Nonparametric Cyclic Correlation Based Detection for Cognitive Radio Systems

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***Abstract*—In this paper a nonparametric cyclic correlation estimator based on complex generalization of sign function is proposed. Theoretical justification for detecting cyclostationary signals is provided. Asymptotic distribution of the estimator under null hypothesis is established. Constant false alarm rate (CFAR) tests based on estimated sign cyclic correlation are derived for single-user and collaborative spectrum sensing. Simulation experiments comparing the proposed method with cyclostationarity based spectrum sensing methods employing the classical cyclic correlation estimator are performed. Nonparametric statistics provide additional robustness when noise statistics are non-Gaussian or not fully known. Simulations demonstrate the reliable performance and robustness of the proposed nonparametric spectrum sensing method in both Gaussian and non-Gaussian noise environments.**

I. Introduction

Underutilization of many parts of radio frequency spectrum has increased the interest in dynamic spectrum allocation. Cognitive radios have been suggested as an enabling technology for dynamic allocation of spectrum resources. Spectrum sensing used for finding free spectrum is a key task in cognitive radio systems. It enables agile spectrum use and interference control. Recently, there has been increasing interest on developing low complexity and reliable spectrum sensing methods for detecting the presence of primary users. Collaborative sensing by multiple secondary users allows for mitigating the effects of shadowing and fading.

It is important to consider the robustness of the spectrum sensing algorithms as well. Motivation for this is that several measurement studies have shown that in many outdoor and indoor frequency bands the noise distribution has heavier tails than Normal distribution. For example, in [1] indoor measurements in the industrial, scientific, and medical (ISM) bands showed the impulsive nature of the noise and interference due to, e.g., microwave ovens and electrical motors in electrical devices, such as elevators, etc. As an example of outdoor measurements, impulsive noise measurements in a digital television band have been reported in [2]. For more experimental measurement results, see [1], [2], and the references therein. Moreover, there may be multiple interferers present contaminating the primary signal we wish to detect.

Some of the most promising spectrum sensing algorithms exploit the cyclostationarity property of communication signals. Cyclostationarity based spectrum sensing algorithms have been proposed in [3], [4], [5]. These algorithms do not require any explicit assumptions on the data or noise distributions. They are based solely on the asymptotic distributions of the cyclic correlation estimators. Nevertheless, these algorithms are not necessarily highly robust. For example, in case the noise distribution has heavier tails than Normal distribution, the convergence of the classical cyclic correlation estimator slows down and the performance of the algorithms deteriorates.

Robust cyclic correlation estimators have been considered in [6] where estimators stemming from M-estimation are proposed. Both the proposed M-estimators as well as the trimmed mean estimator are found to reduce the influence of outliers (highly deviating observations). Simulation experiments demonstrate significant improvement of performance of the robust estimators in the face of non-Gaussian, heavy tailed noise.

In this paper a cyclic correlation estimator based on complex generalization of the sign function is proposed. It is shown that the cyclostationarity property used in the detector is preserved under sign function. Asymptotic distribution of the estimator under the null hypothesis is derived. The test statistics for single-user and collaborative spectrum sensing schemes are proposed. The proposed methods are based on nonparametric statistics making them highly attractive in real applications where noise and interference statistics may not be fully known. No additional nuisance parameters such as scale need to be estimated unlike in the robust methods in [6]. Furthermore, nonparametric detectors achieve a fixed false alarm rate under all conditions satisfying the nonparametric null hypothesis.

Introduction to main techniques of nonparametric signal detection has been given in [7]. Different multivariate sign and rank concepts, corresponding covariance matrices and their statistical properties have been presented in [8]. Complex sign function employed in this paper could be considered to be the bivariate spatial sign function. Cyclic spectrum estimation

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algorithms based on correlating signal with its sign have been considered in [9] with the interest of reducing computational complexity. The signs of the real and imaginary parts are considered separately there.

This paper is organized as follows. Section II introduces the sign cyclic correlation estimator. The distribution of the sign cyclic correlation for a complex noise process is determined in Section III. In Section IV the testing problem is formulated as a hypothesis test. The test statistics and their distributions under the null hypothesis are defined. Simulation examples are given in Section V. Concluding remarks are given in Section VI. Theoretical results showing that the proposed detector is applicable for cyclostationary signals are provided in the Appendix.

II. Sign cyclic correlation

The spatial sign function for complex valued data x is defined as [7], [8]

$$S(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0. \end{cases} \tag{1}$$

We define the sign cyclic correlation estimator as

$$\hat{R}_S(\alpha, \tau) = \frac{1}{M} \sum_{t=1}^{M} S(x(t)) S(x^*(t+\tau)) e^{-j2\pi\alpha t}, \tag{2}$$

where M is the number of observations and α is the cyclic frequency. In the Appendix it is shown that the periodicity of autocorrelation function is preserved for a circularly symmetric complex Gaussian process in spite of the sign function.

In (2) it has been assumed that the signal has zero mean. Otherwise an estimate for the mean (using a robust estimator) has to be obtained and removed from the received signal before employing the estimator.

A symbol rate estimator can now be defined as

$$\alpha_0 = \arg \max_{\alpha \in (0, \frac{1}{2}]} ||\hat{\boldsymbol{r}}_S(\alpha)||^2 \tag{3}$$

where $||\cdot||$ denotes the Euclidean vector norm and $\hat{\boldsymbol{r}}_S(\alpha)$ is a vector containing the estimated sign cyclic correlations for different time delays $\tau_1, \ldots, \tau_N$, i.e.,

$$\hat{\boldsymbol{r}}_S(\alpha) = [\hat{R}_S(\alpha, \tau_1), \ldots, \hat{R}_S(\alpha, \tau_N)]^T. \tag{4}$$

However, in order to define a CFAR test for the presence of cyclostationarity at a given cyclic frequency, the distribution of the estimator needs to be established. In the next section, the distribution of the sign cyclic correlation estimator is determined for independent and identically distributed (*i.i.d.*) zero-mean circular noise process. Nonparametric performance is achieved for all *i.i.d.* circular zero-mean noise probability distribution functions. Note, however, that circularity is not required from the primary user signal.

III. Distribution of the sign cyclic correlation estimator

The number of observations M is typically large (in the order of several thousands) in cognitive radio applications. Hence, applying the central limit theorem to infer the distribution of the sign cyclic correlation estimator is well justified in a realistic scenario. According to the central limit theorem the distribution of the sign cyclic correlation estimator approaches Normal distribution as M goes to infinity. Thus, a Normal distribution approximation can be used for large M. Consequently, only the mean and the variance of the estimators need to be determined in order to fully specify the asymptotic distribution. Validity of the central limit theorem approximation will be assessed by simulations in Section V.

In the following it is assumed that $x(t) = n(t)$ where $n(t)$ is *i.i.d.* zero-mean circular noise process. That is, only noise is considered to be present. In that case, the mean of $\hat{R}_S(\alpha, \tau)$ is given by (assuming that $\tau \neq 0$)

$$\begin{aligned} E[\hat{R}_S(\alpha, \tau)] &= \frac{1}{M} \sum_{t=1}^{M} E[S(n(t)) S(n^*(t+\tau))] e^{-j2\pi\alpha t} \\ &= \frac{1}{M} \sum_{t=1}^{M} E[S(n(t))] E[S(n^*(t+\tau))] e^{-j2\pi\alpha t} \\ &= 0, \ \forall\alpha, \ \forall\tau \neq 0 \end{aligned} \tag{5}$$

where the second equality follows from independence of the noise samples. The last equality follows from the fact that noise is circular, i.e., $S(n(t)) = e^{j\theta}$ where θ has a uniform distribution between 0 and 2π.

Since the mean of $\hat{R}_S(\alpha, \tau)$ is zero, the variance of $\hat{R}_S(\alpha, \tau)$ is given by

$$\begin{aligned} &\mathrm{Var}(\hat{R}_S(\alpha, \tau)) = E[(\hat{R}_S(\alpha, \tau))(\hat{R}_S(\alpha, \tau))^*] \\ &= E\left[\left(\frac{1}{M} \sum_{t=1}^{M} S(n(t)) S(n^*(t+\tau)) e^{-j2\pi\alpha t} \right) \cdot \right. \\ &\quad \left. \cdot \left(\frac{1}{M} \sum_{t=1}^{M} S(n(t)) S(n^*(t+\tau)) e^{-j2\pi\alpha t} \right)^* \right] \\ &= \frac{1}{M^2} \sum_{t=1}^{M} \sum_{k=1}^{M} E[S(n(t)) S(n^*(k)) \cdot \\ &\quad \cdot S(n^*(t+\tau)) S(n(k+\tau)))] e^{-j2\pi\alpha(t-k)} \\ &= \frac{1}{M^2} \left(\sum_{t=1}^{M} E[|S(n(t))|^2 |S(n^*(t+\tau))|^2] \right. \\ &\quad + \sum_{t=1}^{M} \sum_{\substack{k=1 \\ k \neq t}}^{M} \underbrace{E[S(n(t)) S(n^*(k)) S(n^*(t+\tau)) S(n(k+\tau))]}_{=0} \cdot \\ &\quad \left. \cdot e^{-j2\pi\alpha(t-k)} \right) \\ &= \frac{1}{M}, \ \forall\alpha, \ \forall\tau \neq 0. \end{aligned} \tag{6}$$

IV. Hypothesis testing

Testing for the presence of a second-order cyclostationary signal can be seen as testing whether the estimated sign cyclic correlation $\hat{R}_S(\alpha, \tau)$ is different from zero or not for the cyclic frequencies of the signal. Hence, the hypothesis testing problem for testing the presence of a second-order cyclostationary signal for a given cyclic frequency α may be formulated as [3]

$$\begin{aligned} H_0 &: \hat{\boldsymbol{r}}_S(\alpha) = \boldsymbol{\epsilon}(\alpha),\ \forall \{\tau_n\}_{n=1}^N \\ H_1 &: \hat{\boldsymbol{r}}_S(\alpha) = \boldsymbol{r}_S(\alpha) + \boldsymbol{\epsilon}(\alpha),\ \text{for some } \{\tau_n\}_{n=1}^N \end{aligned} \tag{7}$$

where $\boldsymbol{\epsilon}(\alpha)$ is the estimation error and $\hat{\boldsymbol{r}}_S(\alpha)$ is a vector containing the estimated sign cyclic correlations for different time delays $\tau_1, \ldots, \tau_N$,

$$\hat{\boldsymbol{r}}_S(\alpha) = [\hat{R}_S(\alpha, \tau_1), \ldots, \hat{R}_S(\alpha, \tau_N)]^T. \tag{8}$$

From Section III, it follows that under the null hypothesis (assuming an *i.i.d.* circularly symmetric noise process)

$$\hat{R}_S(\alpha, \tau) \sim N_C(0, \frac{1}{M}),\ \forall \alpha,\ \forall \tau \neq 0 \tag{9}$$

where $N_C(\cdot, \cdot)$ denotes the complex Normal distribution.

Now, define the test statistic for the sign cyclic correlation based test for a single secondary user (SU) as

$$\lambda = M||\hat{\boldsymbol{r}}_S(\alpha)||^2. \tag{10}$$

The null hypothesis is rejected if $\lambda > \gamma$ where γ is the test threshold defined by $p(\lambda > \gamma | H_0) = p_{fa}$. Here p_{fa} is the constant false alarm rate parameter of the test.

It follows that under the null hypothesis λ is chi-square distributed with N complex degrees of freedom. The probability distribution function of a chi-square distributed random variable with N complex degrees of freedom is given by

$$f(z) = \frac{1}{(N-1)!} z^{N-1} e^{-z},\ z > 0 \tag{11}$$

which is a gamma distribution with integer parameters N and 1.

The proposed detector is a single cycle detector. However, wireless communication signals typically exhibit cyclostationarity at multiple cyclic frequencies. Multicycle extensions similar to the ones proposed in [4] may be obtained as well in order to take into account the rich information present in wireless communication signals.

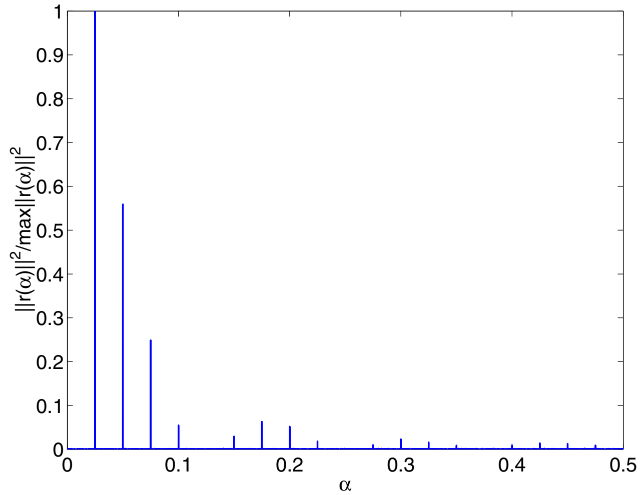
A. Multiple secondary users

Assuming that the test statistics of the secondary users are independent given H_0 or H_1, the single-user test statistics can be combined as follows

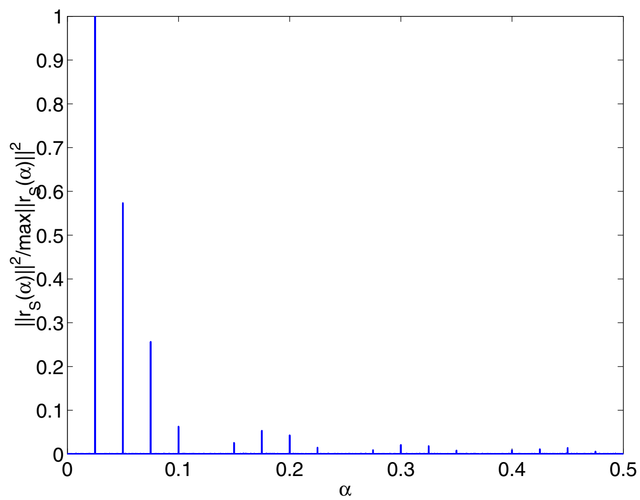
$$\lambda_L = \sum_{i=1}^{L} \lambda^{(i)} \tag{12}$$

where L is the number of collaborating secondary users and $\lambda^{(i)}$ denotes the sign cyclic correlation test statistic of the ith user. Since the single-user test statistic λ has a quadratic form it is the log-likelihood under the null hypothesis. Hence, the sum of the single-user test statistics in (12).

Under the null hypothesis λ_L is chi-square distributed with LN complex degrees of freedom.



(a) Classical cyclic correlation



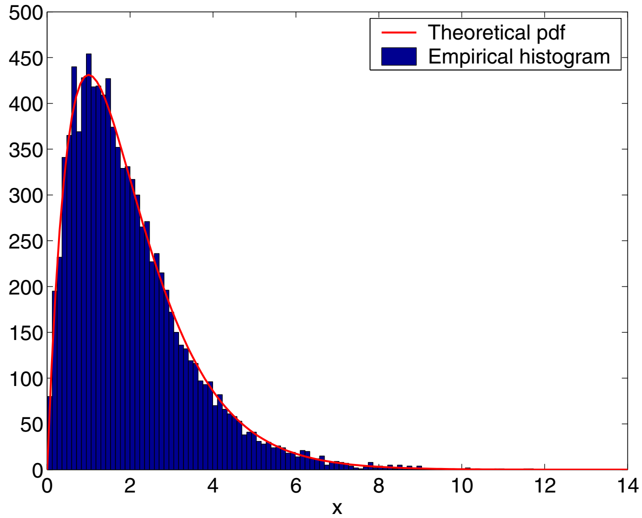
(b) Sign cyclic correlation

Figure 1. Normalized squared modulus of the (a) classical cyclic correlation (b) sign cyclic correlation. The signal is an OFDM signal with symbol frequency of 0.025. The sign non-linearity preserves the cyclic frequencies.

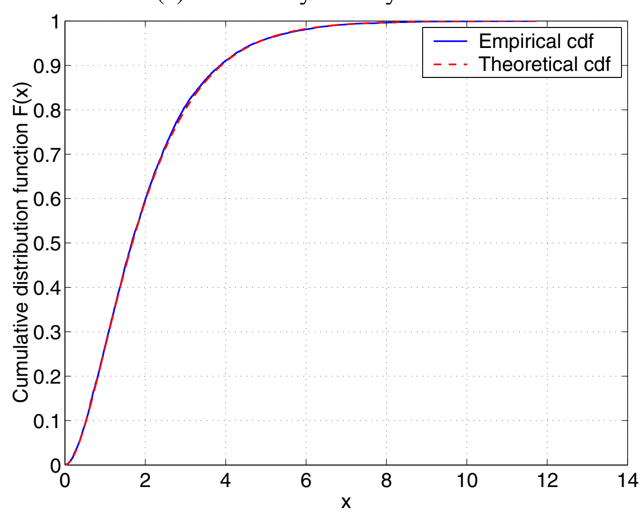
V. Simulation examples

In this section the performance of the proposed single cycle sign cyclic correlation based detector is compared to the single cycle detectors proposed in [4] for both single and multiple secondary users. However, first we show using an example that sign non-linearity preserves the cyclic frequencies for an OFDM signal. Fig. 1 shows the normalized squared modulus of the cyclic correlation for an OFDM signal for classical and sign estimators. It can be seen that the cyclic frequencies are preserved by the sign non-linearity.

Secondly, the validity of the central limit theorem approximation is considered. Fig. 2 plots the probability density



(a) Probability density function



(b) Cumulative distribution function

Figure 2. Validity of the central limit theorem approximation. (a) Probability density function and (b) cumulative distribution function of the test statistic $\lambda = M||\hat{r}_S(\alpha)||^2$ for a contaminated Gaussian process $0.95N_C(0, \sigma^2) + 0.05N_C(0, 25\sigma^2)$. The number of observations $M = 100$. Measured empirical distribution is very accurately approximated by the distribution derived using central limit theorem.

and cumulative distribution functions of the test statistic $\lambda = M||\hat{r}_S(\alpha)||^2$ for a circularly symmetric i.i.d. contaminated complex Gaussian process $0.95N_C(0, \sigma^2)+0.05N_C(0, 25\sigma^2)$. The number of observations $M = 100$. Two random lags between 1 and 20 observations and a random cyclic frequency in the interval $[0.05, 0.5]$ were employed. The histogram and empirical cumulative distribution functions have been obtained from 10000 experiments. Measured empirical distribution is very accurately approximated by the derived theoretical distribution thus confirming the central limit theorem approximation.

The first test signal is an orthogonal frequency division multiplexing (OFDM) signal. The OFDM signal is a DVB-T signal with a Fast-Fourier transform (FFT) size $N_{FFT} = 8192$

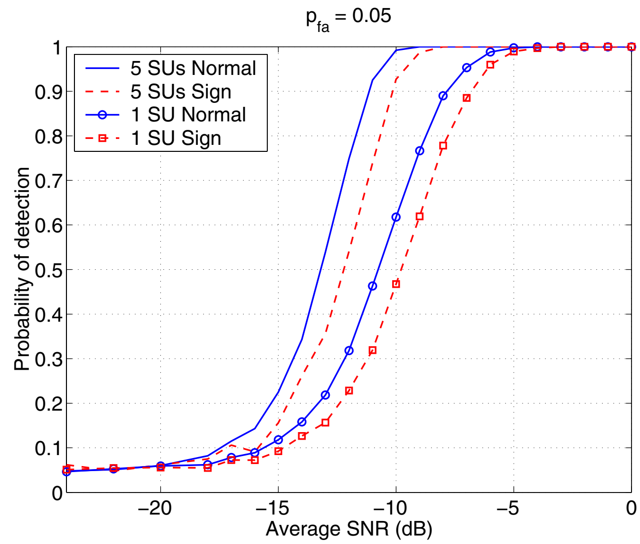


Figure 3. Probability of detection vs. Average SNR (dB) in a Rayleigh fading channel for 1 and 5 secondary users. Additive noise is Gaussian. Signal is an OFDM signal (DVB-T). Sign cyclic correlation based detectors suffer small performance degradation compared to methods based on classical cyclic correlation estimator in Gaussian noise for the OFDM signal.

and a cyclic prefix of $N_{cp} = 1024$ samples. The symbol length is defined as $T_S = N_{FFT} + N_{cp}$. Number of employed subcarriers is 6817. Subcarrier modulation was 64-QAM. The length of the signal is 3 OFDM symbols ($\approx$ 3 ms). The signal was sampled at the Nyquist rate. Thus, the oversampling factor with respect to the symbol rate is $N_{FFT} + N_{cp}$.

OFDM signal is cyclostationary with respect to the symbol frequency. Thus, the detection is performed at the symbol frequency. In addition, all the detectors employ two time lags $\pm N_{FFT}$. The cyclic autocorrelation of the OFDM signal peaks for these time lags [10].

Fig. 3 depicts the performance of the detectors in a Rayleigh fading channel (ETSI EN 300 744 V1.5.1 (2004-11)) and additive white Gaussian noise as a function of the average signal-to-noise ratio (SNR). The SNR is defined as $\mathrm{SNR} = 10\log_{10}\frac{\sigma_x^2}{\sigma_n^2}$ where σ_x^2 and σ_n^2 are the variances of the transmitted signal and the noise, respectively. The channel is normalized to have an expected gain of 1. False alarm rate is 0.05 (the same false alarm rate is used in all of the following simulations, as well). All the simulation curves in the figures are averages over 1000 independent experiments. It can be seen that employing the sign non-linearity causes performance degradation.

Fig. 4 shows the performances for the DVB-T signal in a more impulsive noise environment. The noise has a contaminated Gaussian distribution $0.95N_C(0, \sigma^2) + 0.05N_C(0, 25\sigma^2)$. The robustness of the non-parametric sign cyclic detector compared to the normal cyclic detector can be clearly seen.

The second test signal is a quadrature phase shift keying (QPSK) signal with root raised-cosine pulse shaping with excess bandwidth of 0.2. The length of the signal was 1500

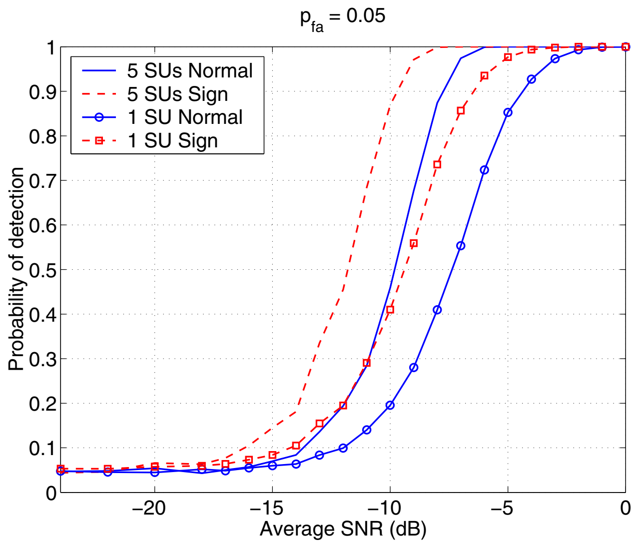


Figure 4. Probability of detection vs. Average SNR (dB) in a Rayleigh fading channel for 1 and 5 secondary users. Additive noise has a contaminated Gaussian distribution $0.95N_C(0,\sigma^2) + 0.05N_C(0, 25\sigma^2)$. SNR is defined with respect to σ^2. Signal is an OFDM signal (DVB-T). The sign cyclic correlation based detectors are more robust against impulsive noise.

symbols. The signal was four times oversampled. Detection was performed at the symbol frequency using the following time delays $\pm 1, \pm 2$ samples.

Figs. 5 and 6 depict the performances of the detectors for the QPSK signal in a frequency flat Rayleigh fading channel for Gaussian and contaminated Gaussian $0.95N_C(0,\sigma^2) + 0.05N_C(0, 25\sigma^2)$ noise distributions, respectively. In this case, the sign cyclic correlation detector outperforms the normal cyclic detector even in the non-impulsive noise environment. This is due to the fact that QPSK is a constant modulus signal. Hence, hard-limiting the amplitude does not result in information loss. Furthermore, unlike sign cyclic correlation based detector the conventional cyclic detector requires estimation of the covariance matrix of the estimator. This can be considered as nuisance parameter whose estimation may result in a small performance loss, especially, for small number of observations. In the impulsive noise environment the robustness of the sign cyclic correlation based detector is clearly observed.

VI. Conclusion

In this paper a nonparametric sign cyclic correlation based spectrum sensing method for cognitive radio systems has been proposed. It has been shown that the sign cyclic correlation function remains periodic for circularly symmetric complex Gaussian processes which makes it applicable to cyclostationary signals. Asymptotic distributions of the test statistics under the null hypothesis have been derived. Tests for single-user and collaborative detection have been developed. Simulation experiments show that the proposed nonparametric spectrum sensing method has highly robust performance in non-Gaussian impulsive noise environments as well as very good performance in Gaussian noise environments.

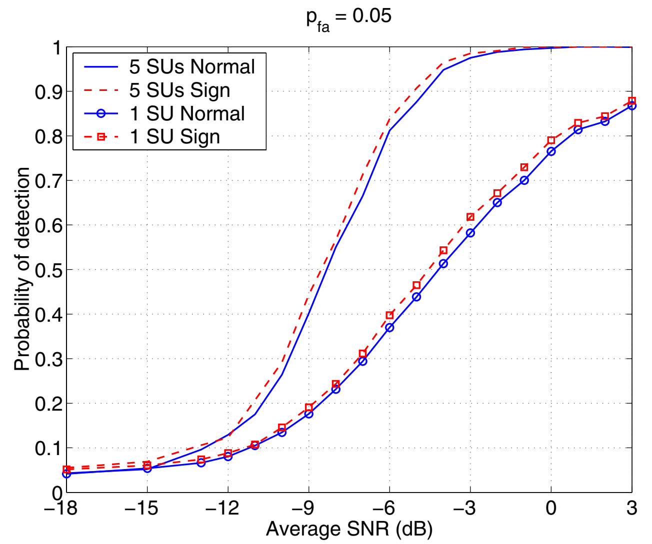


Figure 5. Probability of detection vs. Average SNR (dB) in a frequency flat Rayleigh fading channel for 1 and 5 secondary users. Additive noise is Gaussian. Signal is a QPSK signal with root raised-cosine pulse shaping. Nonparametric sign cyclic correlation based detectors slightly outperform the methods based on classical cyclic correlation estimator even in Gaussian noise for the QPSK signal.

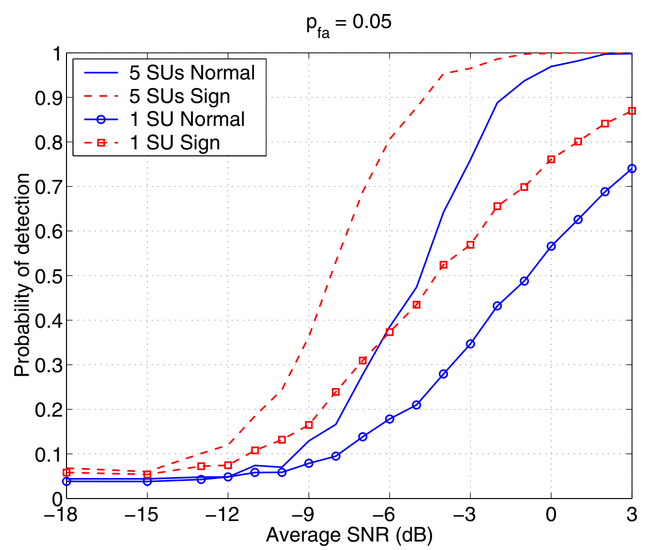


Figure 6. Probability of detection vs. Average SNR (dB) in a frequency flat Rayleigh fading channel for 1 and 5 secondary users. Additive noise has a contaminated Gaussian distribution $0.95N_C(0,\sigma^2) + 0.05N_C(0, 25\sigma^2)$. SNR is defined with respect to σ^2. Signal is a QPSK signal with root raised-cosine pulse shaping. Due to the impulsive nature of the noise the robust sign cyclic correlation based spectrum sensing methods clearly outperform the methods based on classical cyclic correlation estimator.

Appendix

Considering the effects of spatial sign non-linearity for a general cyclostationary process is not an easy task. However, certain results that apply for high SNR cases can be defined for certain type of inputs. Here, we will consider complex Gaussian input process. Although many of the communication signals may not be Gaussian, the Gaussian process is still a very important special case. For example, an OFDM signal is well approximated by a Gaussian process for a sufficiently large number of subcarriers.

Assume that the input signal to the hard-limiting sign non-linearity is a zero-mean circularly symmetric complex Gaussian process. Then it follows that the normalized complex autocorrelation function of the hard-limited process $y(t) = S(x(t))$ is given by [11], [12]

$$\begin{aligned}\rho_{yy}(t,\tau) &= E[S(x(t))S^*(x(t+\tau))] \\ &= \frac{\pi}{4}\rho_{xx}(t,\tau)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;|\rho_{xx}(t,\tau)|^2\right),\end{aligned} \tag{13}$$

where $\rho_{xx}(t,\tau)$ is the normalized autocorrelation function of $x(t)$, i.e., $\rho_{xx}(t,\tau) = E[x(t)x^*(t+\tau)]/E[x(t)x^*(t)]$. ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gaussian hypergeometric function given by the following series representation

$$\begin{aligned}{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;|\rho|^2\right) &= \frac{\Gamma(2)}{\Gamma^2(\frac{1}{2})}\sum_{n=0}^{\infty}\frac{\Gamma^2(\frac{1}{2}+n)}{\Gamma(2+n)}\frac{|\rho|^{2n}}{n!} \\ &= 1+\frac{1}{8}|\rho|^2+\frac{3}{64}|\rho|^4+\frac{25}{1024}|\rho|^6+\ldots\end{aligned} \tag{14}$$

From (13) it can be seen that the phase of the autocorrelation function is unaltered by the spatial sign non-linearity. Furthermore, we can make the following approximations:

$$\rho_{yy}(t,\tau) \approx \rho_{xx}(t,\tau) \tag{15}$$

when $\rho_{xx}(t,\tau)$ is close to unity, and

$$\rho_{yy}(t,\tau) \approx \frac{\pi}{4}\rho_{xx}(t,\tau) \tag{16}$$

when $\rho_{xx}(t,\tau)$ is close to zero.

Hence, the periodicity of the autocorrelation function of a circularly symmetric complex Gaussian process is preserved by the spatial sign non-linearity (although the autocorrelation may be attenuated). Note, however, that the spatial non-linearity may also cause periodicities that do not exist in the original autocorrelation function due to the second and higher terms in (14).

References

[1] K. L. Blackard, T. S. Rappaport, and C. W. Bostian, "Measurements and Models of Radio Frequency Impulsive Noise for Indoor Wireless Communications," *IEEE J. Select. Areas Commun.*, vol. 11, no. 7, pp. 991–1001, Sep. 1993.

[2] M. G. Sanchéz, L. de Haro, M. Calvo, A. Mansilla, C. Montero, and D. Oliver, "Impulsive Noise Measurements and Characterization in a UHF Digital TV Channel," *IEEE Trans. Electromagn. Compat.*, vol. 41, no. 2, pp. 124–136, May 1999.

[3] A. V. Dandawaté and G. B. Giannakis, "Statistical Tests for Presence of Cyclostationarity," *IEEE Trans. Signal Process.*, vol. 42, no. 9, pp. 2355–2369, Sep. 1994.

[4] J. Lundén, V. Koivunen, A. Huttunen and H. V. Poor, "Spectrum Sensing in Cognitive Radios Based on Multiple Cyclic Frequencies," in *Proc. 2nd Int. Conf. on Cognitive Radio Oriented Wireless Networks and Communications*, Orlando, FL, USA, Jul. 31–Aug. 3, 2007.

[5] J. Lundén, V. Koivunen, A. Huttunen and H. V. Poor, "Censoring for Collaborative Spectrum Sensing in Cognitive Radios," in *Proc. 41st Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Nov. 4-7, 2007.

[6] T. E. Biedka, L. Mili, and J. H. Reed, "Robust Estimation of Cyclic Correlation in Contaminated Gaussian Noise," in *Proc. 29th Asilomar Conf. on Signals, Systems, and Computers*, vol. 1, Pacific Grove, CA, USA, Oct. 30–Nov. 2, 1995, pp. 511–515.

[7] S. A. Kassam, "Nonparametric Signal Detection," in *Advances in Statistical Signal Processing*, H. V. Poor and J. B. Thomas, Eds., vol. 2, pp. 66–91, JAI Press Inc., 1993.

[8] S. Visuri, V. Koivunen, and H. Oja, "Sign and Rank Covariance Matrices," *J. Statistical Planning and Inference*, vol. 91, no. 2, pp. 557–575, Dec. 2000.

[9] W. A. Gardner, R. S. Roberts, "One-Bit Spectral-Correlation Algorithms," *IEEE Trans. Signal Process.*, vol. 41, no. 1, pp. 423–427, Jan. 1993.

[10] M. Öner and F. Jondral, "Air Interface Identification for Software Radio Systems," *Int. J. Electron. Commun.*, vol. 61, no. 2, pp. 104-117, Feb. 2007.

[11] I. S. Reed, "On the use of Laguerre Polynomials in Treating the Envelope and Phase Components of Narrow-Band Gaussian Noise," *IRE Trans. Inform. Theory*, vol. IT–5, pp. 102–105, Sep. 1959.

[12] G. Jacovitti and A. Neri, "Estimation of the Autocorrelation Function of Complex Gaussian Stationary Processes by Amplitude Clipped Signals," *IEEE Trans. Inform. Theory*, vol. 40, no. 1, pp. 239–245, Jan. 1994.