ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING PERFORMANCE IN DELAY AND DOPPLER SPREAD CHANNELS[1]

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Abstract—Orthogonal frequency division multiplexing (OFDM) is examined via digital computer simulation. Bit error performance of OFDM in a doubly spread channel is examined, and the effects of varying the guard interval and the number of carriers are demonstrated. The use of simulation as a design tool when doubly spread channel performance cannot be modeled mathematically is demonstrated. Simulations reveal that the designer has to strike a balance between the number of carriers and the guard interval in order to optimize performance and resource utilization.

I. INTRODUCTION

Orthogonal frequency division multiplexing is being intensely investigated for a number of possible applications including digital television broadcast, CD-quality broadcast, personal communications services (PCS), and asymmetrical digital subscriber lines. In PCS applications, in particular, it is envisioned that if OFDM modulation is used it may be subject to both delay and Doppler spread multipath.

OFDM consists of parsing a serial data stream into several parallel streams, each operating at a lower symbol rate than the serial stream, then modulating each parallel stream onto orthogonal carriers. This serves to divide the wideband channel into several different subchannels, with each subchannel operating at a lower symbol rate than the overall symbol rate dictated by the symbol rate of the serial stream.

OFDM enables the system designer to combat intersymbol interference (ISI) by increasing the OFDM symbol period without decreasing the overall symbol rate. A delay spread channel with a given maximum excess delay impacts a smaller fractional portion of the extended OFDM symbol, resulting in reduced ISI. OFDM also accommodates use of a guard interval to combat ISI, which will be covered in more detail later. Unfortunately, if Doppler spread is also present, the OFDM symbol period extension is achieved at the expense of packing additional carriers into the available channel bandwidth which results in increased intercarrier interference (ICI) accompanied by a higher error floor.

In this paper performance results are presented for OFDM systems utilizing various carrier modulation schemes which are subject to both delay and Doppler spread. The results are generated by a simulation model which utilizes a delay-Doppler spread multipath channel simulation [1].

II. OFDM MODEM IMPLEMENTATION

Refer to Fig. 1 [2] where an OFDM transmitter section is depicted, and consider a serial bit stream at a rate of R_b bit/s. A data encoder maps each set of $\log_2(M)$ of these bits to a symbol selected from the signal space of the desired subcarrier modulation scheme, where $\log_2(\cdot)$ represents the logarithm to the base 2, and M represents the number of different possible symbols. This produces a serial symbol rate of $R_s = R_b / \log_2(M)$ symbol/s, each with a duration of $T_s = 1/R_s$ s. A serial-to-parallel converter parses these serial symbols into groups of N symbols. Each symbol from the serial-to-parallel converter keys one of N orthogonal subcarriers. The keyed subcarriers are summed and transmitted, then a new set of N symbols is read in and the process is repeated. Each summation of the N subcarriers represents an OFDM symbol, and the wideband channel is now divided into N individual subchannels. The duration of each OFDM symbol is now $T_{\mathrm{OFDM}} = NT_s$ s, but the overall data rate remains constant because there are N subchannels with each subchannel carrying data at a rate of R_s / N serial symbol/s.

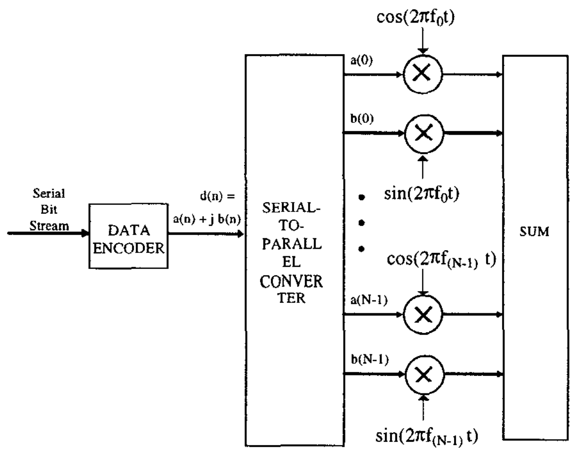

Fig. 1 Transmitter section of OFDM modem [2]

[1] Supported in part by Integrator Corporation, Colorado Springs, CO and in part by the Office of Naval Research under Contract N00014-920-J01761/P00004.

0-7803-3659-3/97 $10.00 ©1997 IEEE

The stream of OFDM symbols at the receiver, depicted in Fig. 2 [2], is applied to a bank of mixers and integrators where the OFDM symbols are demodulated and detected. Present on each given subcarrier is the original serial symbol that keyed the subcarrier at the transmitter. Present on each line going into the mixers at the receiver in Fig. 2 is a sum of orthogonal sinusoids and cosinusoids, and in the absence of noise and channel distortion they have the form $a(0)\cos(2\pi f_0 t)+b(0)\sin(2\pi f_0 t)+,\ldots,+a(N-1)\cos(2\pi f_{N-1}t)+b(N-1)\sin(2\pi f_{N-1}t)$. Consider separation of the in-phase component on the subcarrier at frequency f_0. The in-phase mixer at frequency f_0 multiplies the entire quantity above by $\cos(2\pi f_0 t)$. The product is then integrated: $\int_0^{NT_s} \mathrm{d}t$. Because the carriers are orthogonal, the integration operation yields nonzero values only for the in-phase subcarrier at frequency f_0, and the desired data symbol, $a(0)$, is extracted. The recovered data symbols are applied to a parallel-to-serial converter, and then decoded to produce the transmitted data bits. Note that despite the fact that the subchannel spectra overlap, the orthogonality of the subcarriers still enables separation of the individual subchannels.

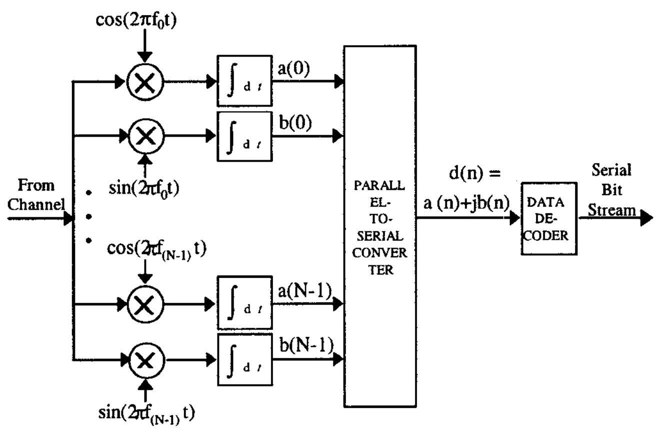

Fig. 2 Receiver section of OFDM modem [2]

A. *The Guard Interval*

To combat ISI, a guard interval is often inserted between successive OFDM symbols at the transmitter and discarded prior to demodulation. Any ISI that impacts the guard interval of the OFDM symbols is discarded with the guard interval. The guard interval increases the bandwidth by a factor of G_T / T_{OFDM}, where G_T is the time devoted to the guard interval. When the OFDM modem is implemented by means of the discrete Fourier transform (DFT), as in the next section, a cyclic prefix serves as an effective guard interval. In this case, the last G samples of each OFDM symbol are copied and cycled to the beginning of the given OFDM symbol. This cycling technique makes the linear convolution imparted by the channel appear as circular convolution to the DFT process at the receiver [3].

B. *DFT Implementation*

Implementation of an OFDM modem by means of the DFT originated in a paper by Weinstein and Ebert [4] which also included an analysis of the effects of linear channel distortion as well as signal design criteria. The use of the DFT makes for efficient implementation of an OFDM system because, for a sufficiently large number of subcarriers, which translates directly to DFT points, it is well known that the computation of the DFT is made even more efficient with the use of the fast Fourier transform (FFT). A block diagram of a DFT-based OFDM modem in baseband form is shown in Fig. 3, which also forms a basis for the simulation model.

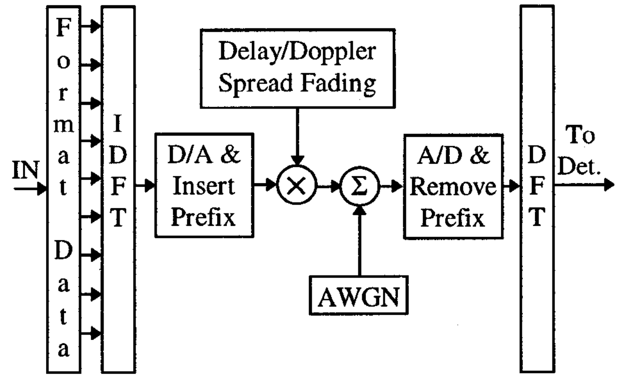

Fig. 3 Block diagram of DFT-based OFDM modem showing channel perturbations simulated

Note that the DFT implementation circumvents the need for banks of mixers at the transmitter and receiver. The DFT technique ensures that the transmitted subcarriers are orthogonal without the use of filter banks to maintain spectral integrity or the threat of oscillator frequency drift.

III. PERFORMANCE RESULTS IN DOPPLER/DELAY SPREAD RAYLEIGH FADING CHANNELS

The goal of this paper is to characterize the performance of OFDM utilizing two carrier modulation formats, both coherent and noncoherent in delay and Doppler spread channels. These include M-ary differential phase shift keying (MDPSK), and M-ary quadrature-amplitude modulation (MQAM). For the QAM investigation, pilot correction is used to adjust decision boundaries to compensate for the fading. To facilitate the discussion, we introduce the ratio of reciprocal maximum Doppler frequency (f_D) to OFDM symbol period called *DSR*, and define it as follows

$$DSR = \frac{1}{f_D T_{\mathrm{OFDM}}}. \quad (1)$$

A. *Purely Doppler Spread Channel Performance*

Typical results for bit error rate (BER) versus the ratio of bit energy to noise power spectral density (E_b / N_0) are shown in Fig. 4 for a 64-carrier OFDM system with 4-DPSK signaling used on the subcarriers in a purely Doppler spread channel. Perfect phase recovery at the receiver is assumed, which was simulated by subjecting the data to only the channel envelope variations. The simulation results are compared to theoretical flat fading performance for 4-DPSK modulation in a single-carrier system [5].

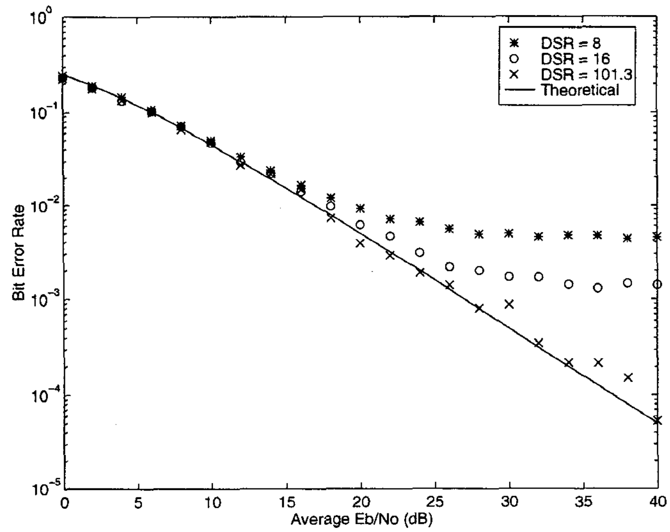

Fig. 4 Performance of 64-carrier OFDM system in Doppler spread Rayleigh fading channel; 4-DPSK

The error floor effect in Fig. 4 is evident for the two smaller *DSR* values, where the channel varies quickly relative to the OFDM symbol period resulting in more severe ICI. For the largest *DSR* value, the slowly varying channel yields performance that closely approximates flat fading.

Similar results are shown in Fig. 5 for 16-QAM subcarrier modulation and a 16-carrier OFDM system where a time domain pilot is used. Once again, perfect phase recovery at the receiver is assumed.

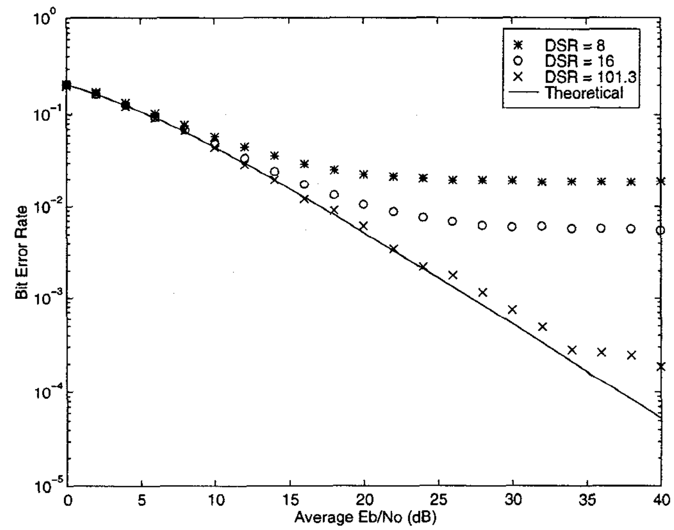

Fig. 5 Performance of 16-carrier OFDM system in Doppler spread Rayleigh fading channel; pilot-assisted 16-QAM

The time domain pilot correction scheme developed for the results in Fig. 5 consists of inserting a pilot at the center of each OFDM symbol at the transmitter. The pilot is extracted at the receiver prior to demodulation, where it is assumed that the OFDM symbol surrounding the pilot underwent the same channel impairments. Each OFDM symbol is corrected by a factor necessary to return its associated pilot to the level that was inserted at the transmitter. In Fig. 5, perfect pilot recovery is assumed, which was simulated by not allowing the pilot to be corrupted by Gaussian noise.

Once again, the error floor in Fig. 5 is evident for the two lower *DSR* values. The lower *DSR* values imply a faster varying channel, which would be expected to degrade the estimate provided by the pilot and worsen the performance. As will be discussed later, the performance degradation is ironically attributable almost entirely to ICI, which dominates the effects of the estimation error provided by the pilot.

B. *Effects Due to ICI and Evaluation of Pilot Correction*

The results here for Doppler spread can be matched with those given in [6] where they considered, theoretically and by simulation, the error floor due to ICI only. If only the error floor due to ICI as a function of the maximum Doppler frequency is simulated, the results in Fig. 6 are obtained. This case is for a 1024-carrier OFDM system using 16-QAM subcarrier modulation in a purely Doppler spread Rayleigh fading channel. To determine error floors, the signal-to-noise ratio is assumed to be infinite. We also include a comparison of the time domain pilot correction scheme used for the results in Fig. 5 with a frequency domain pilot correction scheme that was developed. The results are compared to theoretical expectations obtained from [6] and simulations performed using an ideal compensation term that performs perfect channel estimates. In this case, the data is subjected to both the channel envelope and phase variations and the pilots correct for both channel impairments.

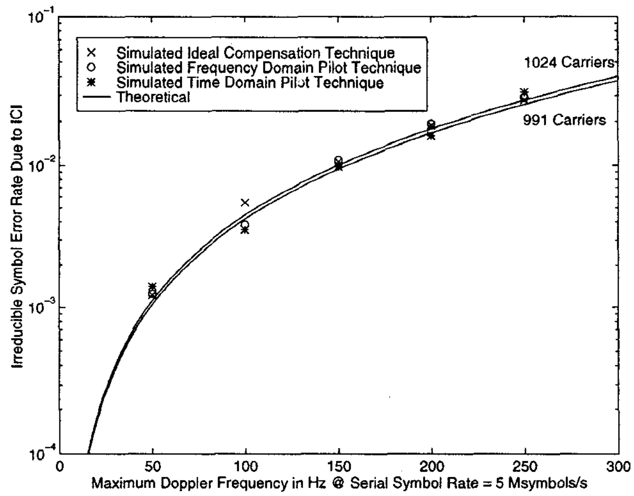

Fig. 6 Effects due to ICI: Comparison of two pilots in Doppler spread Rayleigh fading channel; 16-QAM

The frequency domain pilot technique is implemented by inserting a pilot surrounded by a guard band consisting of 16 frequency bins of null data on each side of the pilot, so that pilot plus guard band totals 33 frequency bins. The guard band serves to protect the pilot from ICI. This combination of pilot and guard band replaces 33 of the QAM data symbols from each set of 1024 QAM data symbols prior to the IDFT operator at the transmitter. The pilot is then recovered at the output of the DFT operator, where it is assumed that the data on the surrounding subcarriers undergo the same channel impairments. The data recovered from each OFDM symbol is adjusted by the same factor necessary to return the associated pilot to its transmitted value. The pilot and guard band are discarded and error analysis is performed on the pilot-corrected data.

In Fig. 6, the pair of theoretical curves represents upper and lower bounds for the theoretical performance of the frequency domain pilot. The curve for 1024 carriers is the theoretical performance for the time domain pilot and ideal compensation. Symbol error rate is shown instead of bit error rate to accommodate comparison with Russell and Stüber's theoretical results, which are in terms of symbol error rate [6].

The frequency domain pilot just demonstrated in Fig. 6 is expected to perform well in a channel with a high coherence bandwidth such as the channel in this case where a delay spread is absent. The fact that the time domain pilot performance is on par with ideal gain compensation and the frequency domain pilot may be rather surprising upon initial inspection. According to (1), the Doppler spread gives rise to *DSR* values ranging from 19.5 at the 250 Hz maximum Doppler frequency to 97.7 at the 50 Hz maximum Doppler frequency. The lower *DSR* values imply a faster varying channel envelope relative to the OFDM symbol duration, which would be expected to degrade the estimate provided by the single pilot sample inserted at the center of each OFDM symbol. However; the lower *DSR* values also increase the ICI effects. These ICI effects dominate the performance degradation mechanism and mask the performance degradation of the time domain pilot associated with the faster varying channel.

C. *Doubly Spread Channel Performance*

The effects of a combination of Doppler and delay spread are investigated next. Delay spread in an OFDM system is combated first by using a guard interval equal to the maximum excess delay of the channel and discarding the guard interval at the receiver, as shown in Fig. 3. Thus, for short delay spreads relative to the OFDM symbol period, the degradation is small since it amounts only to the fraction of power lost due to the guard interval. In more severe channels with longer delay spreads, the use of a simple guard interval is no longer sufficient, and the additional steps of equalization (in the frequency domain [7]) and coding must be taken [3]. With both delay and Doppler spread present, one must seek a compromise between a long OFDM symbol period (to minimize the effects of delay spread) and ICI, which becomes more severe the longer the OFDM symbol period.

An exponential power delay profile is assumed with the 20 dB down point from the direct path power at 15 μs, and a variable guard interval. The subcarrier modulation is ideally-compensated 16-QAM, the serial symbol rate is 1 Msymbol/s, and the maximum Doppler frequency is 100 Hz. The ideal compensation is the same one used for the results in Fig. 6. The signal-to-noise ratio is assumed to be infinite. Fig. 7 shows the effects of varying the guard interval and the number of carriers under these conditions.

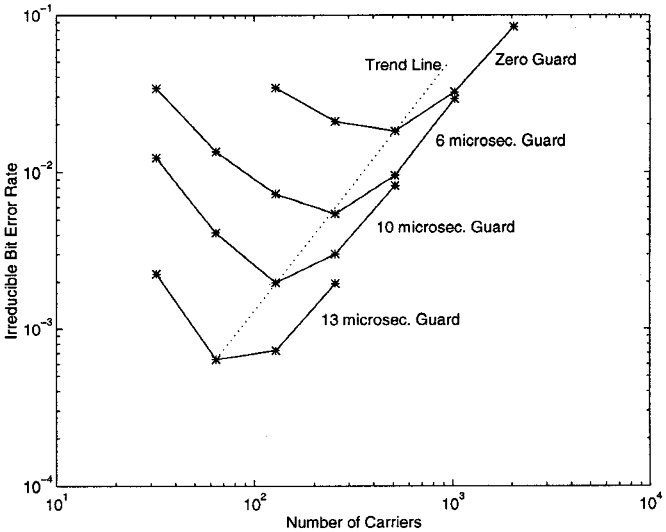

Fig. 7 Effects of varying number of carriers and guard interval: OFDM in doubly spread channel; 16-QAM

In Fig. 7, the solid lines connecting the data points are for the purpose of grouping the data points and should not be misconstrued to imply linearity between the data points or an attempt to interpolate. Likewise, the dotted trend line groups the optimal bit error performance points of the data groups.

Figure 7 demonstrates that an optimal design needs to strike a balance between the number of carriers and the guard interval. Moving from left to right along a solid line, ICI increases due to the increased number of carriers, while ISI decreases because the delay spread impacts a smaller fractional portion of the OFDM symbols. Furthermore, both bandwidth usage and the power penalty imposed by the guard interval decrease when moving from left to right. Moving from top to bottom along an imaginary vertical line that connects data points from different groups, the bit error performance improves because the increased guard interval provides more ISI protection, but bandwidth usage and the power penalty both increase.

Proper selection of the optimum combination of guard interval and number of carriers depends on the required bit error performance, available channel bandwidth, the available transmit power, and the degree of computational burden one is willing to impose by adding more carriers. The proper choice is not always obvious and does not always correspond to the minimum point of one of the solid line groups in Fig. 7. A hypothetical case will demonstrate. Suppose a BER near 2×10^{-3} is acceptable; then the rightmost point of the 13 μs guard interval group and the lowest point of the 10 μs guard interval group are both good candidates for operating parameters. These two operating parameters correspond to $N = \{256, 128\}$, where N is the number of carriers and the first entry of the ordered pair corresponds to the 13 μs guard interval. The percent increase in bandwidth usage and power penalty when compared to a case with zero guard interval are {5.47%, 8.59%}. This implies that the 13 μs guard interval with 256 carriers is probably the better choice. But this does not take into account the rate at which the two sets of parameters approach their error floors, and since each set of parameters is driven to its error floor by different mechanisms, additional simulations are necessary. Fig. 8 shows the estimated BER, arrived at by simulation, of each set of parameters as a function of E_b / N_0.

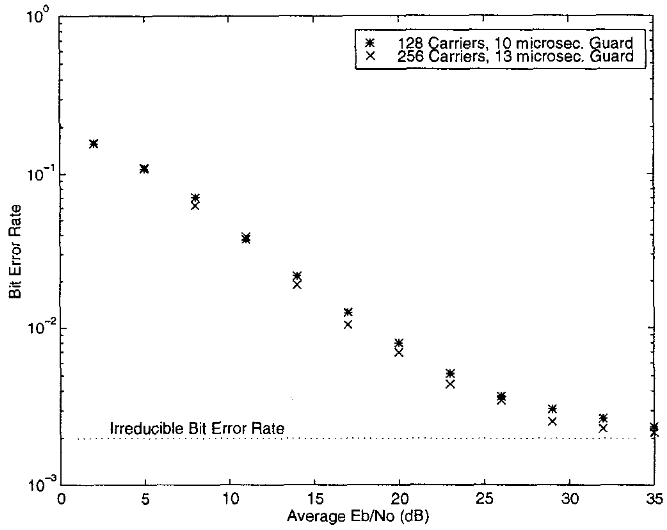

Fig. 8 BER performance for two sets of parameters from Fig. 7: Rightmost point of 13 μs guard interval group and lowest point of 10 μs guard interval group

Examination of Fig. 8 reveals that the case with 256 carriers and a 13 μs guard interval performs slightly better for most values of signal-to-noise ratio, so based on BER performance and bandwidth usage, it is the preferred set of operating parameters. The increased computational burden imposed by the 256-carrier case would also have to be considered in the decision.

IV. CONCLUSIONS

We set out to investigate OFDM performance in delay and Doppler spread channels. The effects of Doppler spread were demonstrated, and the effects of ICI were demonstrated. Two different pilot correction techniques were shown to be effective in a purely Doppler spread channel. The tradeoff between the guard interval and the number of carriers in a delay and Doppler spread channel was also demonstrated. A simple design example illustrated that when theoretical predictions are not available, as in the case of OFDM performance in a delay and Doppler spread channel, resorting to simulations can be effective.

The simulation does not account for hardware limitations, different fading models, nonexponential power delay profiles, cochannel interference, and carrier frequency acquisition. These areas impact OFDM performance significantly, but are left for future investigations.

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