Exploiting Multiuser Diversity with Only 1-Bit Feedback

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***Abstract*— In a system with n users, the sum-rate capacity of the downlink channel grows as $\log \log n$, assuming optimal scheduling. However, optimal scheduling requires that the downlink channel state information (CSI) for all users be fully available at the base station. In this paper we show that the same capacity growth holds even if the feedback rate from the mobiles to the base station is reduced to one bit. We propose a simple scheduling method to achieve this multiuser capacity and furthermore we show that by a judicious choice of the one-bit quantizer, not only the growth rate, but also most of the capacity of a fully informed system can be preserved.**

I. Introduction

Diversity, in its various forms, provides advantages for communication in fading wireless channels. The usual forms of diversity in single-user channels include time, frequency, and space diversity. In a multi-user environment with multiple independent wireless links, it is highly probable that at any given point in time, at least one of those links has high quality. This advantage is called *multiuser diversity.* Obviously, multiuser diversity requires the base station to know the channel coefficients for all users, which is usually estimated at the mobiles and fed back to the base station.

In this work, we address the question of the quantity of information required at the base station. In particular, we show that with only one bit of feedback per user, it is possible to achieve the optimal capacity growth rate (with the number of users). We present a scheduling algorithm that provides this optimal growth rate with modest computational requirements and only a minimal overall loss in capacity. We also address the choice of optimal threshold that maximizes the sum-rate capacity which has not been explored in the previous works. The results proved in this paper answers the question raised in [1] and the answer is that only one-bit per user per block transmission is required to achieve the growth rate and a significant part of the multiuser diversity capacity.

The notion of multiuser diversity was raised by Knopp and Humblet [2] for the uplink, where they mentioned that the best strategy is to always transmit to the user with the best channel. Tse [3] provided similar result for the downlink. Bender *et al.* [4] examined practical aspects of downlink multi-user diversity in the context of IS-95 CDMA standard. Viswanath, Tse and Laroia [5] examined this problem for the downlink and presented a method of opportunistic beamforming via phase randomization. Hochwald, Marzetta and Tarokh [6] investigate the problem of scheduling and rate feedback in the case of MIMO channels. Sharif and Hassibi [7] generalized the opportunistic beamforming of [5] to the case where mobiles also have multiple antennas.

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Some of these works consider the question of the required feedback rate, but to our knowledge, only [6] and [1] explicitly consider the question of quantifying the required feedback. However, they do not consider capacity growth rates, nor optimize the quantization to minimize capacity loss. In this work we present, effectively, a one-bit quantization strategy and the associated scheduling algorithm that guarantees optimal capacity growth rate. The idea of exploiting multiuser gain by limited feedback was first proposed in [7]. In [1] and [8] the idea of thresholding for reducing the feedback load required to exploit multiuser diversity has been proposed, however, as will be discussed in Section III, their scheduling scheme and the amount of information fed back to the base station are different from our scheme. In particular, our method guarantees optimal growth rate with number of users via one-bit fixed-rate feedback, while[1], [8] requires a variable-rate feedback of real-valued numbers and, to our understanding, it has not been proved to guarantee optimal growth rates.

A brief note on notation: $\mathbb{E}[\ ]$ refers to expected value of a random variable, $\gamma \approx 0.577$ is the Euler-Mascheroni constant and $e \approx 2.718281$ is the base of natural logarithm. We use $a_n \overset{\circ}{=} b_n$ to denote the asymptotic equivalence of a_n and b_n defined as: $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$. We use the natural logarithm throughout this paper so the capacity unit is in Nats/Sec/Hz.

II. System Model

We consider a multi-user cellular network with n users who receive data from the base station. We assume the block fading model for each user's channel. The channel state information of each user is assumed to be fully known to that user, and it is constant over a coherence interval of length T. We assume a SISO case in which all users and also the base station is equipped with only one antenna. We conjecture that the extension of our algorithm to the multi-antenna case will yield similar results. Under the block-fading assumption, we have the following model for received signal for each user:

$$y_i(t) = \sqrt{\rho_i} h_i s_i(t) + \nu_i(t) \tag{1}$$

In the above model, we assume that $s_i(t) \in \mathbb{C}^T$ is the vector of transmitted symbols of the i^{th} user at time t with power constraint $\mathbb{E}[\|s_i(t)\|^2] = T$, and $y_i(t) \in \mathbb{C}^T$ is the received signal of the i^{th} user at time t, $\nu_i(t) \sim \mathcal{CN}(0, I_T)$ is the i.i.d. complex Gaussian noise, h_i is the channel gain of the i^{th} user, which is assumed to be zero mean circularly symmetric complex Gaussian random variables with unit variance per dimension. We also assume that users have mutually independent channel gains. Moreover we assume a homogeneous network in which all users have the same SNR, i.e. $\rho_i = \rho$. We also assume that for each user there exits a low-rate but reliable and delay-free feedback channel to the base-station.

III. Scheduling via Single-bit Feedback

In this section we present a simple scheduling algorithm that requires only a single bit of feedback at the base station.

A. Scheduling Mechanism

The base-station sets a threshold α for all users. Each user compares the absolute value of their channel gain to this threshold. Whenever the channel gain exceeds the threshold, a "1" will be transmitted to the base station; otherwise a "0" will be transmitted. The base station receives feedback from all users and then randomly picks a user whose feedback bit was set to one for data transmission.[1] If all the feedback bits received by the base-station are zero, then no signal is transmitted in that interval.[2]

Our work is distinct from that of Gesbert and Alouini [1], [8] in the following manner. Even though the idea of thresholding the users' channel SNR's has also been mentioned by Gesbert and Alouini, the requirements for their scheduling scheme are considerably different from ours. In particular, their method requires the users that have channel gains above a certain threshold to report those channel gains to the base station. This requires a feedback channel of variable-rate, but more importantly, a feedback channel that must still accommodate the transmission of real-valued variables back to the base station. So even though in their scheme, fewer parameters than before are transmitted, still the rate is considerable. In comparison, we are interested in a strictly limited-rate feedback scenario.

[1]The scheduling to users with favorable channels may also be implemented via round robin. In long run, both these strategies have the same average throughput per user. However, the round-robin version may be more appealing from a fairness point of view.

[2]In this case the base station can also randomly pick a user for data transmission, although for large number of user this has vanishing advantage over no transmission when all the received feedback bits are "0".

B. The Sum-Rate Capacity

Upon receipt of each set of feedback bit, the base-station only transmits to users whose channel gain is above the threshold α. Let $p = Pr[|h_i|^2 > \alpha] = e^{-\alpha}$ since the channel gains are all mutually independent, the probability of having k feedback bits equal to one obeys a binomial law, i.e.

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

the ergodic capacity upon receiving k ones by the base-station is:

$$\begin{aligned} \overline{C}_k &= \sum_{i=1}^{k} Pr[\text{the } i^{th} \text{ best user is selected}]\ C_i \\ &= \frac{1}{k}\sum_{i=1}^{k} C_i \end{aligned} \tag{3}$$

where $C_i = \int_0^\infty \log(1+\rho x) dF_i(x)$ and $F_i(x)$ is the CDF of the i^{th} highest absolute value of all channel gains. In other words if $\{X_1, \ldots, X_n\}$ is a permutation of $\{|h_1|^2, \ldots, |h_n|^2\}$ such that $0 \le X_n \le \cdots \le X_1$, then $F_i(x) = Pr[X_i < x]$. When the channel gains are iid, it can be shown that [9]:

$$F_i(x) = \sum_{l=0}^{i-1} \binom{n}{l} (F(x))^l (1-F(x))^{n-l} \tag{4}$$

where $F(x) = 1 - e^{-x}$ is the CDF of $|h_i|^2$ for $i = 1, \ldots, n$. Thus the sum-rate capacity of the network with one-bit feedback can be formulated as:

$$C_{1\_bit} = \sum_{k=1}^{n} p_k \overline{C}_k \tag{5}$$

C. Optimal Threshold

The sum-rate capacity is a function of ρ, p and n. On the other hand the threshold α is uniquely determined by p from the following formula, because the channel magnitude squared obeys an exponential law.

$$\alpha = -\log p \tag{6}$$

In order to find the optimal threshold we choose p such that the sum-rate capacity C_{1_bit} is maximized. The cost function $C_{1\_bit}(p)$ is a weighted sum of functions of the form $p^k(1-p)^{n-k}$ which are all concave over the interval $[0, 1]$, hence C_{1_bit} is a concave function of p. Therefore it has a unique maximum over the interval $[0, 1]$. To calculate the value of p that maximizes the sum-rate capacity, we must solve $\frac{\partial C_{1\_bit}(p)}{\partial p} = 0$ for p. By differentiating Eq. (5) with respect to p we get:

$$\sum_{k=1}^{n} (k - np) p_k \overline{C}_k = 0 \tag{7}$$

A closed form solution to this equation is in general not tractable. However, a numerical solution is possible with $O(n)$ complexity.

D. Extension to MIMO Systems

The effectiveness of multiple antennas at both transmit and receive side has been demonstrated in the past few years [10]. MIMO systems lead to increased capacity and/or the reliability of a wireless link. However, as mentioned in [6], increasing the number of antennas at transmit or receive side *hardens* the channel, meaning that the channel will have little variability. It is further shown that the statistics of the mutual information is Gaussian in the asymptote of large number of antennas, and a scheduling algorithm is proposed to achieve multiuser diversity gain. In this scheduling mechanism the users send their instantaneous channel capacity to the base-station, which then offers a wireless link to the user with the highest capacity. In [7] a scheduling mechanism based on random beamforming has been proposed for MIMO broadcast channel, which is a generalization of opportunistic beamforming of [5]. In this scheme, only the signal to interference plus noise ratio of each user (instead of full CSI) is sufficient to achieve the double logarithmic growth of sum-rate capacity of a fully informed multiuser network. However in both these methods, a real number must be sent to the base-station, requiring substantial rate.

We conjecture that, similar to the method presented in this paper, quantizing each of the above parameters (instantaneous capacity in [6] and signal to interference plus noise ratio in case of [7]) by an optimally chosen threshold can still capture a significant part of the multiuser diversity gain. Further investigation of this conjecture is a subject of future research.

IV. Asymptotic Analysis

In order to explore the asymptotic behavior of the sum-rate capacity we first need to prove some preliminary results:

Lemma 1: Let $\{X_i\}_{i=1}^{n}$ be a sequence of positive iid random variables with finite mean μ_n and finite variance σ_n^2, also $\mathbb{E}[\log^2(X_n)] < \infty$, if $\lim_{n\to\infty} \frac{\sigma_n}{\mu_n} = 0$, then:

$$\log(\mu_n) - \mathbb{E}[\log(X_n)] \longrightarrow 0$$

as $n \to \infty$.

Proof: See the appendix.

Lemma 1 states that if the probability measure associated with the random variable X_n is well concentrated around its mean value for large n, then Jensen's inequality for $\log()$ is asymptotically tight. Note that X_n can be either a *discrete* or a *continuous* random variable.

When channel state information is fully available at the base station, the base station only transmits to the user with the best channel, hence the ergodic sum-rate capacity of the network can be calculated by the following formula:

$$\begin{aligned} C_{full\_CSI} &= C_1 = \int_0^\infty \log(1+\rho x)dF_1 \\ &= n\int_0^\infty \log(1+\rho x)e^{-x}(1-e^{-x})^{n-1}dx \end{aligned}$$

Let $\mu_1 = \int_0^\infty x dF_1$ and $\sigma_1^2 = \int_0^\infty (x-\mu_1)^2 dF_1$, then it is known [9] that: $\mu_1 = \sum_{i=1}^{n} \frac{1}{i}$ and $\sigma_1^2 = \sum_{i=0}^{n} \frac{1}{i^2}$, therefore $\frac{\sigma_1}{\mu_1} \to 0$ as $n \to \infty$. Combined with Lemma 1, it follows that:

$$\begin{aligned} C_{full\_CSI} &\overset{\circ}{=} \log(1+\rho\mu_1) \\ &\overset{\circ}{=} \log(\log n) + \log\rho. \end{aligned} \quad (8)$$

where $\overset{\circ}{=}$ indicates asymptotic equivalence, as defined earlier.

We are interested to investigate the behavior of the sum-rate capacity of the 1-bit feedback scheduling proposed in Section III in the asymptote of large number of users. This is accomplished via the following result.

Theorem 1: The sum-rate capacity of a wireless network with 1-bit feedback and optimal choice of threshold, behaves as $O(\log(\log n) + \log\rho)$, exactly the same as the sum-rate capacity of a fully informed network.

Proof: See the appendix.

V. Simulation Results

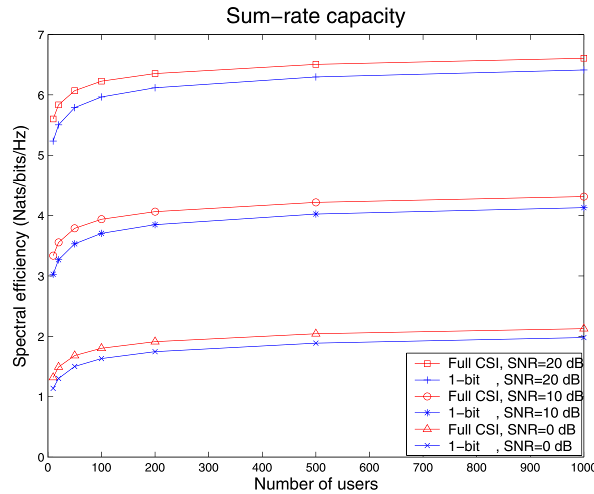

Fig. 1. Comparison of sum-rate capacity for 1-bit and full CSI scheduling for different values of SNR

Fig. 1 shows the simulation results for sum-rate capacity of a SISO network. As it can be seen in the figure, our proposed scheduling, with only 1-bit feedback, has the same double logarithmic growth rate as the fully informed network. The capacity loss is minimal. Scheduling with 1-bit feedback also captures most of the capacity of the fully informed network for a wide range of SNR, thus the scaling law proved in Theorem 1 is verified by the simulation. Fig. 2 shows the optimal threshold for various of SNR values. It can be seen that the optimal threshold scales logarithmically with number of users (in Fig. 2 the x-axis is in logarithmic scale).

VI. Conclusion and Future Work

In this paper we investigate the asymptotic sum-rate capacity of the downlink multiuser network. We show that reducing the CSI feedback to one bit does not have an impact on the scaling law of the sum-rate capacity. Simulation results show the capacity loss is negligible; most of the multiuser diversity gain is retained by a single bit of CSI fed back to the base station. Future work includes the extension of these

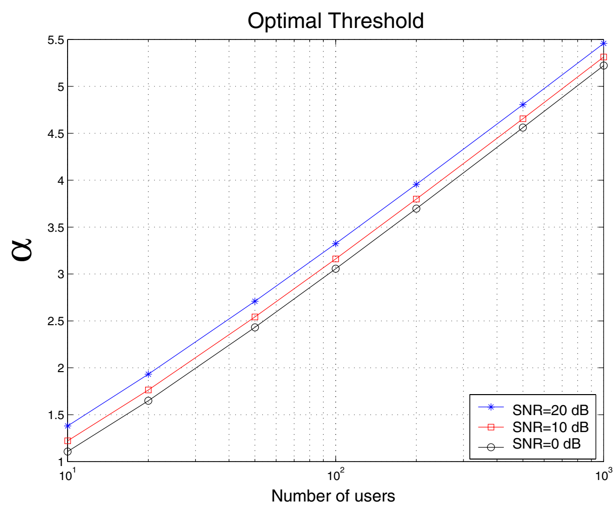

Fig. 2. Optimal threshold vs. number of users for different SNR values

results to the MIMO case. One way of doing this is to utilize the concept of *capacity gain* introduced in [11]. Investigating the conjectures mentioned in Section III-D is also another direction of interest.

VII. Appendix

Proof of Lemma 1: Using Tchebychev inequality for all $\epsilon > 0$ we have:

$$Pr\left[\left|\frac{X_n}{\mu_n} - 1\right| > \epsilon\right] \leq \frac{\mathbb{E}[(x_n - \mu_n)^2]}{\epsilon\mu_n^2} = \frac{1}{\epsilon}\left(\frac{\sigma_n}{\mu_n}\right)^2$$

hence $\frac{X_n}{\mu_n} \xrightarrow{i.p.} 1$, now using the Slutsky's theorem [12] we have

$$\log\left(\frac{X_n}{\mu_n}\right) \xrightarrow{i.p.} 0 \tag{9}$$

On the other hand, for a random variable whose second moment is finite, we have:

$$\begin{aligned}\mathbb{E}[|Z|] &= \int_0^\infty z f_{|Z|}(z)dz \\ &= -z\left(1 - F_{|Z|}(z)\right)\Big|_{z=0}^\infty + \\ &\qquad \int_0^\infty \left(1 - F_{|Z|}(z)\right)dz \end{aligned}\tag{10}$$

$\mathbb{E}[Z^2] = \int_0^\infty |z|^2 f_{|Z|}(z)dz < \infty$ therefore we should have $\lim_{z\to\infty} z^2 f_{|Z|}(z) = 0$ otherwise the integral does not converge. Using L'Hopital rule we also have:

$$\lim_{z\to\infty} z(1 - F_{|Z|}(z)) = \lim_{z\to\infty} z^2 f_{|Z|}(z) = 0$$

Thus from Eq. (10) we can conclude:

$$\mathbb{E}[|Z|] = \int_0^\infty \left(1 - F_{|Z|}(z)\right)dz = \int_0^\infty Pr[|Z| > z]dz$$

now let $Z = \log\left(\frac{X_n}{\mu_n}\right)$, then :

$$\begin{aligned}\mathbb{E}[Z^2] &= \mathbb{E}[(\log X_n - \log \mu_n)^2] \\ &\leq \mathbb{E}[(\log X_n)^2] + (\log \mu_n)^2 < \infty \end{aligned}\tag{11}$$

therefore:

$$\begin{aligned}|\mathbb{E}[\log X_n] - \log \mu_n| &\leq \mathbb{E}\left[\left|\log\left(\frac{X_n}{\mu_n}\right)\right|\right] \\ &= \int_0^\infty Pr\left[\left|\log\left(\frac{X_n}{\mu_n}\right)\right| > a\right]da \end{aligned}\tag{12}$$

but Eq. (9) says that for all $a > 0$, $Pr[|\log(\frac{X_n}{\mu_n})| > a] \to 0$ as $n \to \infty$. We know, $Pr[|\log(\frac{X_n}{\mu_n})| > a] \leq 1$, hence using the *dominated convergence theorem* we can conclude that $\mathbb{E}[\log(X_n)] - \log(\mu_n) \to 0$, Q.E.D.

Proof of Theorem 1: Eq. (7) can be re-written as:

$$C_{1\_bit} = \frac{1}{np}\sum_{k=1}^{n} kp_k\overline{C}_k \tag{13}$$

For a p satisfying Eq. (13) we have:

$$\begin{aligned}C_{1\_bit} &= \frac{1}{np}\sum_{k=1}^{n} kp_k\overline{C}_k \\ &= \frac{1}{np}\sum_{k=1}^{n} kp_k\left(\frac{1}{k}\sum_{i=1}^{k} C_i\right) \\ &= \frac{1}{np}\sum_{k=1}^{n}\sum_{i=1}^{k} p_k C_i \\ &= \sum_{i=1}^{n}\left(\frac{1}{np}\sum_{k=i}^{n} p_k\right) C_i \end{aligned}\tag{14}$$

we notice that $\pi_i = \frac{1}{np}\sum_{k=i}^{n} p_k,\ i = 1,\ldots,n$ is a valid p.m.f. because $\sum_{i=1}^{n}\pi_i = 1$, hence:

$$\begin{aligned}C_{1\_bit} &= \sum_{i=1}^{n}\pi_i C_i \\ &= \sum_{i=1}^{n}\pi_i\int_0^\infty \log(1+\rho x)dF_i \\ &= \int_0^\infty \log(1+\rho x)d\left(\sum_{i=1}^{n}\pi_i F_i\right) \\ &= \int_0^\infty \log(1+\rho x)dF_\pi \end{aligned}\tag{15}$$

where $F_\pi = \sum_{i=1}^{n}\pi_i F_i$ is a mixture probability measure of all order statistics of the exponential family. Now we show that F_π satisfies the required condition for Theorem 1.

$$\mu_\pi = \sum_{i=1}^{n}\pi_i\mu_i \tag{16}$$

where $\mu_i = \int_0^\infty x dF_i(x)$ is the mean of the i^{th} order statistics of the exponential family, and

$$\begin{aligned}
\sigma_\pi^2 &= \int_0^\infty (x-\mu_\pi)^2 dF_\pi(x) \\
&= \int_0^\infty x^2 dF_\pi(x) - \mu_\pi^2 \\
&= \sum_{i=1}^n \pi_i \int_0^\infty x^2 dF_i(x) - \mu_\pi^2 \\
&= \sum_{i=1}^n \pi_i(\sigma_i^2+\mu_i^2) - \mu_\pi^2 \\
&= \sum_{i=1}^n \pi_i\sigma_i^2 + \sum_{i=1}^n \pi_i\mu_i^2 - \mu_\pi^2 \qquad (17)
\end{aligned}$$

where $\sigma_i^2 = \int_0^\infty (x-\mu_i)^2 dF_i$ is the variance of the i^{th} order statistics of the exponential family. It is a known fact (e.g. [9] Section 4.6) that:

$$\mu_i = \sum_{j=i}^n \frac{1}{j} = H_n - H_{i-1}$$

where $H_k = \begin{cases} \sum_{j=1}^k \frac{1}{j} & k>0 \\ 0 & k=0 \end{cases}$ and also,

$$\sigma_i^2 = \sum_{j=i}^n \frac{1}{j^2} = S_n - S_{i-1}$$

where

$$S_k = \begin{cases} \sum_{j=1}^k \frac{1}{j^2} & k>0 \\ 0 & k=0 \end{cases}$$

We have to show that $\frac{\sigma_\pi}{\mu_\pi} \to 0$

$$\begin{aligned}
\mu_\pi &= \sum_{i=1}^n \pi_i\mu_i = \sum_{i=1}^n \pi_i(H_n - H_{i-1}) \\
&= H_n - \sum_{i=1}^n \pi_i H_{i-1} < H_n = \mu_1 \qquad (18)
\end{aligned}$$

it is known [13] that for all $k \geq 1$,

$$\log k + \gamma + \frac{1}{2(k+1)} < H_k < \log k + \gamma + \frac{1}{2k} \qquad (19)$$

using Jensen's inequality we have

$$\begin{aligned}
\mu_\pi &= H_n - \sum_{i=1}^n \pi_i H_{i-1} \\
&> H_n - \sum_{i=1}^n \pi_i H_i \\
&> H_n - \gamma - \sum_{i=1}^n \pi_i \log i - \frac{1}{2}\sum_{i=1}^n \frac{\pi_i}{i} \\
&> H_n - \gamma - \log\left(\sum_{i=1}^n i\pi_i\right) - \frac{1}{2}\sum_{i=1}^n \pi_i \\
&> H_n - \gamma - \frac{1}{2} - \log\left(\sum_{i=1}^n i\pi_i\right) \qquad (20)
\end{aligned}$$

on the other hand

$$\begin{aligned}
\sum_{i=1}^n i\pi_i &= \frac{1}{np}\sum_{i=1}^n i \sum_{k=i}^n p_k \\
&= \frac{1}{np}\sum_{k=1}^n p_k \sum_{i=1}^k i \\
&= \frac{1}{np}\sum_{k=1}^n p_k\left(\frac{k(k+1)}{2}\right) \\
&= \frac{\sum_{k=1}^n k^2 p_k + \sum_{k=1}^n k p_k}{2np} \\
&= \frac{(n-1)p}{2} + 1 \qquad (21)
\end{aligned}$$

from (18), (20) and (21) we get:

$$H_n - \log(np+2-p) - \gamma - \log(2\sqrt{e}) < \mu_\pi < H_n \qquad (22)$$

by inspecting Eq. (7) we also notice that $p_{opt} \overset{\circ}{=} \frac{1}{n}$ because in order to have equality, the number of positive and negative terms in Eq. (7) should be of the same order in the asymptote of large n. Equivalently the optimal threshold α scales logarithmically in the asymptote of large n (this fact can also be seen in Fig. 2 in which the X-axis is in logarithmic scale). Therefore, (22) suggests that

$$H_n - \mu_\pi = \Theta(1) \qquad (23)$$

or,

$$\mu_\pi \overset{\circ}{=} \log n \qquad (24)$$

as $n \to \infty$. On the other hand:

$$\begin{aligned}
\sigma_\pi^2 &= \sum_{i=1}^n \pi_i\sigma_i^2 + \sum_{i=1}^n \pi_i\mu_i^2 - \mu_\pi^2 \\
&< \sigma_1^2 + \mu_1^2 - \mu_\pi^2 \\
&= \sigma_1^2 + (\mu_1+\mu_\pi)(\mu_1-\mu_\pi) \\
&< \sigma_1^2 + 2\mu_1(\mu_1-\mu_\pi) \\
&= S_n + 2H_n(H_n-\mu_\pi) \qquad (25)
\end{aligned}$$

we also notice that $S_n < S_\infty = \frac{\pi^2}{6} < 2$ (here $\pi = 3.1416\ldots$) thus:

$$\sigma_\pi^2 < 2 + 2H_n(H_n - \mu_\pi) \qquad (26)$$

hence from (23), (24) and (26) we have:

$$0 \leq \left(\frac{\sigma_\pi}{\mu_\pi}\right)^2 < \frac{2}{\mu_\pi^2} + 2\left(\frac{H_n}{\mu_\pi}\right)\left(\frac{H_n-\mu_\pi}{\mu_\pi}\right) \to 0 \qquad (27)$$

as $n \to \infty$. Thus we can conclude $\frac{\sigma_\pi}{\mu_\pi} \to 0$ as $n \to \infty$. Also from Eq. (4) it can be seen that CDF of all order statistics of the exponential family can be explained as sum of exponentials hence $F_\pi(x)$ is also consisted of weighted sum of exponential functions therefore its second order logarithmic moment exits, i.e. $\int_0^\infty (\log x)^2 dF_\pi < \infty$. Now we can apply Lemma 1 and Eq. (24) to show that:

$$
\begin{aligned}
C_{1\_bit} &= \int_0^\infty \log(1+\rho x)dF_\pi \\
&\overset{\circ}{=} \log(1+\rho\mu_\pi) \\
&\overset{\circ}{=} \log(\log n) + \log\rho
\end{aligned} \quad (28)
$$

Q.E.D.

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