Some Fundamental Limits on Cognitive Radio

Niels Hoven, Rahul Tandra, and Prof. Anant Sahai
Wireless Foundations, EECS
University of California at Berkeley
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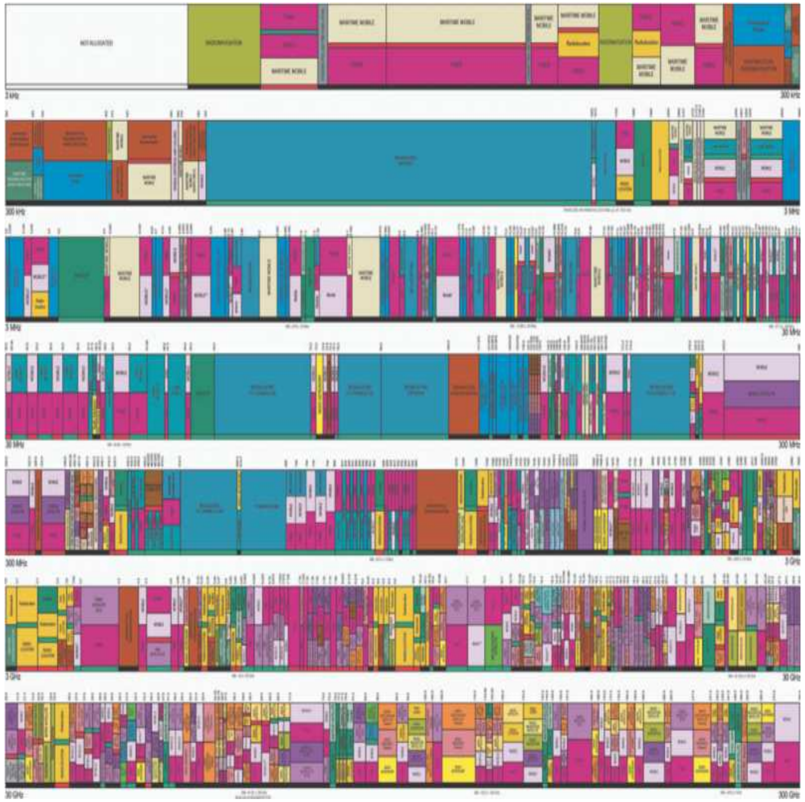
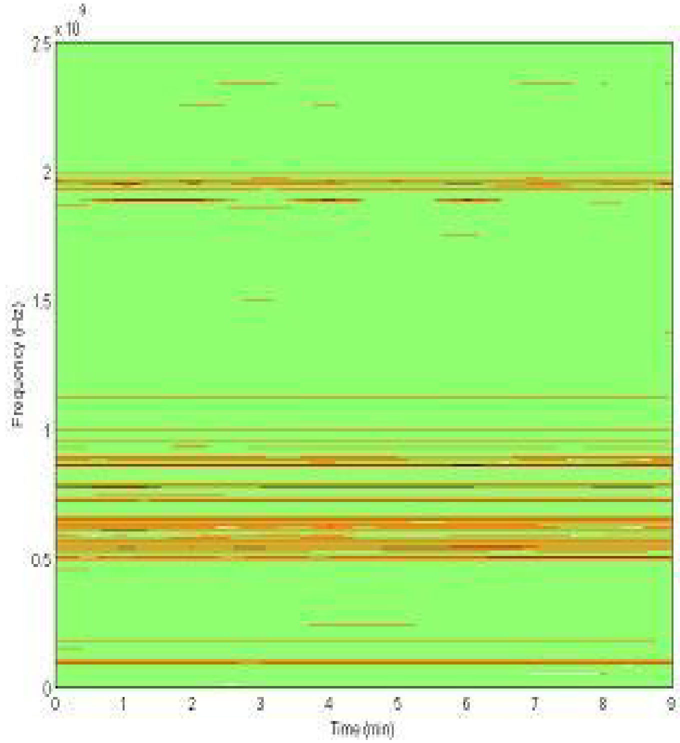
Spectrum: Allocation vs Usage





- Apparent spectrum scarcity
- Actual measurements show that $> 70\%$ of spectrum is unused.
- Enough free spectrum for DVD-res cameras every few feet!

That was then, this is now...

- Primitive analog hardware
- Devices fixed to bands
- Interference a severe challenge
- Long range applications
- Bands allocated by law
- Enforce by licensing devices

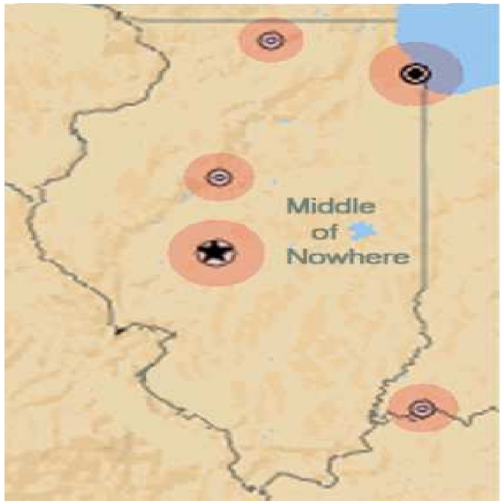
- Digital wideband hardware
- More flexible spectrum view
- Heterogeneous applications
  - Different priorities
  - Range of spatial scales
- Require interoperability
- Enforcement more difficult

What architectures will be needed to better exploit spectrum?

What's the minimal change in regulation?

Cognitive Radio

Justification

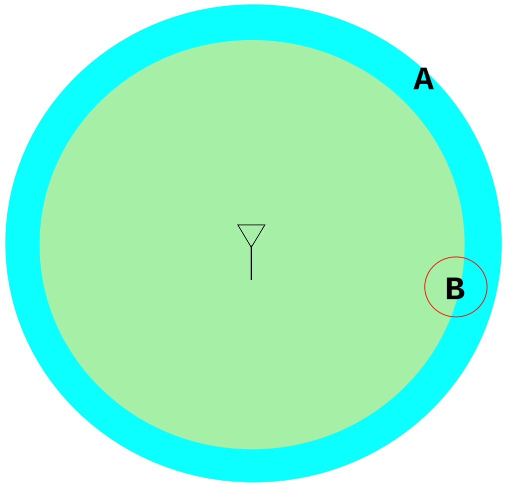


- Wireless interference is primarily a local phenomenon.
- **If a radio system transmits in a band and nobody else is listening, does it cause interference?**

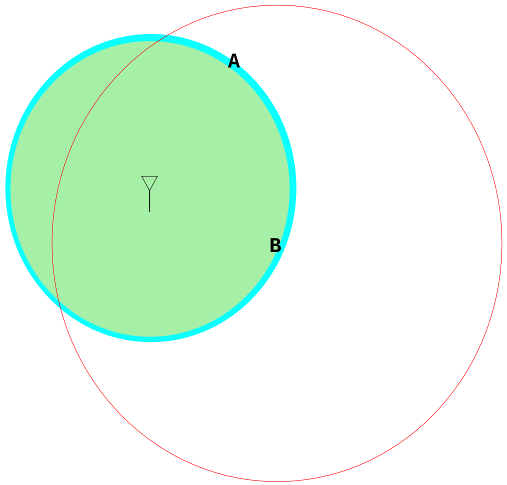
Objectives

- Protect primary users of the spectrum
  - Socially important services may deserve priority on band
  - Legacy systems may not be able to change
- Allow for secondary users to use otherwise unused bands
  - Not the UWB approach: "speak softly but use a wide band"
  - Primary band usage may vary in time
  - May have to scavenge many discontinuous bands
  - May have to coordinate/coexist with other secondary users

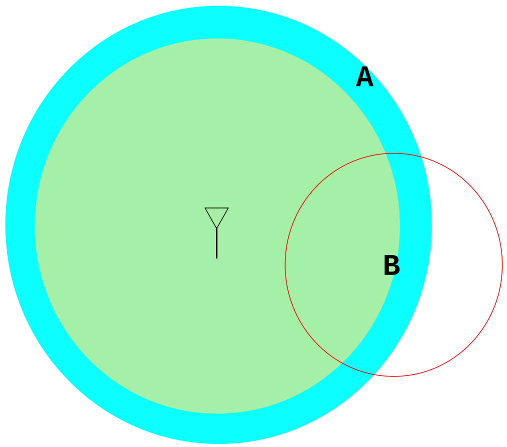
Justification Cont.



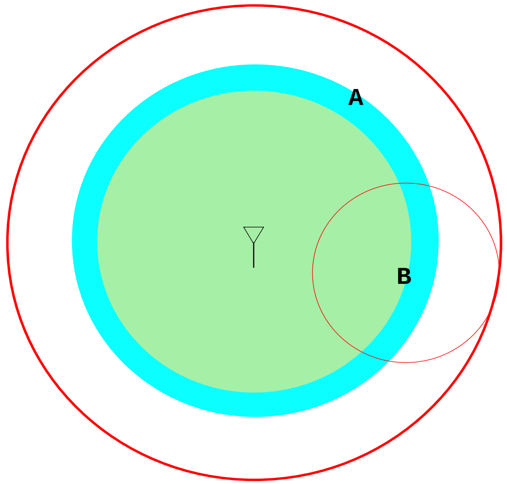
Mice can get close...



The “no talk” zones grow dramatically

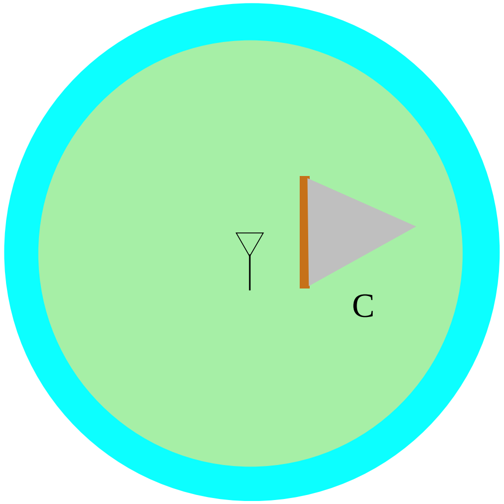


But keep the lions far away!

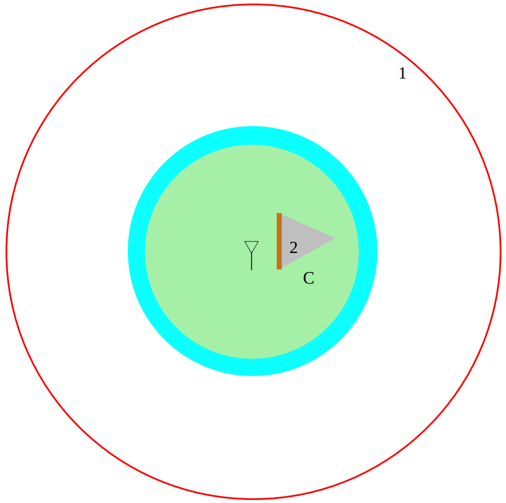


Union of “no talk” zones.

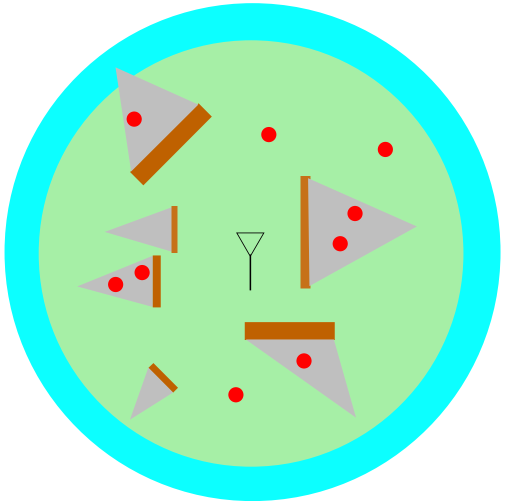
Shadowing



A secondary user might be in a local shadow while his transmissions could still reach an unshadowed primary receiver.



Secondary user can not distinguish between positions (1) and (2) - must be quiet in both.

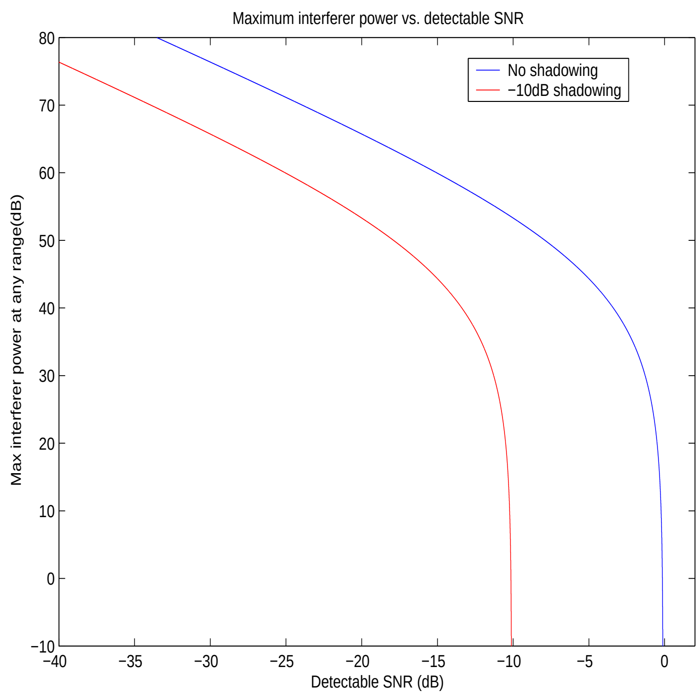


Multiuser diversity should increase our chances of an accurate measurement.

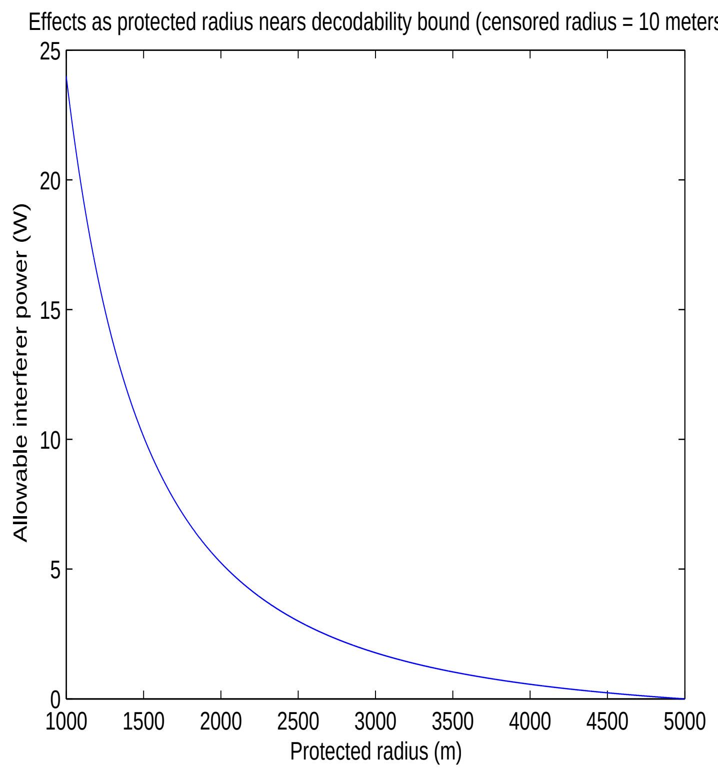
A Fundamental tradeoff
Interferer Power vs Detectable SNR

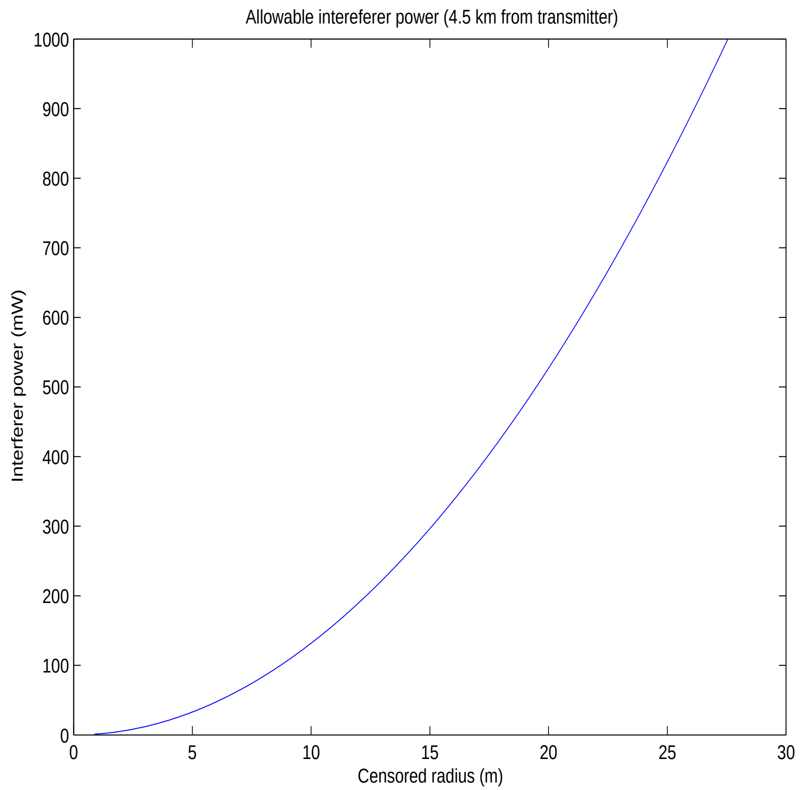
$$P_s = \left(P_p r_p^{-\alpha} 10^{-\frac{\gamma_{dec}+M}{10}} - \sigma^2\right) \left(\left(\frac{P_p}{\sigma^2} \cdot 10^{-\frac{\gamma_{det}-\beta}{10}}\right)^{\frac{1}{\alpha}} - r_p\right)^{\alpha}$$

- Glossary
  - γ_{dec}: Minimum $SINR$ for decodability at the primary receiver.
  - γ_{det}: Minimum SNR at which the secondary can detect the primary transmission.
  - β: SNR loss in detectability due to shadowing.
  - M: Margin of protection given to the primary receivers.



Censored radius vs. interferer power and protected radius





Protecting marginal users forces the cognitive radio to squeak.

Larger censored regions allow the cognitive radios to roar.

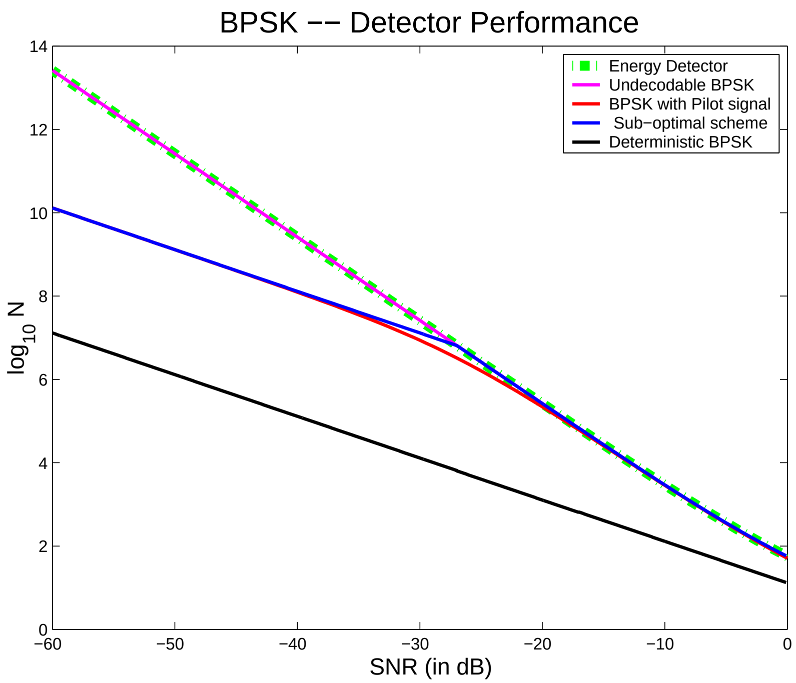
Model

- Hypothesis testing problem: is the primary signal out there?

$$\mathcal{H}_0 : Y[n] = W[n]$$
$$\mathcal{H}_s : Y[n] = W[n] + x[n]$$

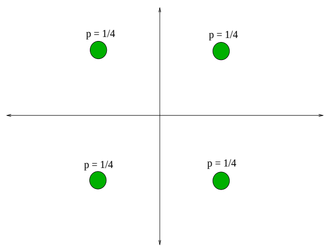
- Moderate P_{fa}, P_{md} targets
- Potentially very low SNR at the detector: will need many samples to distinguish hypothesis
- How long must we listen?

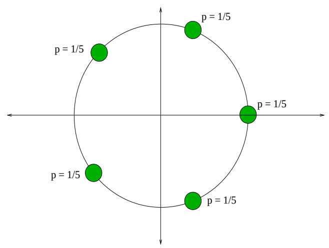
Signal detection

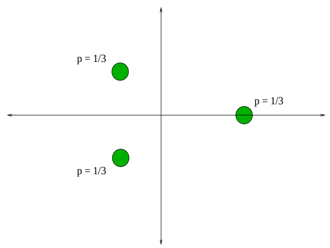


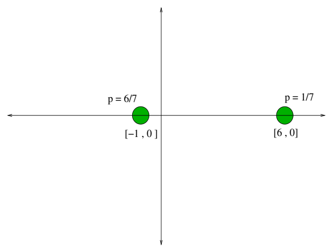
Low SNR

- The optimal detector behaves like an energy detector.
- If one exists, just detecting a pilot signal is nearly optimal.
- Signals without pilots are difficult to detect.

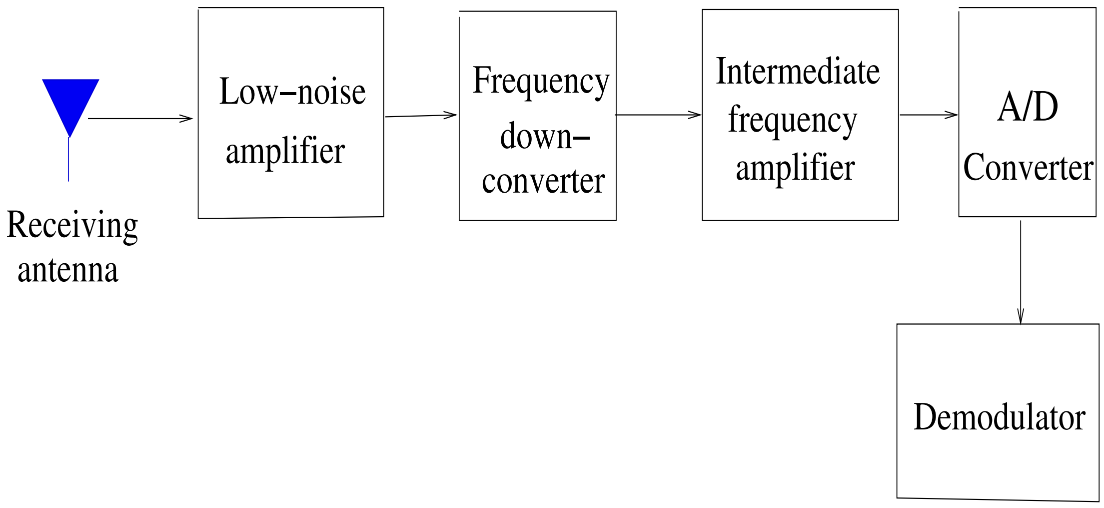








Noise Uncertainty



- In practice there is always uncertainty about the noise.
- Sources of uncertainty:
  - Thermal noise in components (Non-uniform, time-varying)
  - Noise due to transmissions by other users
    * Unintentional (Close-by)
    * Intentional (Far-away)

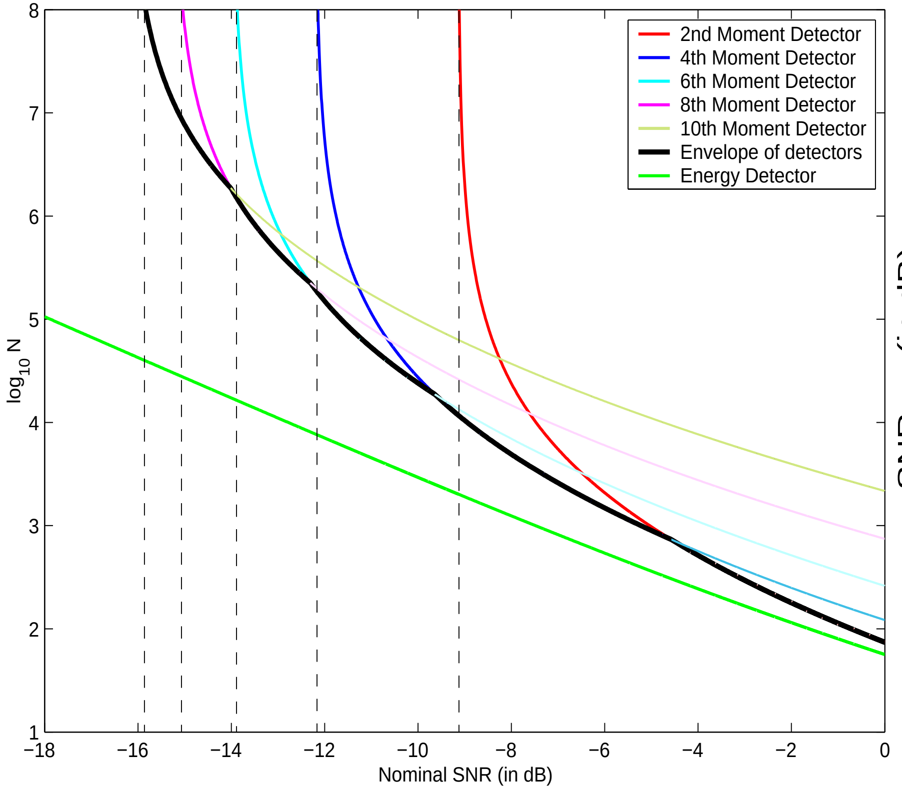
Noise Uncertainty: Conservative Model

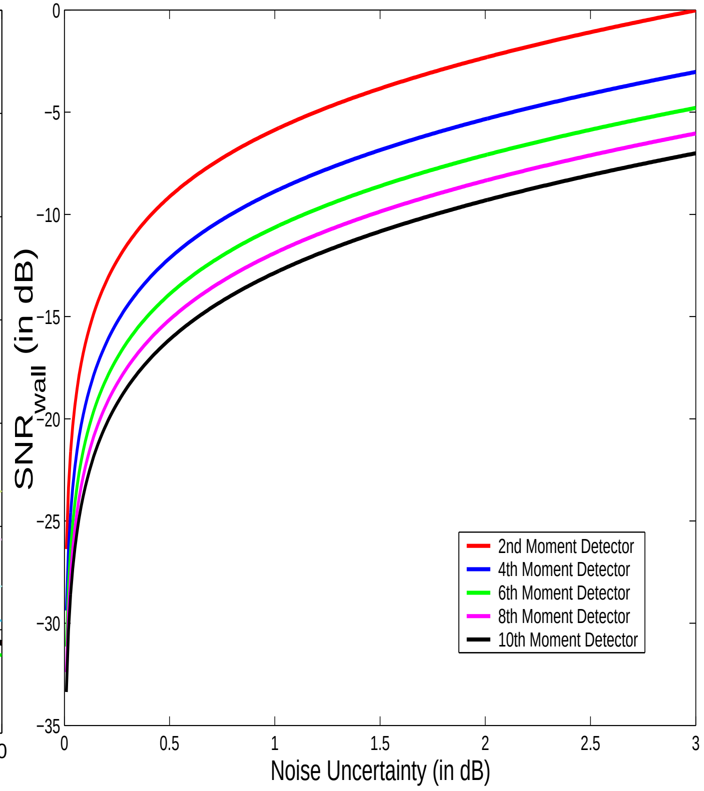
- Noise can be modeled as "Approximately Gaussian" to incorporate uncertainty.
  - Like Gaussian noise, but x dB uncertainty in moments.
  - $\mathbb{E}N^{2k-1} = 0$. [Symmetry property]
  - $\mathbb{E}N^{2k} \in [\mathbb{E}W^{2k}, \alpha\, \mathbb{E}W^{2k}]$, where $W \sim \mathcal{N}(0, \sigma^2)$ and $\alpha = 10^{x/10}$.
- What are the *consequences*?
  - SNR walls
- **Theorem**: For the case of detection of a weak BPSK signal, the '$2k$-th moment detector' encounters a threshold (wall) below which detection is impossible. The threshold for detection as a function of the noise uncertainty x is given by:

$$SNR_{wall}^{2k} = 10 \log_{10} [10^{(x/10)} - 1] - 10 \log_{10} k$$

Noise Uncertainty: Threshold Behavior

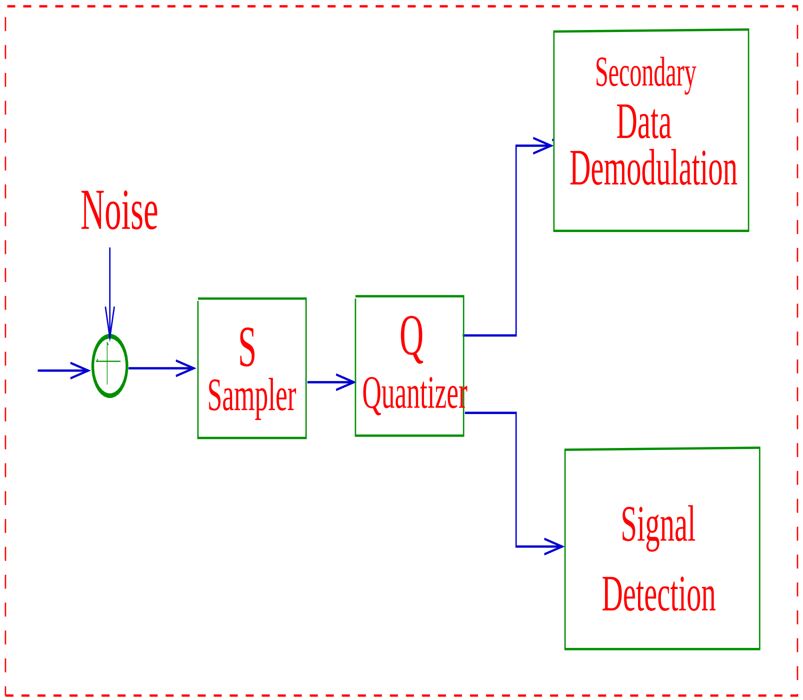
- Moment detector performance
- Noise uncertainty vs SNR wall





Noise uncertainty + Quantization

- Our abstraction



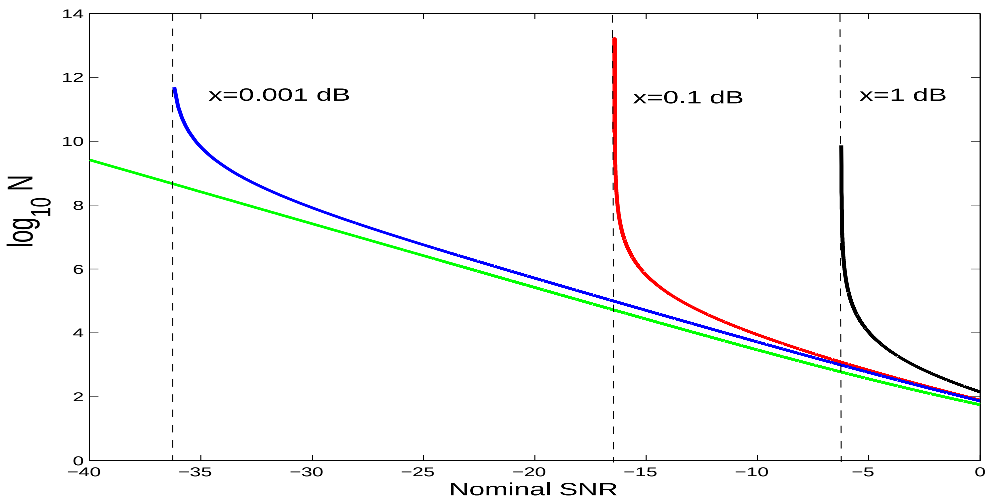
- Things get worse under quantization
- Assumptions:
  - Bounded dynamic range on quantization bins
  - Moment uncertainty model for noise
- There exists an SNR threshold below which detection is *absolutely impossible.*

BPSK example

- Detection can be **absolutely impossible** for 2-bit quantizer
  - Adversarial noise can make the distributions identical under both hypotheses if

$$Q\left(\frac{d_1}{\sigma_0}\right) = \frac{1}{2}\left[Q\left(\frac{d_1+\sqrt{P}}{\sigma_1}\right) + Q\left(\frac{d_1-\sqrt{P}}{\sigma_1}\right)\right]$$

- Wall *always exists* for any detector.



Conclusions

- Cognitive radio can enable significant spectrum reuse.
- To function, we must be able to detect the presence of undecodable signals.
  - Just knowing the modulation scheme and codebooks is nearly useless: stuck with energy detector performance.
  - Even small noise uncertainty causes serious limits in detectability.
  - Quantization makes matters even worse.
- Primary users should transmit pilot signals.
- If not, some infrastructure and/or collaboration will be needed to support cognitive radio deployment.
- Similar limits apply to secondary markets.