Spectrum Sensing Combining Time and Frequency Domain in Multipath Fading Channels

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***Abstract*—Spectrum sensing is one of the key challenges in the cognitive radio network. Primary user signal must be detected reliably in the low signal-to-noise ratio (SNR) regime and in multipath fading environments.**

This paper analyzes effects of time-variant multipath Rayleigh fading channel on cyclostationary characteristics and derives the relationship between cyclostationary statistics of transmitted and received signal. Depending on the distinct feature and the set of tested cyclic frequencies, the test statistics may have different performances. Cyclostationary features of small scale should be selected to be detected taking multipath effects into account. A novel solution to detect cyclostationary features combining time and frequency domain is proposed. Simulation results illustrating the reliability of solution as well as the effects of time-variant multipath fading channel on features are presented.

***Index Terms*—cognitive radio, spectrum sensing, cyclostationarity**

I. Introduction

Recently, Cognitive Radio has been seen as a potential solution to improve spectrum utilization via opportunistic spectrum sharing. It is an intelligent wireless communication system that can sense the radio spectrum in order to find unused frequency bands and use them in an agile manner. One of the key challenges currently facing cognitive radio system is the issue of reliable spectrum sensing.

Cyclostationary feature detectors have been introduced as a complex two dimensional signal processing technique for recognition of modulated signals in the presence of noise and interference [5][6][7]. Recently, they have been proposed for detection of cyclostationary signatures for rendezvous in orthogonal frequency division multiplex (OFDM)-based cognitive radio networks [3][4]. Cyclostationary feature is appealing in good performance at the low SNR regime and distinguishing among different signal types. A key limitation of cyclostationary signatures is the sensitivity to time-variant multipath Rayleigh fading environments.

Communication signals exhibit multiple scales of cyclostationary features due to the symbol or chip rate, training or pilot signals, guard periods and have their own sensitivity to multipath fading. Multiple signatures are introduced to overcome time-variant multipath Rayleigh fading [4] [7]. However, the relationship between cyclostationary statistics of transmitted and received signal in multipath Rayleigh fading environments has not been discussed yet.

In our work, we investigate the effects of time-variant multipath Rayleigh fading channel on cyclostationary characteristics in both time and frequency domain. The relationship between cyclostationary statistics of transmitted and received signal in multipath Rayleigh fading environments is derived. Choosing cyclostationary features of small scale to be detected is also proposed. Then a novel method of cyclostationary feature detection combining time and frequency domain is presented. It extends the methods of [5] and [7] to take into account the rich information present in both time and frequency domain. This method can also serve to identification of signal with unique features in time and frequency domain. Moreover, performance of the detection method as well as distinct sensitivities exhibited by different features in multipath fading channels are illustrated.

This paper is organized as follows. In Section II, there is a short review of cyclostationary statistics. The effects of time-variant multipath Rayleigh fading channel on cyclostationary feature are derived in Section III. Section IV addresses the method of cyclostationary feature detection combining time and frequency domain. Simulation results demonstrating the confidence of detector presence and performance of different features are given in Section VI. Finally, conclusions are drawn.

II. Cyclostationarity

A zero mean complex random process $x(t)$ is characterized by a time varying autocorrelation function $R_x(t,\tau) = E\{x(t)x^*(t+\tau)\}$. It is said to be cyclostationary in wide sense if its autocorrelation is periodic in time t with a given lag τ. Due to the periodicity of the autocorrelation, it has a Fourier series representation as [9]

This work is sponsored by Nokia and in part by the National Basic Research Program of China (973 Program) under grant no. 2007CB310602

$$R_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t} \quad (1)$$

where the Fourier coefficients are

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t,\tau) e^{-j2\pi\alpha t} dt \quad (2)$$

and α is called the cyclic frequency. The function $R_x^{\alpha}(\tau)$ is called the cyclic autocorrelation function (CAF). The cyclic Wiener relation states that the spectral correlation function (SCF) can be obtained from the Fourier transform of the cyclic autocorrelation in (2) [9]

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau \quad (3)$$

which can be seen as a generalization of the conventional power spectral density function. In practical situations, the number of observation samples is limited. Therefore, both the cyclic autocorrelation function and spectral correlation function need to be estimated from a finite set of samples. An estimate of the cyclic autocorrelation may be obtained using T observations as

$$\widehat{R}_x^{\alpha}(\tau) = \frac{1}{T} \sum_{t=1}^{T} x(t) x^*(t+\tau) e^{-j2\pi\alpha t} \quad (4)$$

And we use the spectrally smoothed cyclic periodogram method [1]. Let us define the cyclic periodogram by [2]

$$S_{X_T}^{\alpha}(t,f) = \frac{1}{T} X_T(t, f+\alpha/2) X_T^*(t, f-\alpha/2) \quad (5)$$

where X_T is the short-time Fourier transform defined as follows

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi f u} du \quad (6)$$

The estimated SCF obtained by frequency smoothing of the cyclic periodogram in (5) is

$$S_x^{\alpha}(f) = \lim_{\Delta f\to 0} \lim_{T\to\infty} \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{X_T}^{\alpha}(t,\nu) d\nu \quad (7)$$

III. Cyclostationary Statistics In Time-Varying Multipath Channels

The challenge of cyclostationary detection in cognitive radio involves effects of time-varying multipath channel. In this section we derive effects of time-varying multipath channel on cyclic autocorrelation and spectral correlation function. We assume that cyclostationary features of primary user signal are known to cognitive radio user which is reasonable in this case. Thus some issues dealing with selecting proper set of cyclostationary features to be detected can be obtained.

We define signal model

$$y(t) = \int_{-\infty}^{\infty} h(\tau;t) x(t-\tau) d\tau \quad (8)$$

where $y(t)$ and $x(t)$ are received and transmitted signal respectively, $h(\tau;t)$ represents the equivalent lowpass response of the channel at time t to an impulse at time $t-\tau$. Let time-varying channel impulse response be [10]

$$h(\tau;t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)) \quad (9)$$

where N denotes the number of resolvable multipath components, $\alpha_n(t)$ denotes the amplitude, $\tau(t)$ is the path delay, and $\phi_n(t)$ is the phase shift. We also make the assumption that the in-phase and quadrature components of $h(\tau;t)$ are independent Gaussian processes with the same autocorrelation, a mean of zero, and a cross-correlation of zero. And this channel model has a Rayleigh-distributed amplitude and uniform phase without LOS component.

In order to analyze spectral correlation of received signal under time-varying multipath fading, we characterize autocorrelation of the channel assuming that our channel model is wide-sense stationary (WSS) and uncorrelated scattering (US)

$$A_h(\tau;\Delta t) = E[h^*(\tau;t) h(\tau;t+\Delta t)] \quad (10)$$

Moreover we assume that the delay spread and Doppler spread are separable.

From the definition presented in the preceding section, we can arrive at cyclic autocorrelation function of received signal as

$$\begin{aligned} R_y^{\alpha}(\Delta t) &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} E[y(t) y^*(t+\Delta t)] e^{-j2\pi\alpha t} dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} E[h(\tau;u) h^*(\tau;u+\Delta t)] \\ &\times \; E[x(u-\tau) x^*(u-\tau+\Delta t)] \\ &\times \; e^{-j2\pi\alpha(u-\tau)} e^{-j2\pi\alpha\tau} d\tau du \\ &= \int_{-\infty}^{\infty} A_h^*(\tau;\Delta t) R_x^{\alpha}(\Delta t) e^{-j2\pi\alpha\tau} d\tau \quad (11) \end{aligned}$$

where $R_x^{\alpha}(\Delta t)$ denotes cyclic autocorrelation function of transmitted signal and $A_h^*(\tau;\Delta t)$ denotes complex conjugate of autocorrelation function of $h(\tau;t)$

$$A_h^*(\tau;\Delta t) = E[h(\tau;t) h^*(\tau;t+\Delta t)] \quad (12)$$

Using the Fourier transform of $A_h^*(\tau;\Delta t)$ in variable τ, the cyclic autocorrelation function is given by

$$R_y^{\alpha}(\Delta t) = A_H^*(\alpha;\Delta t) R_x^{\alpha}(\Delta t) \quad (13)$$

where $A_H(\alpha;\Delta t)$ is the Fourier transform of the power delay profile for given Δt [10].

Since the channel response is approximately independent at frequency separations $\Delta t > B_c$, coherence bandwidth, and the time-varying channel decorrelates after approximately channel coherence time T_c, it is possible that the $R_y^{\alpha}(\Delta t)$ approaches zero when frequency separation α exceeds B_c and time separation Δt exceeds T_c. Therefore, when we choose the position of cyclostationary feature for primary user signal detection in time domain, we should take the following conditions into account

$$\begin{aligned} \alpha &< B_c \\ \Delta t &< T_c \end{aligned} \tag{14}$$

Smaller α and Δt lead to better performance of detection in multipath channels.

Now, let us consider the spectral correlation of received signal

$$\begin{aligned} S_y^\alpha(f) &= \lim_{T\to\infty} \frac{1}{T} E[Y_T(t, f+\alpha/2) Y_T^*(t, f-\alpha/2)] \\ &= \lim_{T\to\infty} \frac{1}{T} E[\int_{-T/2}^{T/2} y(u) e^{-j2\pi(f+\alpha/2)u} du \\ &\times \int_{-T/2}^{T/2} y^*(s) e^{j2\pi(f-\alpha/2)s} ds] \end{aligned} \tag{15}$$

Following from the WSS and US properties of $h(\tau;t)$, the spectral correlation function can be simplified as

$$\begin{aligned} S_y^\alpha(f) &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\int_{-\infty}^{\infty} E[h(\tau;u)h^*(\tau;s)] \\ &\times E[x(u-\tau)x^*(s-\tau)] d\tau e^{j2\pi f(u-s)} e^{j2\pi\frac{\alpha}{2}(u+s)} \\ &\times duds \\ & (let: s-u=\Delta t) \\ &= \lim_{T\to\infty} \int_{-T/2}^{T/2}\int_{-\infty}^{\infty} E[h(\tau;u)h^*(\tau;u+\Delta t)] \\ &\times R_x^\alpha(\Delta t) e^{-j2\pi\alpha\tau} e^{j2\pi(f-\frac{\alpha}{2})\Delta t} d\tau d\Delta t \end{aligned} \tag{16}$$

The scattering function [10] for random channels is defined as the Fourier transform of $A_h(\tau;\Delta t)$

$$S_h(\tau;\rho) = \int_{-\infty}^{\infty} A_h(\tau;\Delta t) e^{-j2\pi\rho\Delta t} d\Delta t \tag{17}$$

We can also define $S_H(\Delta f;\rho)$ as the Fourier transform of the scattering function [10]

$$S_H(\Delta f;\rho) = \int_{-\infty}^{\infty} S_h(\tau;\rho) e^{-j2\pi\Delta f\tau} d\tau \tag{18}$$

Now spectral correlation of received signal can be given by

$$\begin{aligned} S_y^\alpha(f) &= \lim_{T\to\infty} \int_T S_H(\alpha, -\Delta t) R_x^\alpha(\Delta t) e^{-j2\pi(\frac{\alpha}{2}-f)\Delta t} d\Delta t \\ &= S_H(\alpha, f-\frac{\alpha}{2}) \circledast S_x^\alpha(\frac{\alpha}{2}-f) \end{aligned} \tag{19}$$

where $\circledast$ denotes convolution.

Considering effect of channel coherence bandwidth, we obtain that if cyclic frequency $\alpha >> B_c$, coherence bandwidth, the autocorrelation goes to zero. Even if the transmitted signal should exhibit cyclostationarity at cyclic frequency α, the spectral correlation will be vanished after passing through this time-varying multipath channel.

In the light of the discussion above, it is evident that small scale of correlation which satisfies condition (14) is appropriate when we select cyclostationary characteristics for detection to overcome time-varying multipath effect.

IV. Cyclostationary Feature Detection Combining Time and Frequency Domain

In this section, a new approach of cyclostationary detection combining features in time and frequency domain is presented.

Communication signals which will be used in cognitive radio exhibit cyclostationarity at multiple cyclic frequencies instead of just a single one. For example a signal that has cyclic frequency related to the symbol rate is typically cyclostationary at all integer multiples of the symbol rate. Besides it may also exhibit cyclostationarity related to guard periods, cyclic prefix (CP) and pilot carriers. In such cases sensitivities of these presented cyclic frequencies to channel quality may be distinct. And they provide rich information for us to detect and identify communication signal.

In the following we extend the test based on second-order cyclic statistics to cyclic frequencies in time and frequency domain. For time domain test, from (4) we obtain that

$$\widehat{R}_x^\alpha(\tau) = R_x^\alpha(\tau) + \Delta_x^\alpha(\tau) \tag{20}$$

where the latter term is estimation error which is consistent [8].

The $1\times 2N$ row vector consisting of cyclic autocorrelation estimates at the cyclic frequency α for a number of lags $\tau_1, ..., \tau_N$ is given as

$$\begin{aligned} \widehat{\mathbf{r}}_x^\alpha(\tau) = [&Re\{\widehat{R}_x^\alpha(\tau_1)\}, ..., Re\{\widehat{R}_x^\alpha(\tau_N)\}, \\ &Im\{\widehat{R}_x^\alpha(\tau_1)\}, ..., Im\{\widehat{R}_x^\alpha(\tau_N)\}] \end{aligned} \tag{21}$$

Defining the row vector of the true value of the cyclic autocorrelation $\mathbf{r}_x^\alpha(\tau)$ and the estimation error vector $\mathbf{\Delta}_x^\alpha(\tau)$ in a similar fashion, we can write

$$\widehat{\mathbf{r}}_x^\alpha(\tau) = \mathbf{r}_x^\alpha(\tau) + \mathbf{\Delta}_x^\alpha(\tau) \tag{22}$$

It can be shown that the estimation error is asymptotically normal distributed [8],

$$\lim_{T\to\infty} \sqrt{T}\mathbf{\Delta}_x^\alpha(\tau) = \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_x) \tag{23}$$

where $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_x)$ is a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma}_x$.

Given that the hypothesis H_0 represents the case where $x(t)$ does not exhibit cyclostationarity with the cyclic frequency α and H_1 represents the case where $x(t)$ does exhibit cyclostationarity the following binary hypothesis testing problem can be formulated:

$H_0: \forall\{\tau_n\}_{n=1}^N \Rightarrow \widehat{\mathbf{r}}_x^\alpha(\tau) = \mathbf{\Delta}_x^\alpha(\tau)$

H_1 : for some $\{\tau_n\}_{n=1}^N \Rightarrow \widehat{\mathbf{r}}_x^\alpha(\tau) = \mathbf{r}_x^\alpha(\tau) + \mathbf{\Delta}_x^\alpha(\tau)$

The asymptotic complex normality of $\widehat{\mathbf{r}}_x^\alpha(\tau)$ allows the formulation of the following generalized likelihood function as the test statistic for the binary hypothesis test [8]:

$$\mathcal{T} = T\widehat{\mathbf{r}}_x^\alpha(\tau)\widehat{\mathbf{\Sigma}}_x^{-1}\widehat{\mathbf{r}}_x^{\alpha(T)}(\tau) \tag{24}$$

Under null hypothesis, the distribution of the test statistic converges asymptotically to a central χ^2 distribution with

$2N$ degrees of freedom. Moreover, relying on the asymptotic nomality and consistency of spectral correlation function, frequency domain test can be developed in a similar way. The cyclic test statistic in frequency domain has similar property as that in time domain[8].

Now in order to extend the test for the presence of second-order cyclostationarity at cyclic frequencies of interest $\alpha_t \in \mathcal{A}_t$ and $\alpha_f \in \mathcal{A}_f$ in time and frequency domain respectively, we formulate the hypothesis testing as follows where $\mathcal{A}_t$ denotes the set of cyclic frequencies in time domain while $\mathcal{A}_f$ the set of cyclic frequencies in frequency domain.

$H_0 : \forall \alpha_t \in \mathcal{A}_t$ and $\forall \{\tau_n\}_{n=1}^N$ and $\forall \alpha_f \in \mathcal{A}_f$ and $\forall \{f_n\}_{n=1}^N$
$\Rightarrow \widehat{\mathbf{r}}_x^\alpha(\tau) = \mathbf{\Delta}_x^\alpha(\tau)$ and $\widehat{\mathbf{s}}_x^\alpha(f) = \mathbf{\Delta}_x^\alpha(f)$

H_1 : for some $\alpha_t \in \mathcal{A}_t$ and for some $\{\tau_n\}_{n=1}^N$ or for some $\alpha_f \in \mathcal{A}_f$ and for some $\{f_n\}_{n=1}^N$
$\Rightarrow \widehat{\mathbf{r}}_x^\alpha(\tau) = \mathbf{r}_x^\alpha(\tau) + \mathbf{\Delta}_x^\alpha(\tau)$ or $\widehat{\mathbf{s}}_x^\alpha(f) = \mathbf{s}_x^\alpha(f) + \mathbf{\Delta}_x^\alpha(f)$

For this hypothesis testing problem, we propose the following test statistic:

$$\mathcal{T}_s = \sum_{\alpha_t \in \mathcal{A}_t} \mathcal{T}_t + \sum_{\alpha_f \in \mathcal{A}_f} \mathcal{T}_f \tag{25}$$

where $\mathcal{T}_t$ denotes the test statistic in time domain test and $\mathcal{T}_f$ the test statistic in frequency domain test [8].

This test statistic calculates the sum of the cyclostationary statistic over the cyclic frequencies of interest. Since the sum of independent χ^2 random variables is also χ^2 distributed, $\mathcal{T}_s$ is asymptotically $\chi^2_{2N(N_{\alpha_t}+N_{\alpha_f})}$ distributed under the null hypothesis, where $(N_{\alpha_t} + N_{\alpha_f})$ is the number of cyclic frequencies taking time and frequency domain into consideration.

Communication signals exhibit cyclostationarity introduced by CP, symbol rate or pilot carriers. Unique features can be created in time and frequency domain for signal and their test statistics may have different performance. This detection method can exploit cyclostationiarity of time and frequency domain simultaneously and serve to identification signal type.

V. Simulations

In the following, the confidence of the proposed method has been investigated. The effects of time-variant multipath fading channel on different signatures are demonstrated as well.

OFDM signals are generated using quadrature phase shift keying (QPSK) modulated random data symbol. A 64-bin IFFT is used. We select cyclostationary features introduced by CP and pilot carriers for time and frequency domain respectively. Simulations are performed for both non-multipath and multipath fading channels. In the latter case, we choose COST 207 bad urban model which contains six fading paths with their respective powers and delays for a carrier frequency 900 MHz. The data are passed through a channel with different SNR. The SNR is measured by the ratio of the power of OFDM symbol to the power of noise. Detection is carried out by 60 OFDM symbol observations.

Fig.1 and Fig.2 show the performance of cyclostationary features of different scale in both addictive white Gaussian noise (AWGN) and multipath propagation channel. Probability of detection P_d vs. SNR for a fixed false alarm rate $P_f = 0.05$

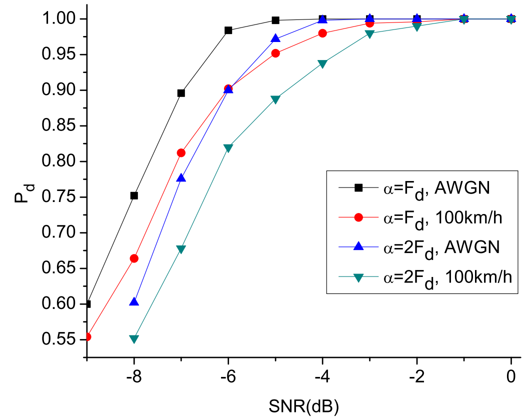

Fig. 1. P_d vs SNR for cyclostationary features of different scale in time domain and $P_f = 0.05$

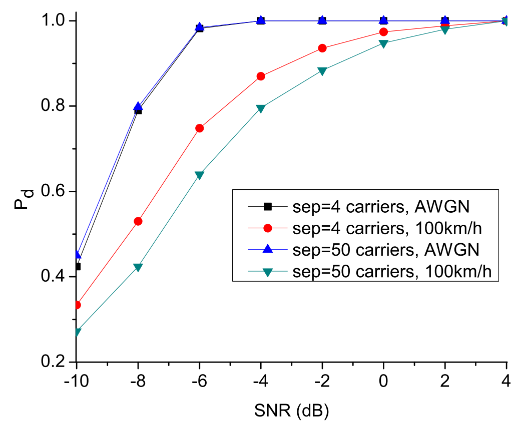

Fig. 2. P_d vs SNR for cyclostationary features of different scale in frequency domain and $P_f = 0.05$

is plotted. In Fig.1 We choose a fixed lag $\tau = T_u$ and cyclic frequencies $\alpha = F_d$ and $2F_d$ to be detected where T_u denotes useful symbol duration and F_d denotes OFDM symbol rate. In Fig.2, Two sets of separation between pilot carriers with same value are taken into consideration: $sep = 4$ carriers and $sep = 50$ carriers. We could observe that feature of a smaller scale $\alpha = F_d$ in Fig.1 and $sep = 4$ carriers in Fig.2 have better performance.

Fig.3 and Fig.4 indicate proposed detection method performance in both AWGN and multipath channel and compare it with single and multiple cyclic frequencies detector where $\alpha_t = F_d$ denotes cyclic frequency introduced by CP and α_f cyclic frequency introduced by pilot carriers. In Fig.3, the proposed method outperforms single cyclic frequency detector. In Fig.4, $\alpha_t = F_d$ and $2F_d$ are selected in time domain test. Performance of this detector approaches that of multiple cyclic frequencies detector in time domain and is much better than that of multiple cyclic frequencies detector in frequency

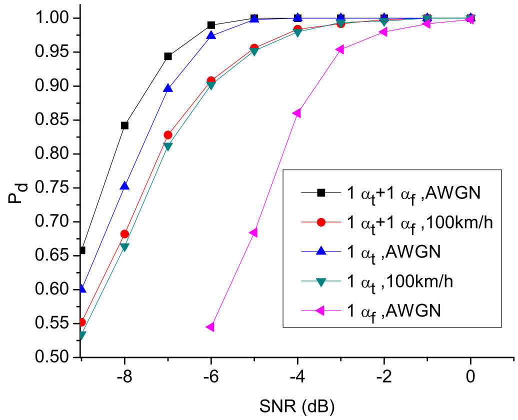

Fig. 3. P_d vs SNR for comparing proposed detection method with single cyclic frequency detectors and $P_f = 0.05$

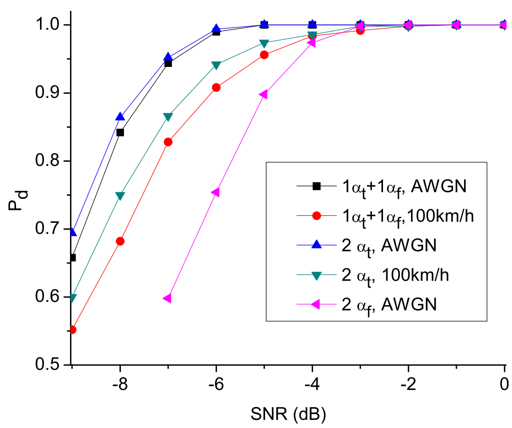

Fig. 4. P_d vs SNR for comparing proposed detection method with other multiple cyclic frequencies detectors and $P_f = 0.05$

domain. Moreover, the detector combining features in time and frequency domain has less complexity than multiple cyclic frequencies detector in time domain. It can also aid in identifying unique features in time and frequency domain simultaneously.

VI. Conclusion

This paper has analyzed effects of time-varying multipath channel on cyclostationary characteristics of OFDM-based waveforms. Cyclostationary features of small scale have been proposed to be detected considering multipath effects. A new method of detecting cyclostationarity combining time and frequency domain have been presented, which can aid in identification of multiple features in two domains. Simulation results demonstrating the confidence of detector have been presented. Desirable performance of the proposed detector has been verified. We have also investigated distinct sensitivities to noise and multipath of several cyclostationary features. The results have indicated that features selected with small scale to be detected improve performance considerably.

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