Improved Channel Estimation Using Noise Reduction for OFDM Systems

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Abstract- **A new channel estimation algorithm for OFDM systems is proposed in this paper based on a noise reduction method. The performance of a least squares (LS) channel estimator is improved by using the colored noise characteristics in the LS estimation of the channel impulse response (CIR). The noise, which is in the CIR (signal) subspace, is estimated and suppressed based on the correlation between two parts of noise, which are within and outside the signal subspace. Knowledge of the CIR statistical parameters is not needed in the proposed estimation algorithm and its performance is insensitive to CIR parameters as long as the CIR length is not longer than the cyclic prefix interval.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has emerged as a transmission technique that mitigates intersymbol interference (ISI) by extending the symbol duration with a time guard interval which is equal to or longer than the channel impulse response length (or channel delay spread). Also, frequency-selective channel has been converted to a number of parallel flat channels in the OFDM system due to adopting the orthogonal multicarrier technique implemented by fast Fourier transform (FFT). The multicarrier nature of OFDM gives the capability to this technique to overcome the complexity of time equalization method by using a simple frequency equalizer.

Channel estimation is required in a coherent OFDM system which provides a spectrally efficient modulation in comparison with the non-coherent method. Generally, the proposed OFDM channel estimation techniques can be divided to three categories. Time processing methods are used in the first category in order to estimate the channel impulse response (CIR) based on its deterministic or stochastic parameters [1-3]. Since the CIR length should not be longer than the cyclic prefix interval in OFDM system design, the minimum mean square error (MMSE) and the least squares (LS) channel estimation methods are modified in [1] based on processing only a limited number of channel taps in the time domain. Minn and Bhargava have extended the idea by considering only the most significant channel taps [2]. The proposed method in [2] improves the channel estimation performance only when the channel is a sparse one. The number of the most significant channel taps is determined by the minimum description length criterion (MDL) in [3] for a channel estimation method based on a parametric channel model.

The second category focuses on channel estimation in the frequency domain. Edfors *et al* have proposed a MMSE channel estimation method in [4] based on subspace frequency domain processing which uses the singular value decomposition technique. The channel covariance matrix in the frequency domain needs to be known in the proposed method [4]. Third channel estimation category considers both time and frequency characteristics of the channel in order to estimate the channel [5]. These methods need more knowledge about the channel characteristics and they are more complex.

The contribution of this paper is to improve channel estimation of OFDM systems by estimating and suppressing the part of noise, which is in the CIR (signal) subspace. The proposed channel estimation methods in [1] are based on eliminating the noise, which is outside the CIR subspace and then estimating the CIR by using its statistical parameters. However, channel estimation can be improved by considering the correlation between the noise within the signal subspace and the noise which is outside the signal subspace. Since the statistical parameters of channel need not to be known in the proposed algorithm, the performance of estimator becomes more robust to variations of the CIR characteristics.

The rest of the paper is organized as follows. In section II, the LS estimator and its modified version are presented based on a discrete model of an OFDM system. The new channel estimation algorithm is developed in section III based on a noise reduction method. Computer simulations and performance evaluations are presented in section IV and conclusions are drawn in section V.

II. SYSTEM DESCRIPTION

The discrete model of an OFDM system is shown in Fig. 1. $s(k)=[s_1(k),\dots,s_N(k)]^T$ is an OFDM symbol where $(.)^T$ represents the transpose operation, $h(k)$ is channel impulse response, $\zeta(k)$ is a zero mean additive white Gaussian noise and $Y(k)=[y_1(k),\dots,y_N(k)]^T$ is the received OFDM

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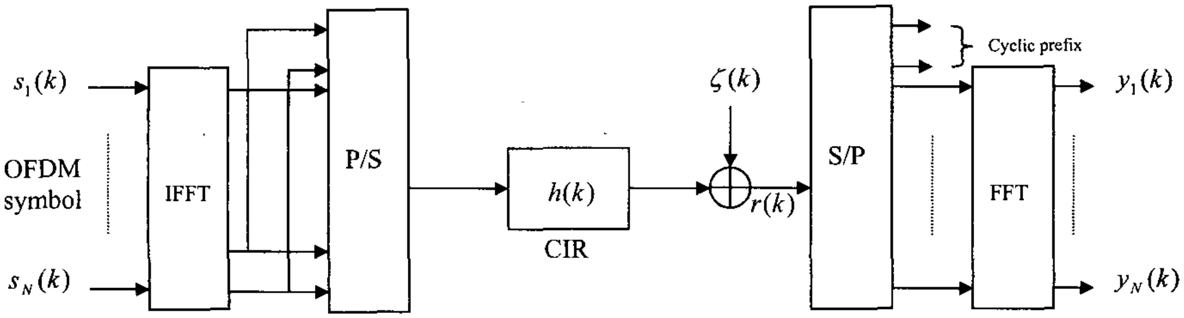

Fig. 1. A discrete model for an OFDM system.

symbol at time *k* after removing cyclic prefix (guard interval) and taking an N-point FFT. *Y*(*k*) can be expressed as

$$Y(k) = \mathbf{X}(k)H + Z(k) \qquad (1)$$

where $\mathbf{X}(k)=\mathrm{diag}\{s_1(k),\dots,s_N(k)\}$ is a diagonal matrix, $H=[H_1,\dots,H_N]^T$ is an N-point FFT of the CIR, $h(k)$, and $Z(k)=[Z_1(k),\dots,Z_N(k)]^T$ is a zero mean complex Gaussian vector whose autocorrelation matrix is $\mathbf{R}_z = N_0\mathbf{I}_N$ that N_0 is the variance of $Z_i(k)$ for $i=1,\dots,N$ and $\mathbf{I}_N$ is a $N\times N$ identity matrix.

The goal is to estimate H by observing $Y(k)$ based on the assumption that $\mathbf{X}(k)$ is known (full pilot training OFDM symbol is considered). The LS estimate of H is [1]

$$\hat{H}^{(LS)} = \mathbf{X}^{-1}Y = H + \mathbf{X}^{-1}Z \qquad (2)$$

We have dropped the argument "*k*" for the sake of simplicity. The LS estimator can be modified by assuming that the CIR length (or delay spread of channel) is known in OFDM system design [1]. As shown in Fig. 2, the LS estimate, $\hat{H}^{(LS)}$, is improved by taking inverse FFT (IFFT) of it and take into account only the first *L* taps (CIR interval) at the output of IFFT, $\hat{h}_L^{(LS)}=[\hat{h}_1^{(LS)},\dots,\hat{h}_L^{(LS)}]^T$, and then taking N-point FFT by adding *N-L* zero points. The modified LS estimator is a time filtering process, which reduces the noise power by eliminating the part of noise, which is outside the CIR subspace. In this method the improvement of channel estimation depends on the factor *L/N*, smaller *L/N* provides more improvement. We develop a new estimation algorithm in the next section to improve the channel estimation performance further.

III. CHNNNEL ESTIMATION USING NOISE REDUCTION

The performance of the modified LS estimator can be improved by noting that the noise of the modified LS estimation in the time domain (at the output of IFFT) is a colored one. Thus, the part of noise within the CIR (signal) subspace can be estimated based on using the other part, which is outside the signal subspace, as shown in Fig. 3. Note that the part of noise, which is outside the signal subspace, can be obtained by a simple time domain filtering.

Defining $V=[v_1,\dots,v_N]^T$ as

$$V = \mathrm{IFFT}(\mathbf{X}^{-1}Z) = \mathbf{W}^{-1}\mathbf{X}^{-1}Z \qquad (3)$$

where $\mathbf{W}$ is a $N\times N$ Fourier transform matrix, while $w_{i,l}$ is given by

$$w_{i,l} = \frac{1}{\sqrt{N}}e^{-j\frac{2\pi}{N}il} \qquad i,l=0,\dots,N-1$$

The goal is to estimate $V_L=[v_1,\dots,v_L]^T$, the part of noise within the signal subspace, from the part of noise which is outside the signal subspace, $V_{N-L}=[v_{L+1},\dots,v_N]^T$, based on *maximum a posteriori* criterion

$$\begin{aligned}\hat{V}_L &= \arg\max_{V_L}\{p(V_L \mid V_{N-L})\}\\ &= \arg\min_{V_L}\left\{[V_L^H, V_{N-L}^H]\,Q_V\begin{bmatrix}V_L\\ V_{N-L}\end{bmatrix}\right\}\end{aligned} \qquad (4)$$

where $(.)^H$ denotes transposed complex conjugate and Q_V is $N\times N$ matrix which is defined as

$$\begin{aligned}Q_V &= \left(E\left[VV^H\right]\right)^{-1} = \left(\mathbf{W}^{-1}\mathbf{x}^{-1}E\left[ZZ^H\right]\mathbf{x}^{-H}\mathbf{W}^{-H}\right)^{-1}\\ &= \frac{1}{N_0}\mathbf{W}^H\mathbf{x}^H\mathbf{x}\mathbf{W}\end{aligned} \qquad (5)$$

After doing some manipulation, it can be shown that

$$\hat{V}_L = -Q_1^{-1}Q_2V_{N-L} \qquad (6)$$

where $V_{N-L}=[\hat{h}_{L+1}^{(LS)},\dots,\hat{h}_N^{(LS)}]^T$ due to the assumption that

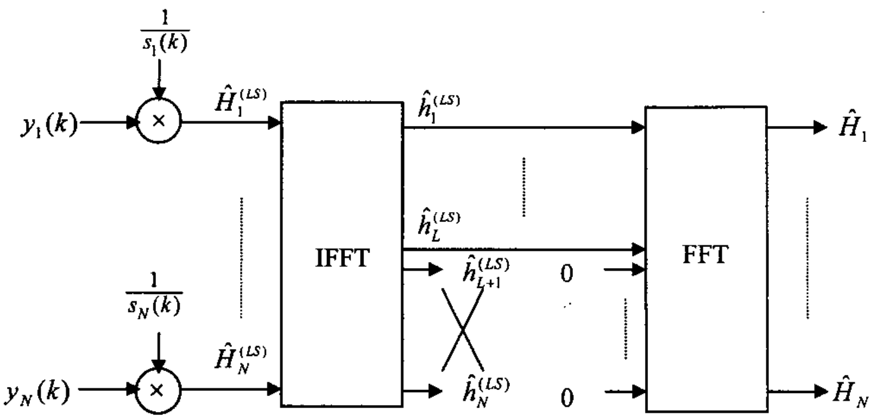

Fig. 2. Block diagram of the modified LS (MLS) channel estimation method.

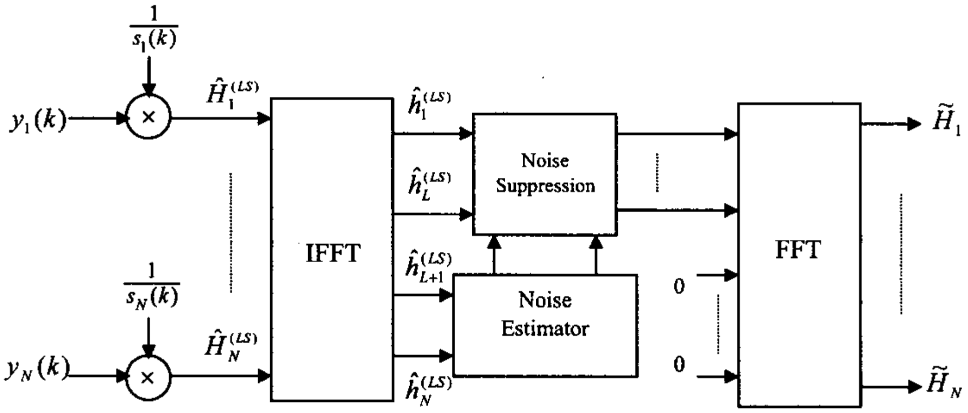

Fig. 3. Block diagram of the least squares channel estimator based on noise reduction (LSNR).

the CIR length is equal to or less than L (see Fig. 3), $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are defined as

$$\boldsymbol{Q}_1 = \begin{bmatrix} q_{1,1} \cdots q_{1,L} \\ \vdots \quad \ddots \\ q_{L,1} \cdots q_{L,L} \end{bmatrix}$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} q_{1,L+1} \cdots q_{1,N} \\ \vdots \quad \ddots \\ q_{L,L+1} \cdots q_{L,N} \end{bmatrix}$$

While $q_{i,j}$ is the *i,j*th element of $\boldsymbol{Q}$ matrix. Improved CIR estimation is obtained by subtracting $\hat{\boldsymbol{V}}_L$ from $\hat{\boldsymbol{h}}_L^{(LS)} = [\hat{h}_1^{(LS)},\ldots,\hat{h}_L^{(LS)}]^T$ and then taking N-point FFT by adding *N-L* zero points as shown in Fig. 3.

IV. SIMULATIONS AND COMPARISONS

The performance of the proposed channel estimation algorithm is evaluated by computer simulation in this section based on the normalized mean square error (NMSE) criterion

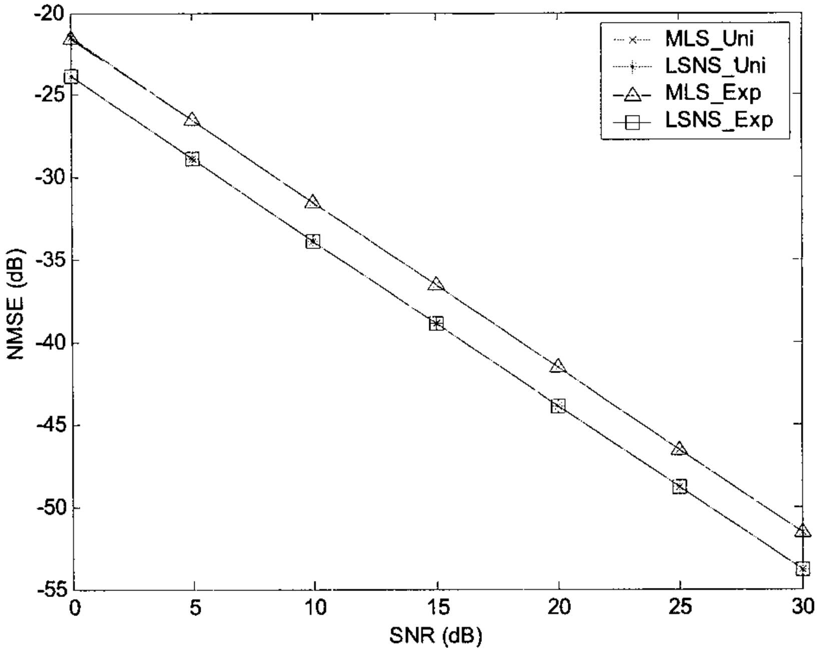

Fig. 4. The performances of the modified LS (MLS) estimation method and the least squares channel estimation using noise reduction algorithm (LSNR) based on the NMSE criterion for uniform and exponential channel models when $L=N/4$.

which is defined as

$$\mathrm{NMSE}_{\hat{H}} = \frac{\sum_{n=1}^{N} \left| H_n - \hat{H}_n \right|^2}{\sum_{n=1}^{N} \left| H_n \right|^2} \tag{7}$$

In order to highlight the improvement of the new channel estimation algorithm, the performances of the LS and the modified LS channel estimation methods are presented as well.

In the simulations, a 16QAM-OFDM system is considered with N=64 subcarriers. A multipath channel model with L taps is used with following impulse response.

$$h(k) = \sum_{l=0}^{L-1} \alpha_l e^{-\beta l} \delta(k-l) \tag{8}$$

where α_l is a zero mean complex Gaussian random process such that $E[\alpha_l \alpha_j] = 0$ for $l \neq j$ and β is exponential decay factor which is selected $\beta = 0$ for uniform channel model and $\beta = 3$ for exponential channel model in the simulations.

Fig. 4 shows the performances of the modified LS (MLS) and the least squares channel estimation using noise reduction algorithm (LSNR) based on the NMSE criterion for uniform and exponential channel models when $L = N/4$. As seen, the new channel estimation algorithm improves the mean square error of the modified LS algorithm by about 2.5dB. Also, as Fig. 4 shows the improved performances of the new algorithm are the same for uniform and exponential channel models.

The performances of LS, MLS and LSNR channel estimation algorithms based on the NMSE criterion are shown in Fig. 5 versus L/N (the number of CIR taps/the number of subcarriers) for uniform and exponential channel models when SNR=20dB. As mentioned before, the difference in performance between LS and MLS algorithms is a function of L/N. However, the difference in performance between MLS and LSNR (the new algorithm) remains approximately constant for L/N<0.5 which can be considered as an upper value in OFDM system design (the cyclic prefix is less than 50% of OFDM symbol interval). As seen in Fig. 5, the improved performance of the new algorithm is approximately the same for uniform and exponential channel models as well.

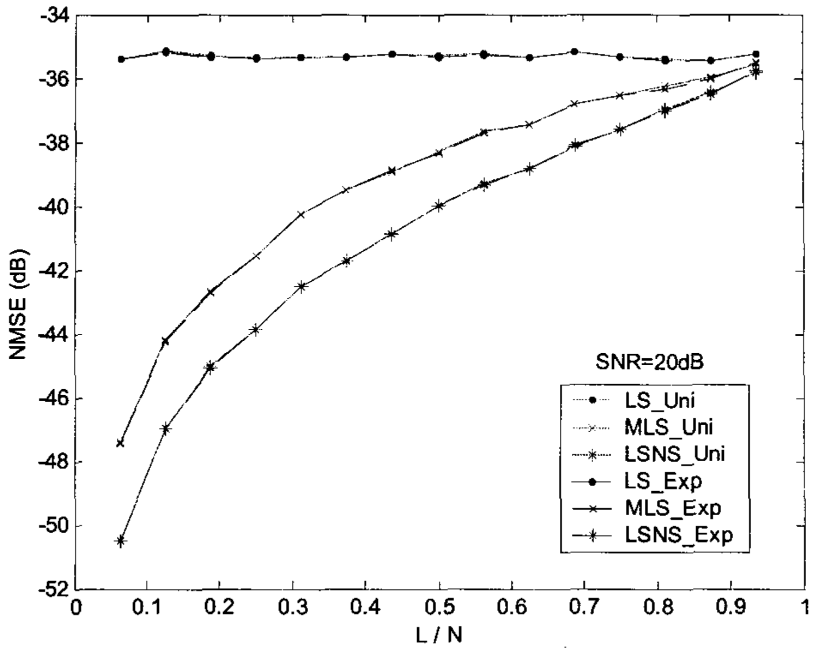

Fig. 5. The NMSE of the LS, MLS and LSNR (the new algorithm) channel estimators verses *L/N* when SNR=20dB.

V. CONCLUSIONS

We have developed a new channel estimation algorithm for OFDM systems based on a noise reduction method. The performance improvement in the new channel estimation algorithm has been achieved by estimating and suppressing the noise of the CIR subspace based on using the correlation between the two parts of noise which are within and outside the CIR subspace. Statistical parameters of channel need not be known in the new channel estimation algorithm and its performance is robust to variations of channel parameters.

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