{2} \leq |t| \leq \frac{T}{2} \\ \frac{1}{T}\left\{1 - e^{(-2ln2/\alpha T)[(T(1+\alpha)/2) - |t|]}\right\}, & \frac{T}{2} \leq |t| \leq \frac{T(1+\alpha)}{2} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

with Fourier transforms $P_r(f)$, $P_{rc}(f)$, and $P_{btrc}(f)$, respectively, where α is the roll-off factor, and $0 \leq \alpha \leq 1$. When $\alpha = 0$, both the raised-cosine and the BTRC pulse coalesce into the rectangular pulse. Fig. 1 shows the frequency functions of these three pulses for $\alpha = 0.2$ and $\alpha = 1.0$.

III. ICI Analysis

Frequency offset, $\Delta f (\Delta f \geq 0)$, and phase error, θ, are introduced during transmission because of channel distortion or

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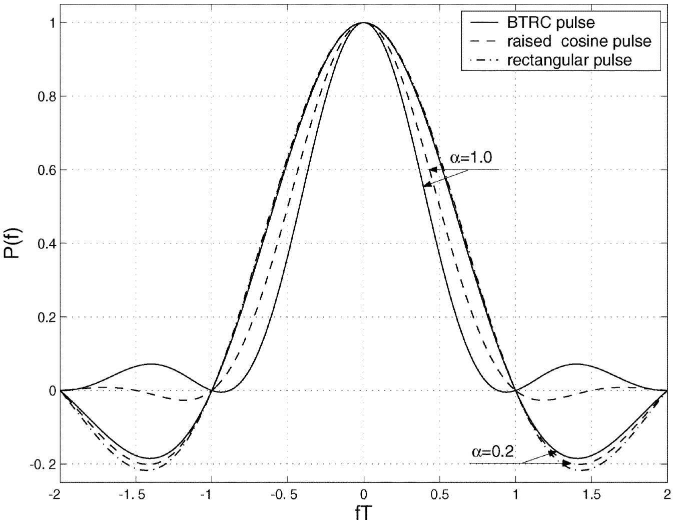

Fig. 1. Frequency functions of rectangular pulse, BTRC pulse, and raised-cosine pulse for roll-off factors $\alpha = 0.2$ and $\alpha = 1.0$.

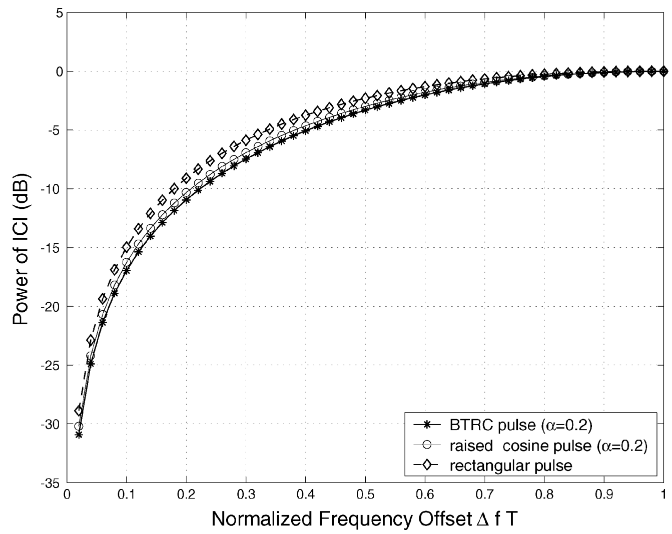

Fig. 2. Comparison of ICI power for different pulse-shaping functions in a 64-subcarrier OFDM system.

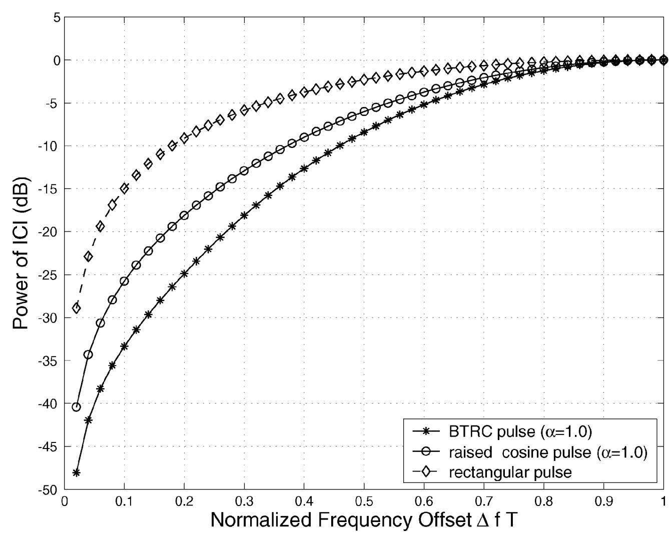

Fig. 3. The ICI power for different pulse shaping functions in a 64-subcarrier OFDM system.

receiver crystal oscillator inaccuracy. The received signal after multiplication by $e^{j[2\pi(-f_c+\Delta f)t+\theta]}$ becomes

$$r(t) = e^{j(2\pi\Delta f t+\theta)} \sum_{k=0}^{N-1} a_k p(t) e^{j2\pi f_k t}. \quad (8)$$

The mth subchannel correlation demodulator, thus, gives the decision variable for transmitted symbol a_m

$$\begin{aligned}\hat{a}_m &= \int_{-\infty}^{+\infty} r(t) e^{-j2\pi f_m t} dt \\ &= a_m e^{j\theta} \int_{-\infty}^{+\infty} p(t) e^{j2\pi\Delta f t} dt \\ &\quad + e^{j\theta} \sum_{\substack{k\neq m \\ k=0}}^{N-1} a_k \int_{-\infty}^{+\infty} p(t) e^{j2\pi(f_k - f_m + \Delta f)t} dt \quad (9)\end{aligned}$$

where the first term in (9) contains the desired signal component, and the second term is the ICI. Combining (3) with (9) gives

$$\hat{a}_m = a_m e^{j\theta} P(-\Delta f) + e^{j\theta} \sum_{\substack{k\neq m \\ k=0}}^{N-1} a_k P\left(\frac{m-k}{T} - \Delta f\right) \quad (10)$$

where $P(f)$ is the Fourier transform of $p(t)$. Hence, the power of the desired signal is

$$\sigma_m = |a_m|^2 |P(\Delta f)|^2 \quad (11)$$

and the ICI power is

$$\sigma_{ICI}^m = \sum_{\substack{k\neq m \\ k=0}}^{N-1} \sum_{\substack{n\neq m \\ n=0}}^{N-1} a_k a_n^* P\left(\frac{k-m}{T} + \Delta f\right) P\left(\frac{n-m}{T} + \Delta f\right). \quad (12)$$

The ICI power depends not only on the desired symbol location, m, and the transmitted symbol sequence, but also on the pulse-shaping function and the number of subcarriers. However, combining (2) with (12) gives the average ICI power, averaged across different sequences as

$$\overline{\sigma_{ICI}^m} = \sum_{\substack{k\neq m \\ k=0}}^{N-1} \left| P\left(\frac{k-m}{T} + \Delta f\right) \right|^2. \quad (13)$$

One sees that the average ICI power for the mth symbol depends on the number of subcarriers and on the spectral magnitudes of the pulse-shaping function at the frequencies $(((k-m)/T) + \Delta f)$, $k \neq m$, $k = 0, 1, \ldots, N-1$. By design, the spectra of the pulses have nulls at the frequency points $((m-k)/T)(m \neq k)$, and hence no ICI occurs when $\Delta f = 0$. For the rectangular pulse, one has (Fig. 1)

$$\left| P_r\left(\frac{k-m}{T} + \Delta f\right) \right| \geq \left| P_{rc}\left(\frac{k-m}{T} + \Delta f\right) \right| \quad (14)$$

$$\left| P_r\left(\frac{k-m}{T} + \Delta f\right) \right| \geq \left| P_{btrc}\left(\frac{k-m}{T} + \Delta f\right) \right|. \quad (15)$$

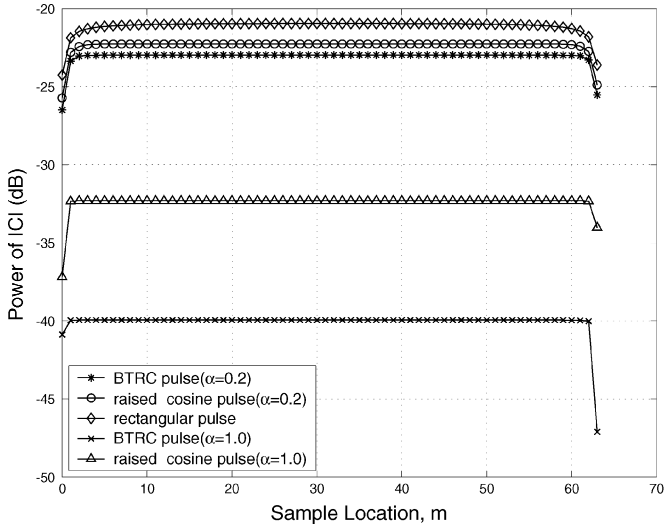

Fig. 4. The ICI power for different sample locations in a 64-subcarrier OFDM system when $\Delta f T = 0.05$.

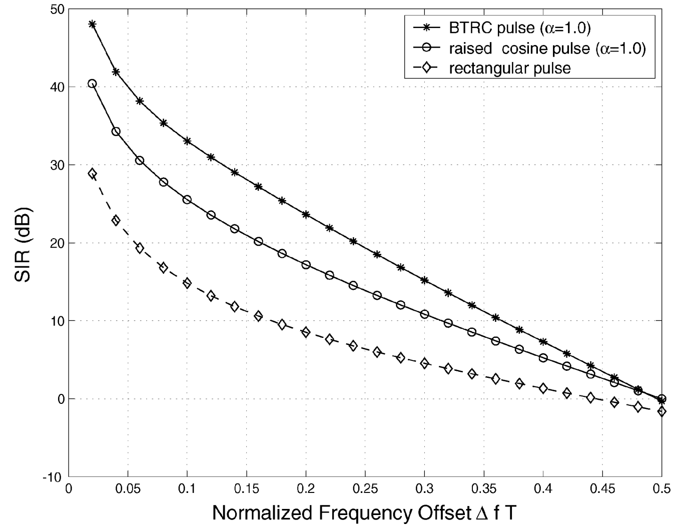

Fig. 5. The asSIR for different pulse-shaping functions in a 64-subcarrier OFDM system.

Therefore, a rectangular pulse-shaped OFDM system always has greater ICI power than a raised-cosine pulse-shaped OFDM system and a BTRC pulse-shaped OFDM system.

Similarly, if the spectrum of one pulse-shaping function has smaller side-lobes than another, then it is expected this pulse-shaping function will lead to less ICI. Fig. 2 compares the ICI power when $\alpha = 0.2$ for the rectangular pulse, raised-cosine pulse and BTRC pulse. Fig. 3 shows a similar comparison for the case of $\alpha = 1.0$. Comparing Figs. 2 and 3, it can be seen that increasing α leads to large ICI power reduction. This is expected since increasing α corresponds to reducing the sidelobes in the spectrum.

Noteworthy, for the same value of α, the BTRC pulse outperforms the raised-cosine pulse. When the normalized frequency offset is 0.05, the BTRC pulse achieves 7.64 dB and 0.70 dB smaller ICI power than the raised-cosine pulse for $\alpha = 1.0$ and $\alpha = 0.20$, respectively. This interesting behavior occurs despite the fact that the tails of $P_{btrc}(f)$ and $P_{rc}(f)$ decay as f^{-2} and f^{-3}, respectively. The BTRC pulse still outperforms the raised-cosine pulse in terms of ICI power reduction because the sum (13) is completely dominated by the closest two terms.

Fig. 4 shows the variation of the ICI power as a function of the sample location, m when $\alpha = 0.2$ and $\alpha = 1.0$. As expected, the ICI power drops for samples located near sample locations 0 and $N-1$, because these samples have fewer interfering samples. When $\alpha = 1.0$, the ICI power is dominated by the nearest two samples, one either side, and drops noticeably only for sample locations 0 and $N-1$, which have only one nearest sample. The superiority of the BTRC pulse is evident.

One can also consider the comparative performances of the different pulses in terms of the average signal power to average ICI power ratio [1], denoted *SIR*. Averaging (11) over all possible transmitted symbols and combining with (13) yields

$$\mathrm{SIR} = \frac{|P(\Delta f)|^2}{\sum\limits_{\substack{k \neq m \\ k=0}}^{N-1} \left| P\left(\frac{k-m}{T} + \Delta f\right) \right|^2}. \tag{16}$$

Fig. 5 compares the SIR for different pulse-shaping functions in a 64-subcarrier OFDM system plotted as functions of the normalized frequency offset, $\Delta f T$. Observe that the BTRC pulse outperforms the raised-cosine pulse. For example, assume that it is desired to maintain a minimum *SIR* of 25 dB. When employing the raised-cosine pulse, the normalized frequency offset must be less than 0.1052. In contrast, the tolerable normalized frequency offset may be as large as 0.1844 when one uses the BTRC pulse.

IV. Conclusion

The employment of the "better than" raised-cosine pulse rather than the raised-cosine pulse gives a substantial improvement in the reduction of ICI caused by frequency offset in an OFDM system, as measured by the average ICI power.

References

[1] T. Pollet, M. V. Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and wiener phase noise," *IEEE Trans. Commun.*, vol. 43, pp. 191–193, Feb./Mar./Apr. 1995.

[2] P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Trans. Commun.*, vol. 42, pp. 2908–2914, Oct. 1994.

[3] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, pp. 365–369, Mar. 1999.

[4] N. C. Beaulieu, C. C. Tan, and M. O. Damen, "A "better than" Nyquist pulse," *IEEE Commun. Lett.*, vol. 5, pp. 367–368, Sept. 2001.

[5] N. C. Beaulieu and M. O. Damen, "A parametric construction of ISI-free pulses," in *Proc. 8th Canadian Workshop on Information Theory*, 2003, pp. 121–124.

[6] N. C. Beaulieu and J. Cheng, "Precise error rate analysis of bandwidth efficient BPSK in Nakagami fading and co-channel interference," *IEEE Trans. Commun.*, vol. 52, pp. 149–158, Jan. 2004.

[7] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2000.