CycloStationary Detection for Cognitive Radio with Multiple Receivers

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***Abstract*—In order to detect the presence of the primary user signal, spectrum sensing is a fundamental requirement to achieve the goal of cognitive radio (CR). This ensures the efficient utilization of the spectrum. Cyclostationary detection is the preferred technique to detect the primary users receiving data within the communication range of a CR user at very low SNR. In this work, we investigate the performance of maximal ratio combining (MRC) based cyclostationary detector to detect a primary user. Using the proposed detection technique, we observed that the MIMO cognitive radio enjoys 6 dB SNR advantage over single antenna when using four receive antennas for all values of probability of detection.**

I. Introduction

The demand of radio-frequency (RF) spectrum is increasing to support the user needs in wireless communication. RF spectrum is scarce resource and requires efficient utilization. Cognitive radio [1], inclusive of software-defined radio, has been proposed as a means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes. The intelligence of cognitive radio (CR) lies on three basic functions: the ability to sense the outside environment; the capacity to learn, ideally in both supervised and unsupervised modes; and finally, the capability to adapt within any layer of the radio communication system [2]. Cognitive radio begins transmission on a piece of spectrum found not utilized by the primary user (PU). This is ensured by sensing the radio environment whether or not PU signal is present and operating on empty spectrum. Subsequently, the transmission from CR should not cause harmful interference to primary user.

To achieve this goal of CR, it is a fundamental requirement that the cognitive user performs spectrum sensing to detect the presence of the PU signal. The sensing of radio environment to determine the presence of primary user is a challenging problem as the signal is attenuated by fading wireless channel. This results in low signal-to-noise ratio (SNR) condition at the CR input, and makes CR susceptible to hidden node problem, wherein CR fails to detect PU signal and begins transmission, thereby, causing potential interference to the PU. To minimize the occurrence of this problem, a detection technique has to achieve a probability of detection close to unity for a specified probability of false alarm and a given SNR.

Many signal detection techniques can be used in spectrum sensing, such as matched filtering, energy detection, and PU signal feature detection with the cyclostationary feature [3]. In [4], author discussed about the detection of the deterministic signal over a flat band-limited Gaussian noise channel. The improvement of PU signal detection performance using different diversity combining schemes are presented in [5] using energy detector.

On the other hand, cyclostationary technique performs better in low SNR region, however with increased complexity compared to energy detection. A signal is said to be cyclostationary (in the wide sense) if its autocorrelation is a periodic function of time with some period. Statistical tests for the presence of cyclostationarity are presented in [6], using both time and spectral domain statistics. In [7], authors have proposed a method in which signatures are embedded in the OFDM signal for distributed rendezvous in dynamic spectrum access networks. In [8] they have studied the performance of cyclostationary signatures in fading channels. In [9], air interface recognition is explored using cyclostationary properties of different air interface signals. In [10], a method has been proposed for detecting a cyclostationary signal using multiple cyclic frequencies. In this work, we study the performance of the detection of the cyclostationary signal using multiple antennas in gaussian noise channel, using time domain cyclic autocorrelation estimates. We demonstrate the theoretical detection performance gains that can be obtained through appropriate signal processing with multiple antenna CRs in comparison to single antenna CRs. Statistics required for performance comparison are obtained from [10].

The rest of the paper is organized as follows. Section II presents the cyclostationary detection technique for multiple cyclic frequency for single antenna CR receiver [10]. The analysis is extended for multiple antenna CR receiver in Sec. III. The numerical results are discussed in Sec. IV. Finally, Sec. V concludes the present work.

II. Cyclostationary Detection in Single Antenna CR Receiver

In this section, we describe the cyclostationary detection technique to detect the PU signal. In these techniques, the cyclic statistics of the signal are obtained from an oversampled signal with respect to the symbol rate or by receiving the signal through multiple receiver. Statistical tests for the cyclostationary detection are derived in [6] for single multiple cyclic frequencies and for multiple cyclic frequencies in [10].

A discrete cyclostationary process has periodic time domain or spectral domain statistics [6]. If $x[n]$ is a wide sense cyclostationary process, then its mean (μ_x) and autocorrelation (R_{xx}) satisfy the following equations,

$$\begin{aligned} \mu_x(n+N_0) &= \mu_x(n)\ \forall\ \ n \\ R_{xx}(n_1+N_0, n_2+N_0) &= R_{xx}(n_1,n_2)\ \forall\ \ n_1,n_2 \end{aligned} \quad (1)$$

for a typical value of N_0, where N_0 is the period of cyclostationary process $x[n]$. Hence, for a cyclostationary process $x[n]$, $R_{xx}(n,k)$ can be represented in the form of a fourier series with respect to time as

$$R_{xx}(n-k/2, n+k/2) = \sum_{\alpha\epsilon\mathcal{A}} R_{xx}(\alpha,k)e^{j2\pi\alpha n} \quad (2)$$

where α is a cyclic frequency, $R_{xx}(\alpha,k)$ is cyclic autocovariance at cyclic frequency α and delay k, $\mathcal{A} = \{\alpha : R_{xx}(\alpha,k) \neq 0\}$, is a set of cyclic frequencies of the process $x[n]$, and the fourier coefficients are given by

$$R_{xx}(\alpha,k) = \lim_{N\to\infty} \frac{1}{(N+1)} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} R_{xx}(n-\frac{k}{2}, n+\frac{k}{2})e^{-j2\pi\alpha n} \quad (3)$$

Now, we present the test statistics required for the detection of PU signal for single cyclic frequency [6] and multiple cyclic frequencies [10]. For the detection of cyclic frequencies, the estimation of cyclic autocorrelation function ($\hat{R}_{xx^*}(\alpha,k)$) can be obtained from expression

$$\hat{R}_{xx}(\alpha,k) = \frac{1}{N_l}\sum_{n=1}^{N_l} x[n]x^*[n+k]e^{-j2\pi\alpha n} \quad (4)$$

where N_l is the number of samples of the process observed.

The cyclic autocovariance estimation in (4) can be decomposed into a vector ($\hat{r}_{xx}(\alpha)$) of real and imaginary parts of $\hat{R}_{xx}(\alpha,k)$ for a candidate cyclic frequency α at different delays of $k_1 \ldots k_L$, as

$$\begin{aligned} \hat{r}_{xx}(\alpha) = \big[&\text{Re}\{\hat{R}_{xx}(\alpha,k_1)\}\ldots\text{Re}\{\hat{R}_{xx}(\alpha,k_L)\}, \\ &\text{Im}\{\hat{R}_{xx}(\alpha,k_1)\}\ldots\text{Im}\{\hat{R}_{xx}(\alpha,k_L)\}\big] \end{aligned} \quad (5)$$

Generally, estimation of cyclic autocorrelation involves some error, so (5) can be represented as,

$$\hat{r}_{xx}(\alpha) = r_{xx}(\alpha) + \hat{\epsilon}_{xx}(\alpha) \quad (6)$$

where $\hat{\epsilon}_{xx}(\alpha)$ is estimation error, $r_{xx}(\alpha)$ is true value of $\hat{r}_{xx}(\alpha)$. Therefore, a hypothesis testing can be formulated based on (6),

$$\begin{aligned} H_0 :&\quad \forall\{k_n\}_{n=1}^{L} \Longrightarrow \hat{r}_{xx}(\alpha) = \hat{\epsilon}_{xx}(\alpha) \\ H_1 :&\quad \text{for some } \{k_n\}_{n=1}^{L} \Longrightarrow \\ &\quad \hat{r}_{xx}(\alpha) = r_{xx}(\alpha) + \hat{\epsilon}_{xx}(\alpha) \end{aligned} \quad (7)$$

if the distribution of $\hat{\epsilon}_{xx}(\alpha)$ is known. In [6], it is shown that $\hat{\epsilon}_{xx}(\alpha)$ has asymptotically normal distribution, where asymptotic distribution of $\hat{\epsilon}_{xx}(\alpha)$ is given as $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma_{xx}}(\alpha))$.

The covariance matrix of $r_{xx}(\alpha)$ is given by $\mathbf{\Sigma_{xx}}(\alpha)$:

$$\mathbf{\Sigma_{xx}}(\alpha) = \begin{bmatrix} \text{Re}\{\frac{\mathbf{Q}+\mathbf{Q}^*}{2}\} & \text{Im}\{\frac{\mathbf{Q}-\mathbf{Q}^*}{2}\} \\ \text{Im}\{\frac{\mathbf{Q}+\mathbf{Q}^*}{2}\} & \text{Re}\{\frac{\mathbf{Q}^*-\mathbf{Q}}{2}\} \end{bmatrix} \quad (8)$$

where the (m,n)-th entries can be computed as

$$\begin{aligned} Q(m,n) &= S_{f_{k_m}f_{k_n}}(2\alpha,\alpha) \\ Q^*(m,n) &= S^*_{f_{k_m}f_{k_n}}(0,-\alpha) \end{aligned} \quad (9)$$

Here $S_{f_{k_m}f_{k_n}}(\alpha,\omega)$ and $S^*_{f_{k_m}f_{k_n}}(\alpha,\omega)$ are unconjugated cyclic and conjugate cyclic spectra of $f(n,k) = x[n]x^*[n+k]$ respectively. These cyclic spectra can be estimated from the smoothed cyclic periodogram as

$$\begin{aligned} \hat{S}_{f_{k_m}f_{k_n}}(2\alpha,\alpha) &= \frac{1}{N_lP}\sum_{s=\frac{-(P-1)}{2}}^{\frac{(P-1)}{2}} W(s) \\ &\quad \times F_{k_m}(\alpha+\frac{2\pi s}{N_l})F_{k_n}(\alpha-\frac{2\pi s}{N_l}) \end{aligned} \quad (10)$$

$$\begin{aligned} \hat{S}^*_{f_{k_m}f_{k_n}}(0,-\alpha) &= \frac{1}{N_lP}\sum_{s=\frac{-(P-1)}{2}}^{\frac{(P-1)}{2}} W(s) \\ &\quad \times F_{k_m}(\alpha+\frac{2\pi s}{N_l})F^*_{k_n}(\alpha+\frac{2\pi s}{N_l}) \end{aligned} \quad (11)$$

where $F_k(\omega) = \sum_{n=1}^{N_l} x[n]x^*[n+k]e^{-j\omega t}$ and W is a spectral window of odd length P. Test statistic used for the detection of the cyclostationarity is $N_l\hat{r}_{xx}(\alpha)\mathbf{\Sigma_{xx}}^{-1}(\alpha)\hat{r}_{xx}(\alpha)^T$ which has χ^2_{2L} distribution (Chi-square distribution with $2L$ degrees of freedom).

Above the detection technique is presented for single cyclic frequency, whereas an extension to this work with regards to the multiple cyclic frequencies is discussed in [10]. Hypothesis testing for a set of cyclic frequencies $\mathcal{A}$ are defined by

$$\begin{aligned} H_0 :&\quad \forall\alpha\ \epsilon\ \mathcal{A} \text{ and } \{k_n\}_{n=1}^{L} \Longrightarrow \hat{r}_{xx}(\alpha) = \hat{\epsilon}_{xx}(\alpha) \\ H_1 :&\quad \text{for some } \alpha\ \epsilon\ \mathcal{A} \text{ and for some } \{k_n\}_{n=1}^{L} \Longrightarrow \\ &\quad \hat{r}_{xx}(\alpha) = r_{xx}(\alpha) + \hat{\epsilon}_{xx}(\alpha) \end{aligned} \quad (12)$$

For this detection problem two statistics have been proposed in [10]:

$$\mathcal{D}_m = \max_{\alpha\epsilon\mathcal{A}} N_l\hat{r}_{xx}(\alpha)\mathbf{\Sigma_{xx}}^{-1}(\alpha)\hat{r}_{xx}(\alpha)^T \quad (13)$$

$$\mathcal{D}_s = \sum_{\alpha\epsilon\mathcal{A}} N_l\hat{r}_{xx}(\alpha)\mathbf{\Sigma_{xx}}^{-1}(\alpha)\hat{r}_{xx}(\alpha)^T \quad (14)$$

In (13), maximum of the test statistics over all the cyclic frequencies is chosen and in (14), sum of the statistics over all cyclic frequencies is chosen as the test statistic. It is shown through simulations [10] that the sum statistic gives better performance.

For a constant false alarm test (CFAR) test, making a decision based on the value of the test statistic requires a value of the threshold, which depends on the false alarm probability chosen and the distribution function of the statistic under the hypothesis H_0. Under hypothesis H_0, asymptotic distribution of the statistics $\mathcal{T}_{xx^*}(\alpha)$ and $\mathcal{D}_s$ are χ^2_{2L} and $\chi^2_{2LN_\alpha}$ respectively, where N_α is the number of cyclic frequencies in $\mathcal{A}$, which is due to the fact that the statistics $\mathcal{T}_{xx^*}(\alpha)$ and $\mathcal{D}_s$ are sum of the squares of $2L$ and $2LN_\alpha$ gaussian random variables.

III. Cyclostationary Detection in Multiple Antenna CR Receiver

There is a recent surge in the interest for using multiple antennas both at the transmitter and receiver for increasing the capacity for wireless channels. In this section, we look at the detection performance of the cyclostationary detector for spectrum sensing in multiple antenna cognitive radio scenario. The diversity gain is achieved for spectrum sensing through cooperative detection [10], where the CR's are spread over a area and they exchange the statistics computed at each CR and hypothesize the presence of the primary user. In this work, we consider that each CR receiver is equipped with multiple antennas and signal are combined at the CR to improve the SNR of the received signal. Details analysis of the present method are discussed below.

Let the CR have N_r receive antennas and $s[n]$ be the transmitted signal. Further assume that $x_i(n)$ be the signal received at the $i^{th}(1 \leq i \leq N_r)$ antenna. There are various ways of combining signals obtained from multiple receivers, and there is a trade off between the performance and complexity for different techniques. Maximal ratio combining (MRC) will give optimum detection performance [11], provided the channel conditions are known.

In MRC scheme, received signals are combined coherently, which results in maximizing the SNR of the combined signal and is given by

$$y[n] = \sum_{i=1}^{N_r} h_i^* x_i[n] \tag{15}$$

where h_i^* is the channel gain for the i^{th} antenna. Under the AWGN channel assumption, received signal at the i^{th} receiver would be $x_i[n] = s[n] + w_i[n]$, and weights involved in the MRC are equal, and signal obtained after MRC is,

$$y[n] = \sum_{i=1}^{N_r} x_i[n] \tag{16}$$

If $s[n]$ is a cyclostationary process then $x_i[n]$ will be a cyclostationary process having same cyclic frequencies as $s[n]$. Since sum of cyclostationary signals which have same cyclic frequency is also cyclostationary with same cyclic frequency [12] and $y[n]$ is also a cyclostationary process with same cyclic frequency. Now we can obtain the cyclic correlation for a set of cyclic frequencies $\mathcal{A}$, and form a vector similar to (5). Hypothesis test can be performed as in (7) and (12), for which covariance matrix can be obtained from (8).

Signals from different receivers are combined using MRC to obtain the resultant signal. As a result we have a single statistic and the asymptotic distribution of the statistic under hypothesis H_0, which is similar to that in the single antenna case. Hence under hypothesis H_0, test statistics $\mathcal{T}_{xx^*}(\alpha)$ and $\mathcal{D}_s$ are sum of the squares of $2L$ and $2LN_\alpha$ gaussian random variables. As a result asymptotic distribution of the statistics $\mathcal{T}_{xx^*}(\alpha)$ and $\mathcal{D}_s$ are χ^2_{2L} and $\chi^2_{2LN_\alpha}$, for single cyclic and multiple cyclic frequencies, respectively.

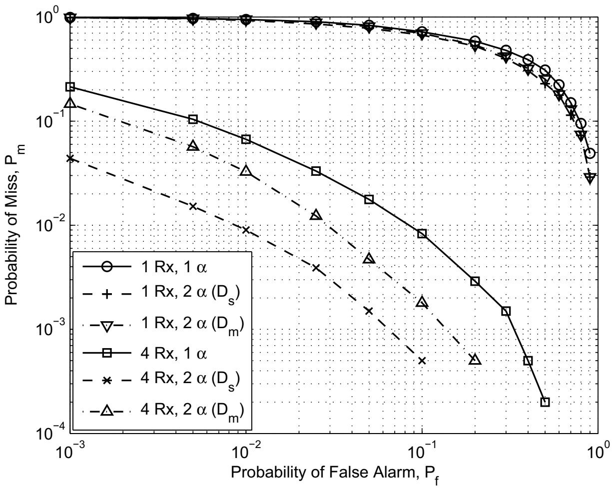

Fig. 1. Complementary ROC curves for diversity schemes based on cyclostationary detector for different cyclic frequencies with SNR -12 dB.

IV. Simulations

In this section, the performance of the cyclostationary detector is discussed. To show the detection performance of multi antenna cognitive radio, we use receiver operating characteristic (ROC) function. More specifically, we quantify the receiver performance by depicting the ROC curve (P_d versus P_f), or equivalently, complementary ROC curve (probability of a miss $P_m = 1 - P_d$, versus P_f) for different detection techniques in both single and multiple antenna scenario. In the present simulation study, we assumed that the PU transmits OFDM signal. To specific, PU uses 32 subcarriers with a guard interval of $1/4$ of the useful symbol data length of the OFDM signal. Further, we assume each subcarrier of OFDM symbol is modulated by 16-QAM and the cyclic frequencies corresponding to an OFDM signal are k/T_s. Here, T_s is OFDM symbol duration, and $k = \{1, 2, \ldots\}$ and delays for which OFDM signal will have nonzero cyclic correlation coefficients are $\pm T_d$, where T_d is length of useful data duration of the OFDM signal [9]. To estimate the cyclic spectra, Kaiser window is used with the window length of 2049, and $\beta = 10$. The SNR of the received signal is given by $10 \log_{10} \frac{\sigma_s^2}{\sigma_n^2}$, where σ_s^2, σ_n^2 are the variance of the signal and noise respectively. In this simulation, we have used the FFT size of 10000 which results in a good resolution of cyclic frequency α. For simulation in case of multiple antenna environment, the number of antennas are considered as $N_r = 4$. Received signal is sampled at the Nyquist rate i.e, $32/T_d$

The complementary ROC curves of the cyclostationary detector are shown in Fig. 1 for single and multiple antennas. As

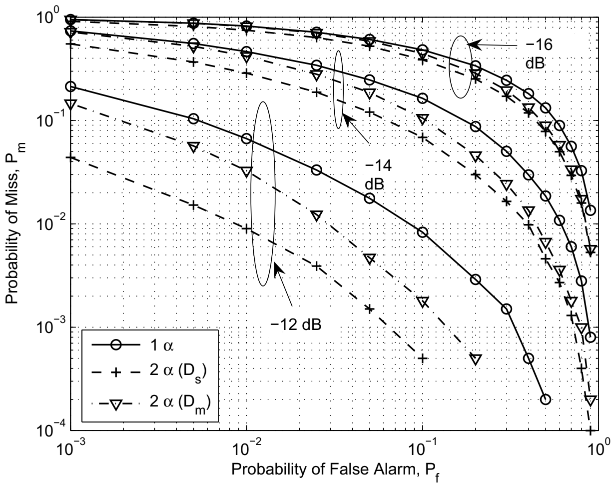

Fig. 2. Complementary ROC curves for MRC based energy detector at different values of SNR for different cyclic frequencies with ($M = 4$).

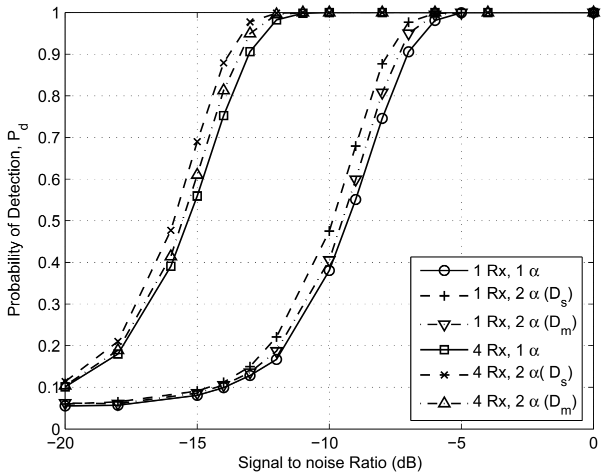

Fig. 3. Probability of detection versus SNR for different diversity schemes based on cyclostationary detector for different cyclic frequencies with $P_f = 0.05$.

evident from the figure, the probability of detection increases with increase in the probability of false alarm. Again, the performance under MRC scheme is superior than the no diversity scheme, for both single and multiple frequencies. The variations of complementary ROC curve with MRC scheme are shown in Fig. 2 for different values of SNR. As expected, the detector performance improves as SNR increases.

Figure 3 illustrates the effect of SNR on detection probability for MRC based cyclostationary detector for $N_r = 4$ and $P_f = 0.05$ for different cyclic frequencies. The MRC based cyclostationary detector offer larger improvement in detection performance at low SNR as well. It can be observed that there is around 6 dB advantage over single antenna when using multiple antennas at SNR -10 dB. Finally, Fig. 4 shows the P_d versus SNR curves for MRC based cyclostationary detector for different value of probability of false alarm.

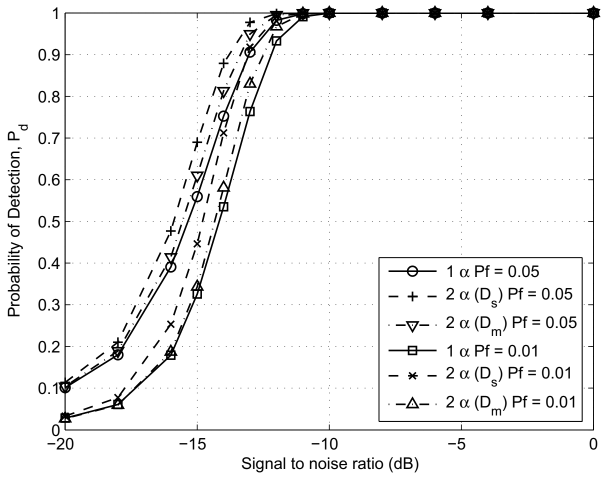

Fig. 4. Probability of detection versus SNR for MRC based energy detector at different values of P_f with $M = 4$.

V. Conclusion

In this paper, we detect the primary user signal with cyclostationary detector in MIMO cognitive radio receiver for multiple cyclic frequency. In our analysis, the cyclostationary detector is based on MRC diversity technique, where cognitive radio is equipped with a maximum of four antennas. Through our simulation, it is observed that MRC based CR can achieve superior performance in comparison with single antenna at particular SNR and P_f. This can be attributed to increase in SNR obtained by combining the signals from multiple antennas.

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