# Spectrum Sensing Based on Goertzel Algorithm

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***Abstract*—The spectrum scarcity, which is a serious problem in wireless communication, shows the extreme importance of spectrum sensing technologies. For the traditional spectrum sensing algorithms, they are very difficult to have a good tradeoff between accuracy and simplicity. This paper is based on the application of Goertzel algorithm to the cyclostationary feature detector, which makes fast-searching and precise-searching of spectrum come true. The result of simulation indicates that it is feasible to get nearly zero Percentage Missed Detect (PMD) when the Signal to Noise Ratio (SNR) is 3. The Goertzel algorithm is capable of computing the spectrum of some appointed frequency, while the Fourier Transform is only allowed to get the spectrum of the whole frequency band. So while detecting whether there exist a primary user in several frequencies rather than the whole band, the proposed algorithm is superior in simplicity of calculation.**

***Keywords-Spectrum sensing; Goertzel; Spectral Correlation Function(SCF); Fast-searching; Pencentage Missed Detect(PMD)***

## I. INTRODUCTION

The current wireless network adopts fixed spectrum-allocated policy. The planning and use of wireless spectral resource are formulated by governmental department, so the use of the resource should be allowed. However, the Federal Communications Commission's (FCC) frequency allocation chart indicates that the wireless spectral resource is in low utilization, especially in the 3-6 MHZ bands. With the study of 2003, the FCC's Spectrum Policy Task Force (SPTF) reported that the allocated spectrum with utilization was ranging from 15% to 85% [1]. Moreover, there is quite different spectrum efficiency in different time and space.

The technology of spectrum sensing is the fundamental of cognitive radios. Traditional spectrum sensing techniques of single node [2] are listed and compared in table I. The table shows that the current spectrum sensing technologies are not perfect enough for both accuracy and simplicity. For example, the energy detector is simple in implementation but suffers from a bad performance at low Signal to Noise Ration (SNR), the cyclostationary feature detector [3][4][5] can differentiate the types of signals but it is highly complexity in implementation, the matched filter require the information of primary user, which is difficult to get when implementing. The Goertzel algorithm [6] does not require to compute the spectral values of the whole frequency band if just several frequency points be required, which can reduce the simplicity in computation. Applying the Goertzel algorithm to the cyclostationary feature detector for spectrum sensing can realize a fast and accurate spectrum-searching.

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TABLE I. Comparions of Various Technologies of Spectrum Sensing

| Detection Method | Applicable scenario | Advantages | Disadvantages |
|---|---|---|---|
| Energy detector | CR nodes do not require the information of primary user | Simplicity and does not require the priori knowledge of primary user signal | Highly susceptible to unknown or variant noise levels; does not differentiate between modulated signals, noise and interference; long time to detect |
| Matched filter | CR nodes require the information of primary user | Requires less time to detect | Require the priori knowledge of primary user signal |
| Cyclostationary feature detector | The primary user signal is cyclostationary | Can differentiate between the noise and modulated signal | Highly complexity |

The paper is organized as follows: Section Ⅱ defines the Spectral Correlation Function(SCF) and proposes a way for spectrum sensing based on it. The related simulation results are given in this section as well. In section Ⅲ, it investigates the Goertzel algorithm and apply it to the SCF for the spectrum sensing. The application scenario and the superiority in computation are also showed in this section. Section Ⅳ presents the simulative results and does performance analysis. Finally, conclusions are presented in section Ⅴ.

## II. SPECTRUM SENSING BASED ON SPECTRAL CORRELATION FUNCTION (SCF)

### A. Definition of SCF

One can define spectral correlation function (SCF) [5] as in (1), where $R_x^{\alpha}(\tau)$ is called cyclic autocorrelation function(CAF) and given in (2), where $\langle \ \rangle_t$ is the mean for t, $x(t)$ is input signal, $\alpha$ is cyclic frequency and $R_x(t,t+\tau)$ is autocorrelation function.

$$S_x^{\alpha}(f) = \int_{-\infty}^{+\infty} R_x^{\alpha}(\tau)\cdot e^{-j2\pi f\tau} d\tau \quad (1)$$

$$R_x^{\alpha}(\tau) = \left\langle R_x(t, t+\tau)\cdot e^{-j2\pi\alpha t} \right\rangle_t \quad (2)$$

In the definition of CAF, the autocorrelation function $R_x(t, t+\tau)$ is multiplied by time-variant factor $e^{-j2\pi\alpha t}$, which makes its phase congruent and realizes coherent integration at different frequency, so the spectral peak can be showed at some cyclic frequencies for the detected signal. Usually, the SCF is two dimensional transform, in general complex valued and the parameter $\alpha$ is called cycle frequency, $f$ called spectral frequency.

*B. The Characteristic of SCF for Different Signal[7]*

*1) Noise Signal:* The SCF of noise signal has spectral valued only in the zero cyclic frequency (when $\alpha = 0$ ); moreover, in the non-zero cyclic frequency , the value of SCF is 0, as in (3).

$$S_n^f(f) = \begin{cases} N(f)/2, \alpha = 0 \\ 0 \quad\quad , others \end{cases} \quad (3)$$

*2) Interference Signal:* e.g. Amplitude modulation signal $i(t) = [i_0 + p_n(t)]\cdot\cos(2\pi f_0 t + Q_0)$ , where $p_n(t)$ (modulated noise) is a stationary random signal with zero-mean and $Q_0$ is the initial phase, $f_0$ is carried frequency. Its SCF is in (4). In terms of the SCF, it shows that the spectral peak values only happen at the zero-cyclic frequency and double-frequency. Therefore, it would not affect the characteristics of other signals in the non-zero cyclic frequency.

$$S_i^{\alpha}(f) = \begin{cases} \frac{1}{4}I_0^2\cdot e^{\pm i2\theta_0}\cdot\delta(f) + \frac{1}{4}I_0^2\cdot e^{\pm i2\theta_0}\cdot S_u(f) \quad ,\alpha = \pm 2f_0 \\ \frac{1}{4}I_0^2[\delta(f-f_0)+\delta(f+f_0)] + \frac{1}{4}[S_u(f-f_0)+S_u(f+f_0)], \alpha = 0 \\ 0 \quad\quad ,others \end{cases} \quad (4)$$

*3) Modulated Signal:* For a BPSK modulated signal, e.g. $s(t) = \sqrt{2}\cdot P\sum_{-\infty}^{+\infty} a_n r(t - nT_c - t_0)\cdot\cos(2\pi f_0 t + Q_0)$ , where P is average power, $Q_0$ is initial phase, $t_0$ is initial time migration, $f_0$ is carried frequency, $a_n$ is the random variable when random distribution is $\pm 1$ , $r(t)$ is a rectangular pulse with unit amplitude and $T_c$ is pulse width. The SCF of BPSK modulated signal is in (5), where $R(f) = T_c \sin c(fT_c)$ is a Fourier Transform of rectangular pulse $r(t)$ .

$$S_s^{\alpha}(f) = \begin{cases} \frac{P}{2T_c}[R(f+\frac{\alpha}{2}+f_0)\cdot R(f-\frac{\alpha}{2}+f_0) + R(f+\frac{\alpha}{2}-f_0)\cdot R(f-\frac{\alpha}{2}-f_0)], \alpha = \frac{K}{T_c} \\ \frac{P}{2T_c}R(f+\frac{\alpha}{2}+f_0)\cdot R(f-\frac{\alpha}{2}-f_0) \quad , \alpha = \frac{K}{T_c} - 2f_0 \\ \frac{P}{2T_c}R(f+\frac{\alpha}{2}-f_0)\cdot R(f-\frac{\alpha}{2}+f_0) \quad , \alpha = \frac{K}{T_c} + 2f_0 \\ 0 \quad ,others \end{cases} \quad (5)$$

*4) Summary*

*a)* For the noise signal, spectral values only appear in the zero-cyclic frequency.

*b)* There are peak spectral values in the zero cyclic frequency and the double cyclic frequency for the interference signals.

*c)* For the modulated signal, there are spectral peaks at $\alpha = \frac{K}{T_c}$ and $\alpha = \pm 2f_0 + \frac{K}{T_c}$ .

*d)* If the signal is added by stationary noise or nearly stationary, there is less influence to user signal at non-zero cyclic frequency. Therefore, it can be used to determine that the detected signal is a user signal or just a noise.

*e)* If the signal is added by modulated noise, it would not affect the characteristics of other signals at non-zero cyclic frequency; in addition, while the carried frequency of interference signal does not equal that of user signal, the characteristics of user signals at double cyclic frequency are not affected either. Therefore, by means of determining the detected signal is a user signal or just an interference signal, it can be determined whether there is a primary user at the detected frequency band.

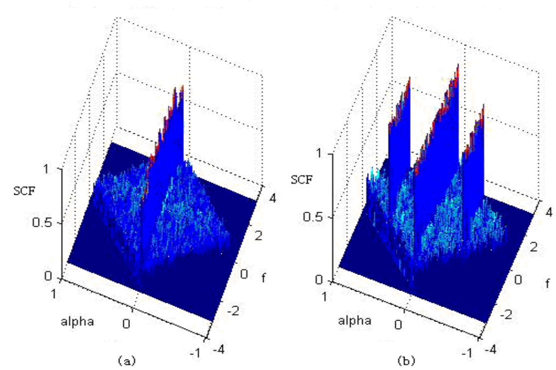


Figure 1. Spectral correlation funciton(SCF) based on FFT. (a) the SCF of noise signal, (b) the SCF of interference signal

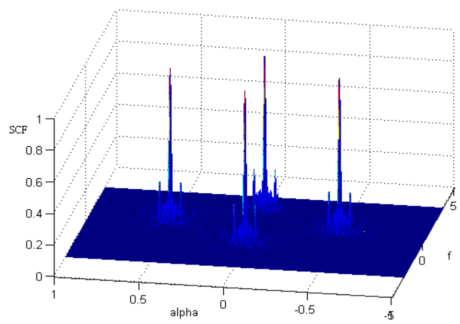


Figure 2. The SCF of the modulated signal based on FFT.

### C. Simulation Results for the SCF based on FFT

*1)* For the noise signal, peak values appear when $\alpha = 0$. For the interference signal, let $f_0 = 0.25Fs$, the peak appears while $\alpha = 0$ and $\alpha = \pm(2f_0)/Fs = \pm 0.5$, as showed in Fig.1.

*2)* For the modulated signal, let $f_0 = 0.25Fs$ and $T_c = 2Fs$; when $K = 0$, the spectral peaks appear at $\alpha = 0$ and $\alpha = \pm(2f_0)/Fs = \pm 0.5$; when $K = \pm 1$, the peak values happen at $\alpha = \pm 0.5 \pm 1/T_c$ and $\alpha = \pm 1/T_c$, as showed in Fig.2.

*3)* Based on the characteristics of SCF for different signals, the spectrum sensing is executed and the result is showed at Fig.6 for comparison with the result of proposed algorithm.

## III. Goertzel Algorithm and its Application for Spectrum Sensing

Actually, the algorithm of Goertzel is a kind of recursive FFT . The recursive procedure of Goertzel algorithm is in (6) (7) (8). The (8) is right the value of DFT for $k$ .

$$U_k(n) = 2\cos(\frac{2\pi k}{N})u_k(n-1) - u_k(n-2) + x(n) \qquad U_k(-1) = 0, U_k(-2) = 0 \quad (6)$$

$$y_k(n) = u_k(n) - w_N^k u_k(n-1) \quad (7)$$

$$X(k) = y_k(N) \quad (8)$$

### A. Apply the Goertzel to SCF

According to the definition of SCF, the CAF is obtained by $R_x(t, t+\tau)$ multiplied factor of $e^{-j2\pi\alpha t}$, and then take mean for t; SCF is the Fourier Transform of CAF. When simulation, it can be carried out as: firstly, Fourier Transform for detected signal is implemented, as in (9); then doing a shift of $\alpha$ units in frequency domain; finally, the operations of autocorrelation and taking mean are carried out to get the result of SCF, the procedure are showed in (10) (11). The Fourier Transform in the definition of SCF can be achieved by utilizing the Goertzel Algorithm.

$$u(K) \triangleq \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}} \qquad v(K) \triangleq \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}} \quad (9)$$

$$R_{uv}^0(\tau) \triangleq \left\langle u(K+\frac{\tau}{2}) v^*(K-\frac{\tau}{2}) \right\rangle_t = R_x^\alpha(\tau) \quad (10)$$

$$S_{uv}^0(f) \triangleq \int_{-\infty}^{\infty} R_{uv}^0(\tau) e^{-j2\pi f\tau} d\tau = S_s^\alpha(f) \quad (11)$$

### B. Application Scenario

Spectrum fast-searching is considered to be an important application such as in the Multi-channel Media Access Control (MMAC) protocol of Ad Hoc. The working flow chart [8] is given in Fig.3; it demonstrates that, its first step is to achieve nodes competing for sending beacon and available channels fast scanning. The fast scanning requires fast speed and high accuracy, so the technology of spectrum sensing based on Goertzel algorithm can be used.

### C. Comparing the Computation of FFT with that of Goertzel

The computation in the FFT is correlative with the length of signal, for doing FFT of a N points signal, it requires to accomplish $2N\log_2 N$ times real multiplications and $3N\log_2 N$ times real additions. However, with the Goertzel algorithm, for a signal of N points, if the spectral values of L points are required, its computation is $N \times L$. As a result, when $L < \log_2 N$, the Goertzel algorithm is superior in computational simplicity.

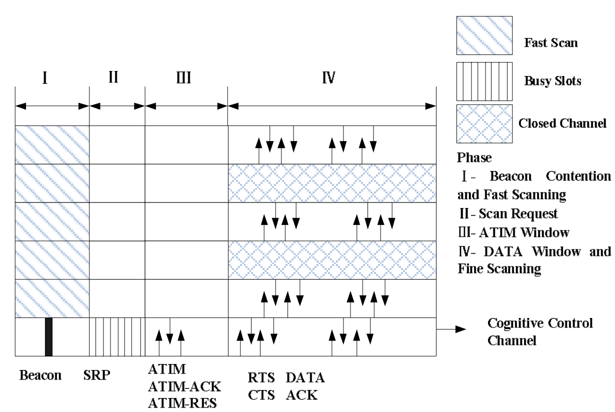


Figure 3. Working flow chart of MMAC.

In the simulation, a signal of 1024 points is applied, assuming to detect whether there is a primary user signal at ten frequency point (e.g. $f = kf_0, k = 1,2,...10$ ), for the FFT, it requires to do 20480 times real multiplications and 30720 times real additions, while just 10240 times are required for the Goertzel algorithm. If just detecting whether there is a primary user at a frequency, 1024 times computations are enough. However, if points of frequency $2\log_2 N$ , the FFT algorithm is more efficient.

## IV. SIMULATION AND PERFORMANCE ANALYSIS

Let the input signal be a 1024 points signal; assuming that ten frequency points should be detected whether there are primary users; applying the Goertzel algorithm to the SCF. For the interference signal, let $f_0 = 0.25Fs$ , as in the Fig.4, the black lines represent the spectral values, which appear at $\alpha = 0$ and $\alpha = \pm(2f_0)/Fs = \pm 0.5$ . For the modulated signal, let $f_0 = 0.25Fs$ and $T_c = 2Fs$ , the values of $\alpha$ for peak values are the same as the algorithm based on FFT, as Fig.5 showed.

Fig.6 shows the Percentage of Missed Detect (PMD) based on FFT and Goertzel algorithm. According to the simulation curve, it nearly achieves zero-PMD when $SNR \geq 2$ for the FFT-based sensing method. As to the PMD based on Goertzel algorithm, it equals 0.01 when SNR=2, and it nearly achieves zero-PMD when $SNR \geq 3$ , which is close to the result of the FFT. For the values of SNR, the transitional area (e.g. SNR=0 and SNR=1) can be avoided in practice, so the cyclostationary feature detector is proved to realize high detecting accuracy at low Signal to Noise Ratio (SNR). The result also reveals that the Goertzel-based algorithm keeps good accuracy as the FFT-based algorithm, but it is much more computationally efficient, which is proved at the front part of the paper.

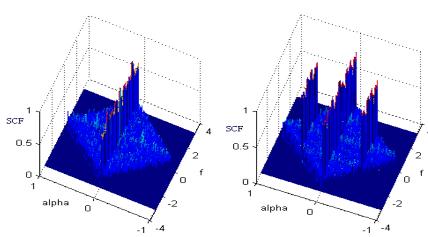


Figure 4. SCF of the noise and interference signal based on Goertzel.

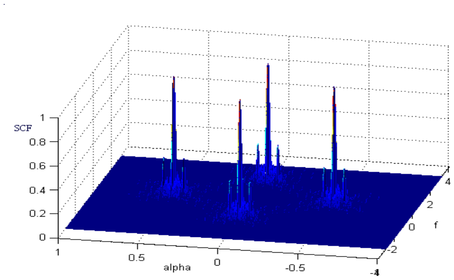


Figure 5. SCF of modulated signal based on Goertzel.

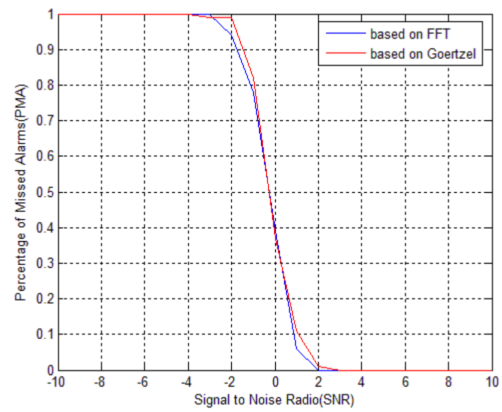


Figure 6. PMD-SNR based on FFT and Goertzel

## V. CONCLUSIONS

The drawbacks of various spectrum sensing technologies for single node restrict their applications: (1) the cyclostationary feature detector is highly accurate and can differentiate the types of signal, but its complexity might diminish its advantages. (2) The energy detector is simply in implementation but suffers from a bad performance at low SNR. (3) The matched filter requires the priori knowledge of primary user signal, which is difficult to get when implementing. Many forces are drawing researchers to improve the spectrum sensing algorithm for fast-searching and high accuracy. This article has applied the Goertzel algorithm to the cyclostationary feature detector to realize effective and accurate spectrum sensing. The authors believe that the new algorithm is important for the success of cognitive radio and more attention needs to be devoted to such areas.

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