

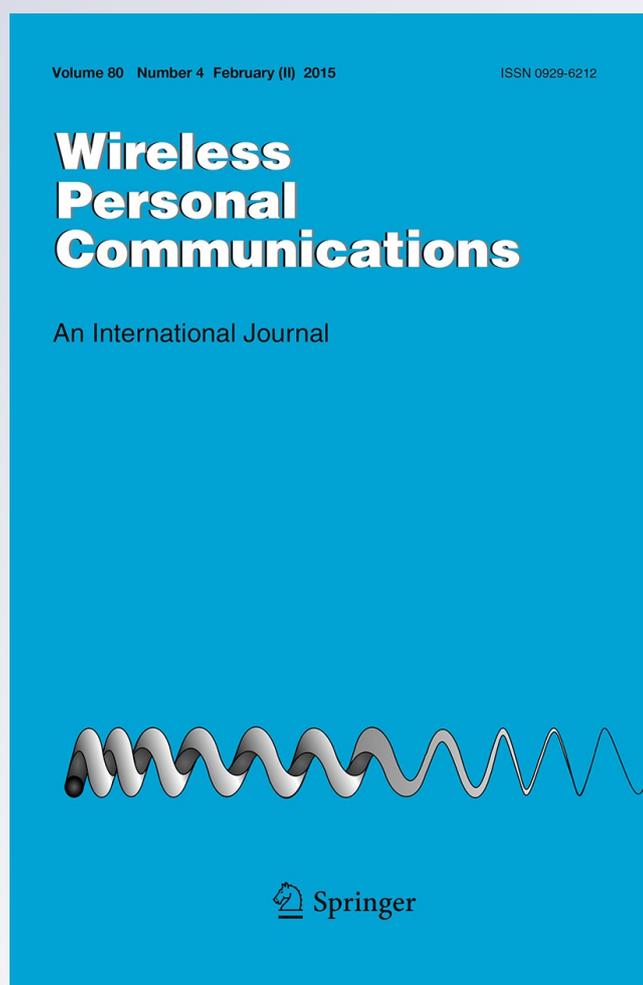
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Wireless Personal Communications
An International Journal

ISSN 0929-6212
Volume 80
Number 4

Wireless Pers Commun (2015)
80:1381-1404
DOI 10.1007/s11277-014-2089-0



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Sensitivity Analysis of OFDMA and SC-FDMA Uplink Systems to Carrier Frequency Offset

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Published online: 27 September 2014
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Abstract Orthogonal frequency division multiple access (OFDMA) and single carrier frequency division multiple access (SC-FDMA) are two technologies for the uplink transmission of present and next generation of broadband wireless systems. This paper studies and compares OFDMA and SC-FDMA in terms of sensitivity to carrier frequency offset (CFO) in the uplink. In order to calculate signal-to-interference ratio (SIR), we use a simple superposition principle approach where the contributions of different users are studied separately. We derive closed-form mathematical expressions for the desired signal, interference terms, and consequently SIR for both OFDMA and SC-FDMA systems in the uplink. It is pointed out that there is a strong relationship between sensitivity analysis to CFO and subcarrier allocation schemes. Also, we prove that the derived expressions for both systems are reduced to very simple forms in interleaved subcarrier allocation. Finally, the theoretical analysis are verified using Monte Carlo simulations in block, interleaved, and block-interleaved subcarrier allocations, and the two systems are compared upon these set of results.

Keywords Orthogonal frequency division multiple access (OFDMA) · Single carrier frequency division multiple access (SC-FDMA) · Carrier frequency offset (CFO) · Signal-to-interference ratio (SIR) · Inter-carrier interference (ICI) · Multiple access interference (MAI) · Inter-symbol interference (ISI)

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1 Introduction

Orthogonal frequency division multiplexing (OFDM) is widely used in many communication systems, e.g. digital audio broadcasting (DAB), digital video broadcasting (DVB), asymmetric digital subscriber lines (ADSL), and IEEE 802.11a/g. In OFDM, a set of equally spaced subcarriers are used for parallel data transmission. Furthermore, orthogonal frequency division multiple access (OFDMA) is a promising technology for broadband wireless communication that has been adopted by the IEEE 802.16e [1]. In OFDMA systems, the available subcarriers are divided into several mutually exclusive subsets assigned to different users for simultaneous transmission. The orthogonality among the subcarriers guarantees intrinsic protection against inter-carrier interference (ICI) and multiple access interference (MAI) [2]. Due to some disadvantages of OFDMA, such as high peak-to-average power ratio (PAPR) and high sensitivity to inevitable multiple carrier frequency offsets (CFOs), a modified form of OFDMA referred to as single carrier frequency division multiple access (SC-FDMA) was proposed by the Third Generation Partnership Project-Long Term Evolution (3GPP-LTE) for the uplink of the next generation cellular broadband wireless systems [3,4]. As in OFDMA, the transmitters in an SC-FDMA system use different orthogonal subcarriers to transmit information symbols. However, they transmit the subcarriers sequentially rather than in parallel [3]. In other words, in an SC-FDMA system the information symbols are spread over corresponding subcarriers using a discrete Fourier transform (DFT) precoder. The SC-FDMA has similar throughput performance and essentially the same overall complexity as OFDMA [3].

Similar to OFDM, OFDMA and SC-FDMA are sensitive to CFO caused by oscillator instabilities and/or wireless channel effects [2,3]. However, this is a more challenging issue in OFDMA and SC-FDMA uplink systems where different users are affected by different CFOs. Mutual orthogonality among the subcarriers in the presence of CFO is destroyed, and ICI and MAI are consequently introduced [2]. In an SC-FDMA system, ICI leads to inter-symbol interference (ISI).

The sensitivity analysis of OFDMA and SC-FDMA systems to CFO in the uplink is closely related to the subcarrier allocation scheme [5]. There are three major allocation schemes, namely block, interleaved, and general schemes [6]. In the block allocation, disjoint blocks of contiguous subcarriers are allocated to distinct users. In the interleaved allocation, subcarriers of each user are equally spaced over the whole transmission bandwidth. Due to the blockwise structure, the block allocation provides more robustness against CFO [6]. However, the block allocation is vulnerable to channel frequency selectivity. On the other hand, the interleaved allocation provides robust performance against channel frequency selectivity since spreading of subcarriers over the full band of transmission leads to maximum frequency diversity gain. Recently, another subcarrier allocation scheme, namely block-interleaved allocation, is under investigation to take advantages of both block and interleaved allocations. In the block-interleaved allocation, each user is allocated by small blocks of contiguous subcarriers that are equidistantly distributed over the transmission bandwidth. The current trend for both OFDMA and SC-FDMA, however, is the general subcarrier allocation scheme where users can select the best available subcarriers [2,3]. The general subcarrier allocation provides more flexibility than block and interleaved allocations, and it can make use of the best available multiuser and channel diversities. The SC-FDMA systems with block and interleaved allocations are also referred to localized FDMA (LFDMA) and interleaved FDMA (IFDMA), respectively [3]. Similar terminology are also used to refer to OFDMA systems.

Although the sensitivity analysis of OFDM and OFDMA systems to CFO has been investigated extensively in the literature [2,5,7–14], less attention has been paid to SC-FDMA

[15–18]. A broad review of synchronization methods in OFDMA is presented in [2]. In [5], one of the pioneering works on the sensitivity analysis of OFDMA uplink system to CFO is presented where the effects of CFO and timing offset have been studied over additive white Gaussian noise (AWGN) channels. Authors of this study obtained analytical expressions for MAI and emphasized its dependency on the subcarrier allocation scheme. The generalization of the analysis in [5] by including frequency selective channels has been performed in [11]. In [12], CFO has been modelled as an independent and identically distributed (i.i.d) random variable, and signal-to-interference-plus-noise ratio (SINR) has been derived as a function of the variance of CFO over different subcarriers. In [13], the combined effects of both CFO and timing offset on signal-to-interference ratio (SIR) have been investigated by considering different conditions on CFO and timing offset. In [14], sensitivity analysis of interleaved OFDMA uplink system to CFO has been studied. Few papers also have studied the SC-FDMA uplink system. In [15], two variants of SC-FDMA systems with block-interleaved subcarrier allocation scheme are considered and compared in terms of sensitivity to CFO. Authors have pointed out that these two variants are not similarly affected by CFO. In order to derive SINR, they have calculated the contribution of each transmitted symbol on each detected symbol. The result is a rather complex analysis. In [16], SIR performance of LFDMA system in the presence of CFO has been addressed and based on the analysis results a phase compensation method has been proposed. In [17], a comparison between OFDMA and SC-FDMA systems with block subcarrier allocation in the presence of large CFOs and timing offsets is presented. In this analysis, only block subcarrier allocation has been considered, and matrix notation has been used where the effect of multiple CFOs and the role of subcarrier allocation scheme lie behind the complex matrix relations. In [18], effects of multiple CFOs in multiple-input multiple-output (MIMO) SC-FDMA systems has been analyzed. Also, In this work matrix notation has been used, and authors have studied interference power seen by different users in order to analyze effects of multiple CFOs. They have compared LFDMA and IFDMA systems in terms of sensitivity to multiple CFOs.

This paper continues and develops the research of the previous works by specializing the sensitivity analysis of OFDMA uplink system for different subcarrier allocation scheme. In the first part of this paper, we present the mathematical analysis of effects of CFOs on OFDMA uplink systems using a simple superposition principle approach where the contributions of different users are considered separately. We derive closed-form mathematical expressions for the desired signal, interference terms, and consequently SIR, in a general subcarrier allocation. Then these are specialized for cases of block, interleaved, and block-interleaved allocations where we derive expressions that to the best of our knowledge are not presented in the previous related works. Interestingly, we will find that the results are reduced to very simple forms in interleaved subcarrier allocation. Such simplifications can be exploited to find more effective techniques for CFO compensation [19]. In the second part of this work, we extend our analysis for SC-FDMA uplink system. On the first thought, it may seem that SC-FDMA and OFDMA uplink systems are similarly affected by CFO; however, as we will see, they are affected by CFO in essentially different ways. For the case of SC-FDMA uplink, also, we will show that the results are reduced to very simple forms in interleaved subcarrier allocation. Simulation results are included in the last part of this paper where it is shown that they are in excellent match with the theoretical analysis in block, interleaved, and block-interleaved subcarrier allocation schemes.

The rest of this paper is organized as follows. The system model of a network that we consider in this paper is presented in Sect. 2. The SIR analysis of OFDMA and SC-FDMA uplink systems are addressed in Sects. 3 and 4, respectively. Simulation results and comparisons are included in Sect. 5, and finally the conclusions are drawn in Sect. 6.

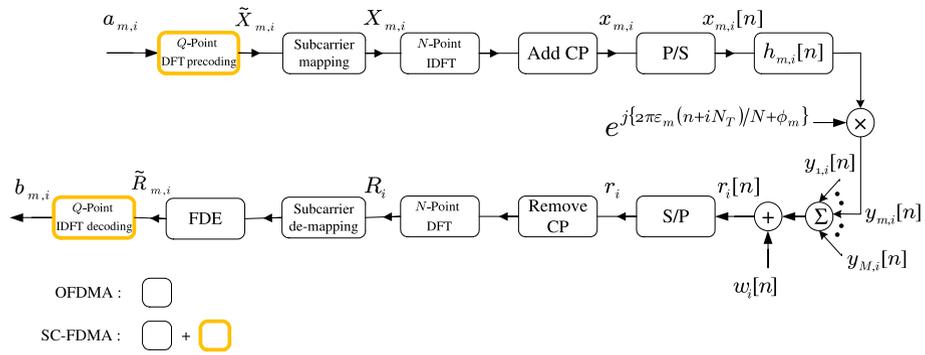


Fig. 1 The baseband equivalent of OFDMA/SC-FDMA uplink system for the i th transmitted block in the presence of CFOs

2 System Model

We consider the uplink of an OFDMA/SC-FDMA system where M active users are communicating with a base station. There are $N = MQ$ subcarriers where Q is the number of subcarriers allocated to each user. The number of subcarriers assigned to distinct users may be different, but here, without loss of generality, we assume that it is the same for all users. The index set of Q subcarriers assigned to the m th user is denoted by \mathcal{I}_m . Clearly $\bigcup_{m=1}^M \mathcal{I}_m = \{0, 1, \dots, N - 1\}$, and $\mathcal{I}_m \cap \mathcal{I}_j = \emptyset, \forall m \neq j$. Baseband discrete time block diagram of OFDMA/SC-FDMA uplink system in the presence of CFO is depicted in Fig. 1. The two systems are different from on the absence/presence of DFT precoding and IDFT decoding blocks for OFDMA and SC-FDMA respectively as highlighted in Fig. 1. In this figure, subscript m denotes the m th user, and subscript i denotes the i th uplink transmitted block. Also, N_g is the number of cyclic prefix (CP) samples, and $N_T = N_g + N$ is the total number of samples for an uplink block. In an OFDMA system, the data stream of each user is divided into blocks of Q symbols, and the i th block of the m th user (i.e., $\tilde{X}_{m,i}$) is assigned to its own subcarriers by the subcarrier mapping unit. Also, in an SC-FDMA system the data stream of each user is divided into blocks of Q symbols, and the i th block of the m th user (i.e., $a_{m,i}$) is precoded by a DFT unit and then the output block is assigned to the corresponding subcarriers by the subcarrier mapping unit. Thus, $X_{m,i}[k], k \in \mathcal{I}_m$ are frequency domain symbols of the m th user allocating to the corresponding subcarriers. We focus on the i th uplink block, and for notational simplicity the subscript i will be removed throughout this paper. The received signal at the base station is

$$r[n] = \sum_{m=1}^M y_m[n] + w[n], \tag{1}$$

where $w[n]$ is the AWGN, and $y_m[n]$ is the received signal from the m th user in the presence of CFO as

$$y_m[n] = e^{j\left\{\frac{2\pi\epsilon_m}{N}(n+iN_T)+\phi_m\right\}} (x_m[n] * h_m[n]), \tag{2}$$

where ϕ_m is the phase offset, and ϵ_m is the CFO normalized to the subcarrier spacing. We also assume that $-0.5 < \epsilon_m < 0.5$. Moreover, $*$ denotes the linear convolution, and $x_m[n]$ is the transmitted signal by the m th user :

$$x_m[n] = \frac{1}{N} \sum_{k \in \mathcal{I}_m} X_m[k] e^{j2\pi k(n-N_g)/N}; \quad n = 0, 1, \dots, N_T - 1. \quad (3)$$

The signal $x_m[n]$ is transmitted through a multipath fading channel which is assumed static over each uplink transmitted block, and $h_m[n]$ is the channel impulse response (CIR) between the m th user and the base station. The channel taps are assumed to be complex-valued and statistically independent circular Gaussian random variables with zero mean (Rayleigh fading) and exponential power delay profile, $E\{|h_m[n]|^2\} = \beta_m e^{-n/L_m}$; $n = 0, 1, \dots, L_m - 1$, where L_m is the CIR order, and β_m is a scaling factor for the average energy of CIR as $\sum_{n=0}^{L_m-1} E\{|h_m[n]|^2\} = \bar{\gamma}_m$. Furthermore, we assume that the users are time-synchronized. It can be shown that if $N_g > \max\{L_m + \theta_m\}$ where $\theta_m = \text{int}(\Delta t_m/T_s)$ is the normalized timing error to the sampling period, then the systems will be quasi-synchronous and the timing errors can be compensated for by the channel equalizer [2].

3 SIR Analysis for OFDMA

In this section, the sensitivity of OFDMA uplink to CFO is studied. We assume that the data streams of different users are independent of each other and also independent of the channel noise. Thus, we can apply the superposition principle to different users due to linearity of the system. At this point, we assume that the data streams of all users except the m th user are zero. In other words, we calculate the effect of desired user's CFO on all the subcarriers. Also, for simplicity the channel noise is ignored in this section. By substituting (2) in (1), we have

$$r[n] = y_m[n] = N c_m[n] e^{j\psi_m} (x_m[n] * h_m[n]), \quad (4)$$

where $c_m[n] \triangleq \frac{1}{N} e^{j2\pi \varepsilon_m n/N}$, and $\psi_m \triangleq 2\pi \varepsilon_m i N_T/N + \phi_m$. After removing CP from $r[n]$ and taking DFT, we have

$$R[k] = e^{j\psi_m} C_m[k] \circledast (H_m[k] X_m[k]); \quad k = 0, 1, \dots, N - 1, \quad (5)$$

where $C_m[k]$, $H_m[k]$, and $X_m[k]$, are the DFT of $c_m[n]$, $h_m[n]$, and $x_m[n]$, respectively, and \circledast denotes circular convolution. It can be shown that

$$C_m[k] = \frac{\sin(\pi(\varepsilon_m - k))}{N \sin(\frac{\pi}{N}(\varepsilon_m - k))} e^{j\pi(\varepsilon_m - k)(N-1)/N} = f_N(\varepsilon_m - k), \quad (6)$$

where $f_N(x) \triangleq \frac{\sin(\pi x)}{N \sin(\pi x/N)} e^{j\pi x(N-1)/N}$.

3.1 Desired Signal, ICI and MAI Power

Equation (5) may be rewritten as

$$R[k] = \overbrace{e^{j\psi_m} C_m[0] H_m[k] X_m[k]}^{\text{desired signal term}} + \overbrace{e^{j\psi_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} C_m[k-r] H_m[r] X_m[r]}^{\text{ICI term}}; \quad (7)$$

for $k \in \mathcal{I}_m$, and

$$R[k] = \overbrace{e^{j\psi_m} \sum_{r \in \mathcal{I}_m} C_m[k-r] H_m[r] X_m[r]}^{\text{MAI term}}; \quad (8)$$

for $k \notin \mathcal{I}_m$. In the absence of CFO, $R[k] = e^{j\phi_m} H_m[k] X_m[k]$, $k \in \mathcal{I}_m$ are the received symbols on the subcarriers of the m th user. Note that this is free of any ICI term. Also, we do not have any interference over subcarriers of other users, i.e. $R[k] = 0$, $k \notin \mathcal{I}_m$. Therefore, in the presence of CFO, the first term in (7) for $k \in \mathcal{I}_m$ is the desired signal, and the second term is the ICI caused by the m th user's CFO. The term for $k \notin \mathcal{I}_m$ in (8) is the MAI caused by the m th user over the k th subcarrier belonging to other users. We denote the average power of these terms by $P_S(k)$, $P_{ICI}(k)$, and $P_{MAI}(m, k)$, respectively. The information symbols of each user are often chosen from a quadrature amplitude modulation (QAM) constellation and are zero mean and uncorrelated. They are directly mapped on the subcarriers assigned to the user so $E\{X_m[r]X_m^*[r']\} = \sigma_m^2 \delta[r - r']$. These symbols are also independent of the channel taps. In addition, it can be shown that $E\{|H_m[k]|^2\} = \bar{\gamma}_m$. Using these facts, the calculation of $P_S(k)$, $P_{ICI}(k)$, and $P_{MAI}(m, k)$, are performed as follows.

For the desired signal power, $P_S(k)$, we have

$$\begin{aligned}
 P_S(k) &= |C_m[0]|^2 E\{|H_m[k]X_m[k]|^2\} \\
 &= |f_N(\varepsilon_m)|^2 \sigma_m^2 \bar{\gamma}_m; \quad k \in \mathcal{I}_m,
 \end{aligned}
 \tag{9}$$

where $\sigma_m^2 = E\{|X_m[k]|^2\}$.

The average ICI power, $P_{ICI}(k)$, is obtained as

$$\begin{aligned}
 P_{ICI}(k) &= E\left\{\left|\sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} C_m[k-r] H_m[r] X_m[r]\right|^2\right\} \\
 &= \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} |f_N(\varepsilon_m - k + r)|^2 \sigma_m^2 \bar{\gamma}_m \\
 &\triangleq P_{ICI}^n(k) \sigma_m^2 \bar{\gamma}_m; \quad k \in \mathcal{I}_m,
 \end{aligned}
 \tag{10}$$

where $P_{ICI}^n(k)$ is defined as ICI power normalized to $\sigma_m^2 \bar{\gamma}_m$. For $k \in \mathcal{I}_m$, $P_{ICI}^n(k)$ is the normalized ICI power caused by the m th user on the k th subcarrier. In general, the normalized ICI power for $k \in \mathcal{I}_m$ depends on the system parameters, the m th user's CFO, and the subcarrier index.

Similarly, MAI power caused by the m th user over the subcarriers of other users is obtained as

$$\begin{aligned}
 P_{MAI}(m, k) &= E\left\{\left|\sum_{r \in \mathcal{I}_m} C_m[k-r] H_m[r] X_m[r]\right|^2\right\} \\
 &= \sum_{r \in \mathcal{I}_m} |f_N(\varepsilon_m - k + r)|^2 \sigma_m^2 \bar{\gamma}_m \\
 &\triangleq P_{MAI}^n(m, k) \sigma_m^2 \bar{\gamma}_m; \quad k \notin \mathcal{I}_m,
 \end{aligned}
 \tag{11}$$

where $P_{MAI}^n(m, k)$ is defined as MAI power normalized to $\sigma_m^2 \bar{\gamma}_m$. For $k \notin \mathcal{I}_m$, $P_{MAI}^n(m, k)$ is the normalized MAI power caused by the m th user over the k th subcarrier which does not belong to this user. Generally, the normalized MAI power also depends on the system parameters, the m th user's CFO, and the subcarrier index.

3.2 SIR Calculation

We use the superposition principle to calculate SIR of different received symbols on the corresponding subcarriers. The signal power is calculated according to (9), and the interference power consists of ICI power caused by the desired user plus MAI powers caused by other users. Using (9)–(11), we obtain

$$SIR(k) = \frac{|f_N(\varepsilon_m)|^2 \sigma_m^2 \bar{\gamma}_m}{P_{ICI}^n(k) \sigma_m^2 \bar{\gamma}_m + \sum_{\substack{m'=1 \\ m' \neq m}}^M P_{MAI}^n(m', k) \sigma_{m'}^2 \bar{\gamma}_{m'}}; \quad k \in \mathcal{I}_m. \quad (12)$$

If the average energy of the CIR is assumed to be unity (i.e., $\bar{\gamma}_m = 1, \forall m$), and perfect power control is assumed (i.e., $\sigma_m^2 = \sigma_X^2, \forall m$), then (12) is simplified to

$$SIR(k) = \frac{|f_N(\varepsilon_m)|^2}{P_{ICI}^n(k) + \sum_{\substack{m'=1 \\ m' \neq m}}^M P_{MAI}^n(m', k)}; \quad k \in \mathcal{I}_m. \quad (13)$$

This result may be written in a more explicit form, if we note from (10) that

$$P_{ICI}^n(k) = \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} |f_N(\varepsilon_m - k + r)|^2; \quad k \in \mathcal{I}_m \quad (14)$$

and from (11),

$$P_{MAI}^n(m, k) = \sum_{r \in \mathcal{I}_m} |f_N(\varepsilon_m - k + r)|^2; \quad k \notin \mathcal{I}_m. \quad (15)$$

3.3 Interleaved Allocation

In the following, we will show that (14) and (15) reduce to very simple forms in interleaved allocation scheme. In this allocation, the index set of the subcarriers assigned to the m th user can be written as $\mathcal{I}_m = \{i_m + qM; q = 0, 1, \dots, Q - 1\}$ where i_m is an integer number within the interval $[0, M - 1]$. Under this condition, it can be proved that the normalized ICI and MAI powers are respectively simplified to (see Appendix 1)

$$P_{ICI}^n(k) = |f_M(\varepsilon_m)|^2 - |f_N(\varepsilon_m)|^2; \quad k \in \mathcal{I}_m \quad (16)$$

$$P_{MAI}^n(m, k) = |f_M(i_m - i_{m'} + \varepsilon_m)|^2; \quad k \in \mathcal{I}_{m'}, m' \neq m. \quad (17)$$

According to (16), the normalized ICI powers on all subcarriers of the m th user are the same, and depend only on ε_m . From (17), we see that the normalized MAI powers caused by the m th user over all subcarriers of the m' th ($m' \neq m$) user are also the same, and depend on ε_m and on the spacing between the subcarriers of the two users. In Fig. 2, the normalized desired signal, ICI, and MAI, powers are shown as a function of normalized CFO for an interleaved OFDMA uplink system with $N = 64$ subcarriers and $M = 4$ active users. As seen from this figure, the MAI for $i_m - i_{m'} = 1$ and $i_m - i_{m'} = M - 1$ are the dominant interference terms in interleaved subcarrier allocation.

3.4 Block Allocation

In this allocation, the index set of the subcarriers assigned to the m th user can be written as $\mathcal{I}_m = \{i_m Q + q; q = 0, 1, \dots, Q - 1\}$ where i_m is an integer within the interval $[0, M - 1]$. For the normalized ICI power on the k th subcarrier, $k \in \mathcal{I}_m$, we obtain (see Appendix 1)

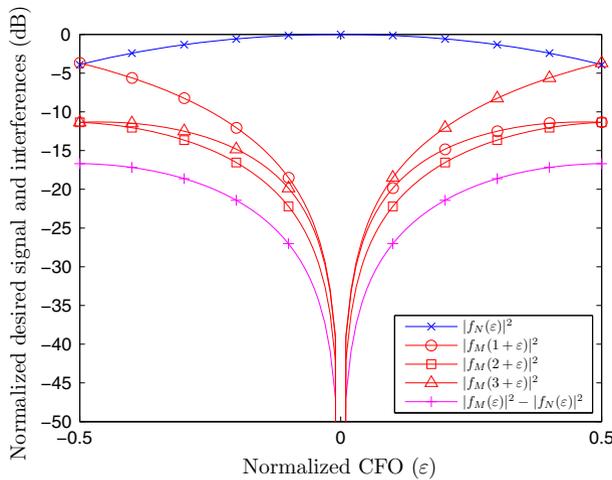


Fig. 2 The normalized desired signal, ICI, and MAI powers, as functions of normalized CFO for an interleaved OFDMA uplink system with $N = 64$ subcarriers and $M = 4$ active users

$$P_{ICI}^n(k) \approx \begin{cases} 1 - |f_N(\varepsilon_m)|^2 - \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi Q}{N}\right)}{N\pi \left(\cos\left(\frac{\pi}{N}(Q-2d-1+2\varepsilon_m)\right) - \cos\left(\frac{\pi Q}{N}\right)\right)}, & 1 \leq d \leq Q-2; \\ |f_N(1 \pm \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi(Q-2)}{N}\right)}{N\pi \left(\cos\left(\frac{\pi(Q-2)}{N}\right) - \cos\left(\frac{\pi}{N}(Q+1 \pm 2\varepsilon_m)\right)\right)}, & d = 0, Q-1, \end{cases} \quad (18)$$

where $d \triangleq k - i_m Q$ is the distance of the k th subcarrier from the beginning of the m th user's block. From (18), we find that the normalized ICI power on the m th user's subcarriers not only depends on ε_m , but also depends on the position of the subcarriers in the block of this user. In a similar way, for the normalized MAI power over the k th subcarrier, $k \notin \mathcal{I}_m$, we have (see Appendix 1)

$$P_{MAI}^n(m, k) \approx \begin{cases} \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi Q}{N}\right)}{N\pi \left(\cos\left(\frac{\pi Q}{N}\right) - \cos\left(\frac{\pi}{N}(Q-2d-1+2\varepsilon_m)\right)\right)}, & Q+1 \leq ((d))_N \leq N-2; \\ |f_N(1 \mp \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi(Q-1)}{N}\right)}{N\pi \left(\cos\left(\frac{\pi(Q-1)}{N}\right) - \cos\left(\frac{\pi}{N}(Q+2 \mp 2\varepsilon_m)\right)\right)}, & ((d))_N = Q, N-1, \end{cases} \quad (19)$$

where $d \triangleq k - i_m Q$, and $((d))_N$ is the value of d reduced to the interval $[0, N-1]$. From (19), we see that the normalized MAI power also depends on ε_m and on the position of the subcarriers with respect to the m th user's block. As seen from (18) and (19), they are more complex compared to corresponding equations of (16) and (17) in interleaved allocation, but still relatively straightforward equations.

3.5 Block-Interleaved Allocation

In the block-interleaved allocation, the index set of the subcarriers assigned to the m th user can be written as $\mathcal{I}_m = \{(i_m + lM)P + p; l = 0, 1, \dots, L - 1, p = 0, 1, \dots, P - 1\}$ where i_m is an integer number within the interval $[0, M - 1]$; L is the number of interleaved segments allocated to each user, and P is the number of contiguous subcarriers in each segment. Each user has Q subcarriers, therefore $LP = Q$. Since, block-interleaved allocation can be viewed as superposition of some interleaved allocation schemes, one can calculate the normalized ICI and MAI powers in this allocation using (16)–(19). In the following, we present the calculation results without proof, as the derivations are straightforward and follow those of the previous equations. For the normalized ICI power on the k th subcarrier, $k \in \mathcal{I}_m$, we have

$$P_{\text{ICI}}^n(k) \approx |f_{MP}(\varepsilon_m)|^2 - |f_N(\varepsilon_m)|^2 + \begin{cases} 1 - |f_{MP}(\varepsilon_m)|^2 - \frac{2 \sin^2(\pi \varepsilon_m) \sin(\frac{\pi}{M})}{MP\pi(\cos(\frac{\pi}{MP}(P-2d-1+2\varepsilon_m)) - \cos(\frac{\pi}{M}))}, & 1 \leq d \leq P - 2; \\ |f_{MP}(1 \pm \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin(\frac{\pi(P-2)}{MP})}{MP\pi(\cos(\frac{\pi(P-2)}{MP}) - \cos(\frac{\pi}{MP}(P+1 \pm 2\varepsilon_m)))}, & d = 0, P - 1, \end{cases} \tag{20}$$

where $d \triangleq ((k))_{MP} - i_m P$. Similarly for the normalized MAI power over the k th subcarrier, $k \notin \mathcal{I}_m$, we obtain

$$P_{\text{MAI}}^n(m, k) \approx \begin{cases} A, & P + 1 \leq ((d))_{MP} \leq MP - 2; \\ B, & ((d))_{MP} = P, MP - 1, \end{cases} \tag{21}$$

$$A = \frac{2 \sin^2(\pi \varepsilon_m) \sin(\frac{\pi}{M})}{MP\pi(\cos(\frac{\pi}{M}) - \cos(\frac{\pi}{MP}(p-2d-1+2\varepsilon_m)))},$$

$$B = |f_{MP}(1 \mp \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin(\frac{\pi(P-1)}{MP})}{MP\pi(\cos(\frac{\pi(P-1)}{MP}) - \cos(\frac{\pi}{MP}(P+2 \mp 2\varepsilon_m)))},$$

where $d \triangleq ((k))_{MP} - i_m P$. In block-interleaved allocation, behavior of ICI and MAI powers over interleaved segments of each user is periodic as in interleaved allocation and their behavior over the adjacent segments of different users is similar to block allocation.

3.6 Generalization

The analysis of SINR can be performed with an approach similar to (13), when AWGN is present. Derivation is straightforward and omitted hence because of lack of space. The result is

$$\text{SINR}(k) = \frac{|f_N(\varepsilon_m)|^2 \text{SNR}_0}{\left(P_{\text{ICI}}^n(k) + \sum_{\substack{m'=1 \\ m' \neq m}}^M P_{\text{MAI}}^n(m', k) \right) \text{SNR}_0 + 1}; \quad k \in \mathcal{I}_m, \tag{22}$$

where $\text{SNR}_0 = \frac{\sigma_s^2}{\sigma_w^2}$ is the received SNR in the absence of CFO, and $\sigma_w^2 = E\{|W[k]|^2\}$ is the AWGN average power on the given subcarrier at the receiver.

The above analysis is also applicable when some users are absent and the number of active users is less than M . In the proposed analysis, the channel equalizer is not considered. It is straightforward to show that the channel equalization using a bank of one-tap multipliers in the frequency domain will not change (22). Indeed, linear channel equalization using a bank of one-tap multipliers affects the desired signal and interferences by the same factor and ,

hence, does not change their power ratio. Therefore, these results are applicable to sensitivity analysis of CFO after equalization.

4 SIR Analysis For SC-FDMA

In this section, we study the sensitivity analysis of SC-FDMA uplink system to CFO. Since there are many common units in the structure of SC-FDMA and OFDMA systems, their corresponding mathematical relations are the same, and in order to not repeat them, we use the same notation for both systems. In an SC-FDMA system, as it was shown in Sect. 2, the symbols of each user are first precoded by a DFT unit, and then the output block is mapped onto the corresponding subcarriers. For the m th user, this mapping is one to one, taking the set $\{0, 1, \dots, Q - 1\}$ and maps onto \mathcal{I}_m . We denote this by $\{0, 1, \dots, Q - 1\} \mapsto \mathcal{I}_m$. The rule of the inverse mapping $\mathcal{I}_m \mapsto \{0, 1, \dots, Q - 1\}$ is expressed by the function $g_m(k), k \in \mathcal{I}_m$ and accordingly symbols of the m th user in the frequency domain, at the input of the N -point IDFT block in Fig. 1, are expressed as

$$X_m[k] = \begin{cases} \frac{1}{\sqrt{Q}} \sum_{u=0}^{Q-1} a_m[u] e^{-j2\pi u g_m(k)/Q}, & k \in \mathcal{I}_m; \\ 0, & k \notin \mathcal{I}_m, \end{cases} \tag{23}$$

where $\{a_m[u]\}_{u=0}^{Q-1}$ are the information symbols of the m th user. Similarly, the information symbols can be reconstructed from the frequency domain symbols using IDFT as follows

$$a_m[u] = \frac{1}{\sqrt{Q}} \sum_{k \in \mathcal{I}_m} X_m[k] e^{j2\pi u g_m(k)/Q}; \quad u = 0, 1, \dots, Q - 1. \tag{24}$$

The symbols of different users are independent of each other and also independent of the channel noise. Therefore, we can again apply the superposition principle to different users due to the linearity of the system. In the following, we assume that the symbols of all users except the m th user are zero, and then we calculate the effect of desired user's CFO on all the received symbols. By this assumption and following similar approach that led to (4) to (6), received frequency domain symbols can be obtained according to (5).

To proceed further, we suppose that the channels between the users and base station are flat fading (similar to [15] and [16]). In other words, the channel order is assumed to be zero for all users, or $L_m = 1, \forall m$. Therefore, $H_m[k] = H_m, \forall k$ where H_m is a complex Gaussian random variable with zero mean and variance of $E\{|H_m|^2\} = \bar{\gamma}_m$. This assumption will ease the theoretical analysis by providing easier frequency domain equalization (FDE) using the same multiplier coefficients for all the subcarriers. For the m th user, the multiplier coefficient of the channel equalizer is denoted by D_m , and for a zero forcing (ZF) equalizer, it is clear that $D_m \cdot H_m = 1$. Although here the theoretical analysis is only performed for flat fading channels, the simulation results will be examined for both flat fading and frequency selective channels in the next section. After channel equalization, the m th user's received symbols are extracted using the IDFT decoding according to (24), and the u th received symbol of the m th user is obtained as

$$b_m[u] = \frac{D_m H_m e^{j\psi_m}}{\sqrt{Q}} \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} C_m[k - r] X_m[r] e^{j2\pi u g_m(k)/Q}; \quad u = 0, 1, \dots, Q - 1. \tag{25}$$

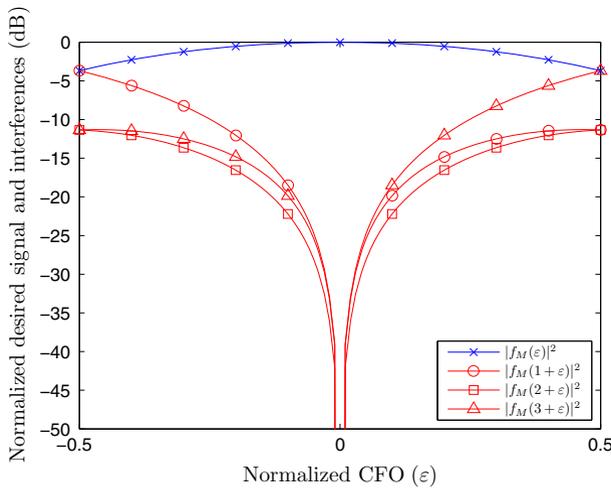


Fig. 3 The normalized desired signal and MAI powers as functions of normalized CFO for an IFDMA uplink system with $N = 64$ subcarriers and $M = 4$ active users

4.1 Desired Signal, ISI, and MAI Power

By substituting (23) in (25) and some straightforward manipulations, (25) can be rearranged as follows

$$\begin{aligned}
 b_m[u] = & \overbrace{a_m[u] \left(\frac{e^{j\psi_m}}{Q} \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} C_m[k-r] e^{j2\pi u(g_m(k) - g_m(r))/Q} \right)}^{\text{desired signal term}} \\
 & + \overbrace{\frac{e^{j\psi_m}}{Q} \sum_{k \in \mathcal{I}_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} C_m[k-r] a_m[v] e^{j2\pi(u g_m(k) - v g_m(r))/Q}}^{\text{ISI term}}. \quad (26)
 \end{aligned}$$

The first term in (26) is the desired signal term, and the second one is the ISI term caused by the m th user's CFO. Note that here, ICI leads to ISI, and, unlike in OFDMA, the information symbols are not directly extracted from frequency domain signal samples. In a similar way, MAI on the u th received symbol of the m' th ($m' \neq m$) user is given by

$$b_{m'}[u] = \overbrace{\frac{D_{m'} H_m e^{j\psi_m}}{\sqrt{Q}} \sum_{k \in \mathcal{I}_{m'}} \sum_{r \in \mathcal{I}_m} C_m[k-r] X_m[r] e^{j2\pi u g_{m'}(k)/Q}}^{\text{MAI term}}. \quad (27)$$

We denote the average power of the desired signal, ISI, and MAI terms, by $P_S(m, u)$, $P_{ISI}(m, u)$, and $P_{MAI}(m, m', u)$, respectively. In order to calculate these average powers, there are two points that we should take note of. First, the information symbols of each user are often chosen from a QAM constellation and are zero mean and uncorrelated. Therefore, $E\{a_m[v] a_m^*[v']\} = \sigma_m^2 \delta[v - v']$. These symbols are also independent of the channel taps, and hence the multiplier coefficients of the channel equalizer. Second, according to (23), it

is easy to show that the transmitted frequency domain symbols are also uncorrelated random variables with zero mean and have the same variance as the information symbols. In other words, $E\{X_m[r]X_m^*[r']\} = \sigma_m^2\delta[r - r']$. For the desired signal power, $P_S(m, u)$, we have

$$\begin{aligned} P_S(m, u) &= E\left\{\left|a_m[u]\left(\frac{e^{j\psi_m}}{Q} \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} C_m[k - r]e^{j2\pi u(g_m(k) - g_m(r))/Q}\right)\right|^2\right\} \\ &= \frac{1}{Q^2} \left| \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} f_N(\varepsilon_m - k + r)e^{j2\pi u(g_m(k) - g_m(r))/Q} \right|^2 \sigma_m^2 \\ &\triangleq P_S^n(m, u)\sigma_m^2, \end{aligned} \tag{28}$$

where $P_S^n(m, u)$ is defined as desired signal power normalized to σ_m^2 . The $P_S^n(m, u)$ is the normalized desired signal power for the u th received symbol of the m th user in the presence of CFO. For the average ISI power, $P_{ISI}(m, u)$, we have

$$\begin{aligned} P_{ISI}(m, u) &= E\left\{\left|\frac{e^{j\psi_m}}{Q} \sum_{k \in \mathcal{I}_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} C_m[k - r]a_m[v]e^{j2\pi(u g_m(k) - v g_m(r))/Q}\right|^2\right\} \\ &= \frac{1}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{k \in \mathcal{I}_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} f_N(\varepsilon_m - k + r)e^{j2\pi(u g_m(k) - v g_m(r))/Q} \right|^2 \sigma_m^2 \\ &\triangleq P_{ISI}^n(m, u)\sigma_m^2, \end{aligned} \tag{29}$$

where $P_{ISI}^n(m, u)$ is defined as ISI power normalized to σ_m^2 . The $P_{ISI}^n(m, u)$ is the normalized ISI power caused by the m th user's CFO on its u th received symbol. In a similar way, the MAI power caused by the m th user on the u th received symbol of the m' th ($m' \neq m$) user is given by

$$\begin{aligned} P_{MAI}(m, m', u) &= E\left\{\left|\frac{D_{m'}H_m e^{j\psi_m}}{\sqrt{Q}} \sum_{k \in \mathcal{I}_{m'}} \sum_{r \in \mathcal{I}_m} C_m[k - r]X_m[r]e^{j2\pi u g_{m'}(k)/Q}\right|^2\right\} \\ &= \frac{1}{Q} \sum_{r \in \mathcal{I}_m} \left| \sum_{k \in \mathcal{I}_{m'}} f_N(\varepsilon_m - k + r)e^{j2\pi u g_{m'}(k)/Q} \right|^2 \sigma_m^2 \bar{\alpha}_{m'} \bar{\gamma}_m \\ &\triangleq P_{MAI}^n(m, m', u)\sigma_m^2 \bar{\alpha}_{m'} \bar{\gamma}_m, \end{aligned} \tag{30}$$

where $\bar{\alpha}_{m'} = E\{|D_{m'}|^2\}$, and $P_{MAI}^n(m, m', u)$ is defined as MAI power normalized to $\sigma_m^2 \bar{\alpha}_{m'} \bar{\gamma}_m$. Indeed, $P_{MAI}^n(m, m', u)$ is the normalized MAI power caused by the m th user's CFO over the u th received symbol of the m' th ($m' \neq m$) user. In general, the normalized desired signal, ISI, and MAI powers, depend on the system parameters, the m th user's CFO, and the given symbol index.

4.2 SIR Calculation

Now, we are ready to use the superposition principle to calculate SIR over different received symbols. The signal power is calculated according to (28), and the interference power consists of the ISI power caused by the desired user's CFO plus MAI powers caused by other users over the given symbol. As a result, the SIR over the u th received symbol of the m th user is equal to

$$SIR(m, u) = \frac{P_S^n(m, u)\sigma_m^2}{P_{ISI}^n(m, u)\sigma_m^2 + \sum_{\substack{m'=1 \\ m' \neq m}}^M P_{MAI}^n(m', m, u)\sigma_m^2 \bar{\alpha}_m \bar{\gamma}_{m'}}. \tag{31}$$

If the average energy of the CIR and the variance of the channel equalizer multiplier coefficients are assumed to be unity (i.e., $\bar{\gamma}_m = 1$ & $\bar{\alpha}_m = 1, \forall m$), and perfect power control is assumed (i.e., $\sigma_m^2 = \sigma_X^2, \forall m$), then (31) reduces to

$$SIR(m, u) = \frac{P_S^n(m, u)}{P_{ISI}^n(m, u) + \sum_{\substack{m'=1 \\ m' \neq m}}^M P_{MAI}^n(m', m, u)}. \tag{32}$$

Generally, $P_S^n(m, u)$, $P_{ISI}^n(m, u)$, and $P_{MAI}^n(m, m', u)$, are calculated according to (28), (29), and (30), respectively, and they take different forms for different subcarrier allocation schemes.

4.3 Interleaved Allocation

As mentioned before, for interleaved case, the index set of subcarriers assigned to the m th user can be written as $\mathcal{I}_m = \{i_m + qM; q = 0, 1, \dots, Q - 1\}$ where i_m is an integer number within the interval $[0, M - 1]$. Furthermore, the inverse rule of the mapping that assigns the output block of the DFT precoder to the corresponding subcarriers is $g_m(k) = (k - i_m)/M$. Under this condition, it can be shown that the normalized desired signal, ISI, and MAI powers, are respectively simplified to (see Appendix 2)

$$P_S^n(m, u) = |f_M(\varepsilon_m)|^2, \tag{33}$$

$$P_{ISI}^n(m, u) = 0, \tag{34}$$

and

$$P_{MAI}^n(m, m', u) = |f_M(i_m - i_{m'} + \varepsilon_m)|^2. \tag{35}$$

From (33)–(35), the normalized desired signal, ISI, and MAI powers, are defined in very simple forms. According to (33), the normalized desired signal power for all received symbols of the m th user are the same and depend only on ε_m . According to (34), there is no ISI on received symbols in interleaved subcarrier allocation, and (35) shows that the normalized MAI power caused by the m th user's CFO over all received symbols of the m' th ($m' \neq m$) user are also the same and depend on ε_m and on the spacing between the subcarriers of the two users. In Fig. 3, the normalized desired signal and MAI powers are shown as a function of normalized CFO for an interleaved SC-FDMA uplink system with $N = 64$ subcarriers and $M = 4$ active users. From Fig. 3, it is seen that similar to interleaved OFDMA, the MAI for $i_m - i_{m'} = 1$ and $i_m - i_{m'} = M - 1$ are the dominant interference terms in the interleaved SC-FDMA system.

4.4 Block Allocation

Calculation of the normalized desired signal, ISI, and MAI powers, for an SC-FDMA system in block allocation is not as simple as it was for an OFDMA system. So, here we can only present some general expressions for these terms. The normalized desired signal, ISI, and MAI powers, in block allocation are given by

$$P_S^n(m, u) = \frac{1}{Q^2} \left| \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} f_N(\varepsilon_m - k + r) e^{j2\pi u(g_m(k) - g_m(r)) / Q} \right|^2, \tag{36}$$

$$P_{ISI}^n(m, u) = \frac{1}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{k \in \mathcal{I}_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} f_N(\varepsilon_m - k + r) e^{j2\pi(u g_m(k) - v g_m(r)) / Q} \right|^2, \tag{37}$$

and

$$P_{MAI}^n(m, m', u) = \frac{1}{Q} \sum_{r \in \mathcal{I}_m} \left| \sum_{k \in \mathcal{I}_{m'}} f_N(\varepsilon_m - k + r) e^{j2\pi u g_{m'}(k) / Q} \right|^2, \tag{38}$$

where $\mathcal{I}_m = \{i_m Q + q; q = 0, 1, \dots, Q - 1\}$ is the index set of the subcarriers assigned to the m th user with i_m as an integer number within the interval $[0, M - 1]$, and $g_m(k) = k - i_m Q$ is the inverse rule of the mapping that assigns the output block of the DFT precoder to the corresponding subcarriers.

4.5 Block-Interleaved Allocation

In this allocation, we can also obtain the normalized desired signal, ISI, and MAI powers, from (36) to (38), respectively, wherein $\mathcal{I}_m = \{(i_m + lM)P + p; l = 0, 1, \dots, L - 1, p = 0, 1, \dots, P - 1\}$, and $g_m(k) = \lfloor \frac{k}{MP} \rfloor P + ((k))_P$.

4.6 Comparison of SC-FDMA and OFDMA

One of the major differences between SC-FDMA and OFDMA is related to their single carrier versus multicarrier intrinsic natures. In multicarrier systems, ISI caused by multipath fading channel is eliminated via a CP of sufficient duration. Therefore, the quality of the channel equalization does not affect the derived expression for SIR of an OFDMA system; however, an appropriate channel equalization is necessary for coherent symbols detection. On the other hand, single carrier systems deal with ISI via time or frequency domain channel equalization. So, the quality of the channel equalization has a significant effect on the SIR performance of an SC-FDMA system. In other words, in an SC-FDMA system some ISI are introduced by CFO, and some ISI are introduced because of channel frequency selectivity and imperfect channel equalization.

Also, we can compare SC-FDMA and OFDMA in different subcarrier allocation schemes. In interleaved allocation, by comparison of (16)–(17) with (33)–(35), we see that the SIR performance of the two systems are very close to each other under the condition that they are evaluated. From Figs. 2 and 3, it is seen that MAI is the dominant interference term for both systems and is identical in them. However, the normalized desired signal power in SC-FDMA is slightly more than that of OFDMA, and the normalized ISI power in SC-FDMA is less than the normalized ICI power in OFDMA. So, we expect that interleaved SC-FDMA for flat fading channel to have a slightly better SIR performance than interleaved OFDMA.

Comparison of SC-FDMA and OFDMA in block allocation through comparison of (18)–(19) with (36)–(38) is difficult, since these expressions have relatively complex forms. However, we can plot them using computer programs under different conditions and for different received symbols. By doing so, we see that for most of the received symbols, ICI is the dominant interference term in an OFDMA system, but in an SC-FDMA system neither ISI nor MAI are dominant. In an SC-FDMA, not only the normalized desired signal power is

significantly more than that of OFDMA, but also the normalized ISI and MAI powers are significantly less than the normalized ICI and MAI powers in an OFDMA. So, we expect that SC-FDMA in block allocation to outperform OFDMA by a significant margin for a flat fading channel. These expected behavior and differences of OFDMA and SC-FDMA are explored numerically in the following section.

5 Simulation Results

In this section the theoretical analysis are examined using Monte Carlo simulations. We consider $M = 4$ active users communicating with a base station in the uplink of a network. The total number of subcarriers is assumed to be¹ $N = 64$ where $Q = 16$ subcarriers are allocated to each user. Furthermore, in the block-interleaved allocation, the number of subcarriers per segment is considered to be $P = 4$. The vectors $\Upsilon_1 = [0.04, 0.015, 0.025, -0.03]$ and $\Upsilon_2 = [0.25, -0.15, 0.20, -0.10]$ are considered as the normalized CFO vectors. Note that in Υ_1 , CFOs are less than 5% of the subcarrier spacing, and in Υ_2 they are beyond 10% of that. For OFDMA systems, simulation and theoretical results are presented for frequency selective channels without channel equalization. The same results are applicable for flat fading channels and frequency selective channels with FDE. The frequency selective channels between the users and base station are assumed to be according to the model presented in Sect. 2, and all of them are considered to be of the same order $L_m = 4, \forall m$.

For SC-FDMA systems, simulation results are examined for both flat fading and frequency selective channels, but theoretical results are presented only for flat fading channels. Furthermore, in the SC-FDMA uplink system, ZF-FDE is performed by ideal channel estimation. Note that in SIR analysis, minimum mean square error (MMSE) FDE and ZF-FDE are equivalent, since the channel noise is not present.

For both systems, the number of CP samples is chosen to be $N_g = 16$, and the average energy of CIR is normalized to unity. In addition, we assume that the power control is perfect. The average performances in ensuing simulations are obtained by 10^5 realizations for each case.

The simulation and theoretical SIR in the presence of the given CFO vectors and over different received symbols are depicted in Figs. 4, 5, 6, 7, 8 and 9. Figures 4 and 5 correspond to interleaved allocation; Figs. 6 and 7 correspond to block allocation, and Figs. 8 and 9 correspond to block-interleaved allocation. As seen from these figures, the simulation results are in excellent match with the corresponding theoretical results. Also, we see that both systems are highly sensitive to CFO, since SIR performances degrade significantly in term of increasing CFOs. From Figs. 4 and 5, it is seen that in interleaved subcarrier allocation, the SIR performance of SC-FDMA system is slightly better than that of the OFDMA system for flat fading channel. The reason is that in this subcarrier allocation, MAI is the dominant interference term, and it is the same for the two systems according to (17) and (32). However, channel frequency selectivity degrades the SIR performance of SC-FDMA system significantly. From Figs. 6 and 7, it is seen that in block subcarrier allocation, the SIR performance of SC-FDMA system is much better than that of the OFDMA system for flat fading channel. Also, in this subcarrier allocation, the SIR performance of SC-FDMA system under frequency selective channel degrades to some extent. By comparing these two figures we observe that both OFDMA and SC-FDMA systems have better SIR performance in block

¹ The number of subcarriers in practical systems are larger than 64. In LTE for example, it ranges from 128 to 2,048. The simulation results for larger number of the subcarriers are approximately the same as the case $N = 64$. We have chosen $N = 64$ to avoid the figures becoming too busy.

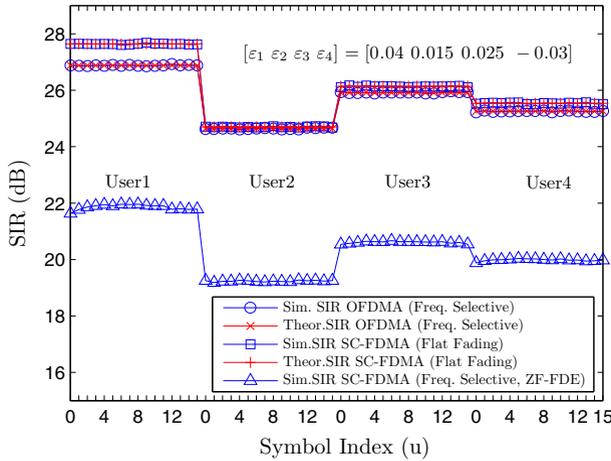


Fig. 4 Received SIR as a function of symbols index in interleaved subcarrier allocation for Υ_1 CFO vector

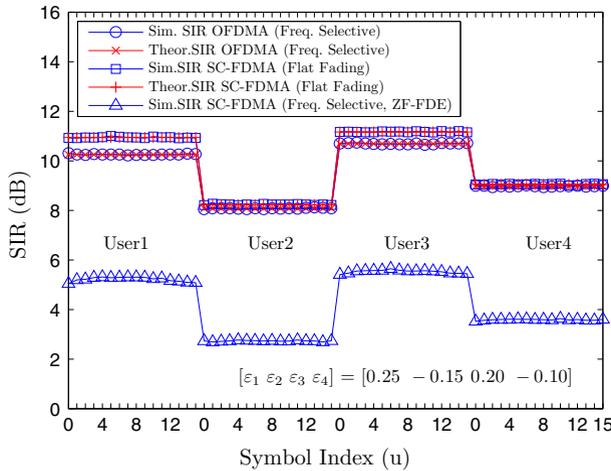


Fig. 5 Received SIR as a function of symbols index in interleaved subcarrier allocation for Υ_2 CFO vector

subcarrier allocation than what they have in interleaved subcarrier allocation. From Figs. 6 and 7, we see that SIR performance in block-interleaved allocation is merely between the SIR performances of block and interleaved allocations. Our observations may be summarized as follows:

- Both OFDMA and SC-FDMA systems are highly sensitive to CFO, and the sensitivity does not decrease by increasing the uplink transmission power, since this also increases the power of the interference terms caused by CFOs.
- The SIR performance of SC-FDMA systems is better than that of OFDMA systems for flat fading channel. However, channel frequency selectivity and imperfect channel equalization degrade the SIR performance of SC-FDMA systems significantly, since it introduces some residual ISI in addition to what is introduced by CFO.

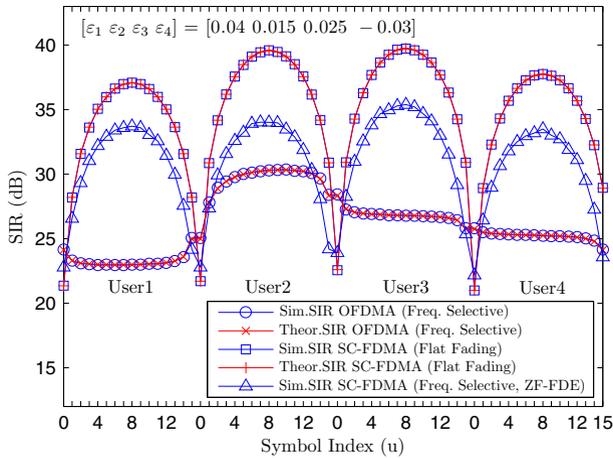


Fig. 6 Received SIR as a function of symbols index in block subcarrier allocation for Υ_1 CFO vector

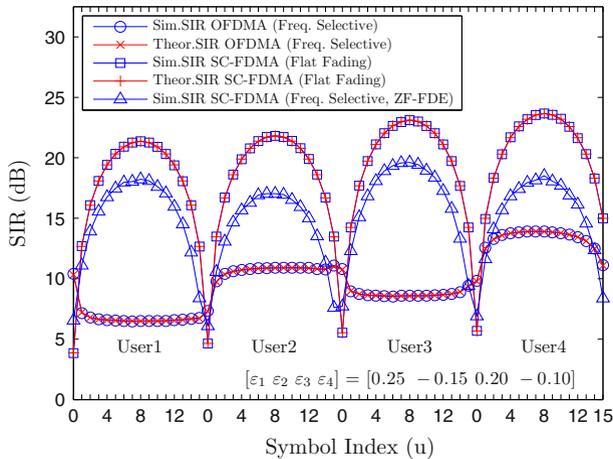


Fig. 7 Received SIR as a function of symbols index in block subcarrier allocation for Υ_2 CFO vector

- In interleaved subcarrier allocation, the desired signal power, the powers of the interference terms, and consequently SIR, are the same over all received symbols of each user for both OFDMA and SC-FDMA systems.
- Both OFDMA and SC-FDMA systems have better SIR performance in block subcarrier allocation than what they have in interleaved subcarrier allocation.

6 Conclusion

This paper presented a mathematical analysis of OFDMA and SC-FDMA systems in terms of sensitivity to CFO in the uplink of a network using superposition principle where the contributions of different users were studied separately. We found exact and closed-form expressions for SIR in different subcarrier allocation schemes including general, block, interleaved, and

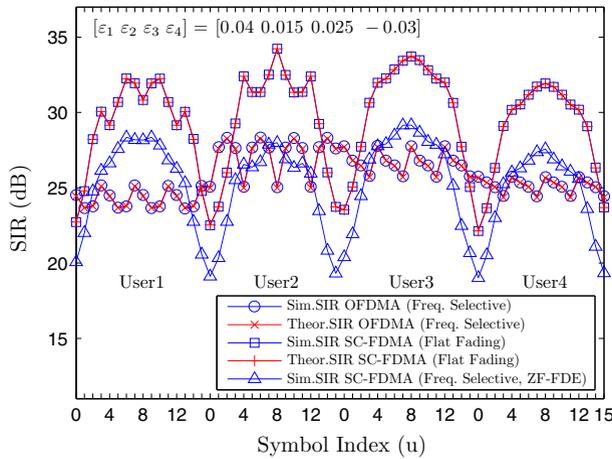


Fig. 8 Received SIR as a function of symbols index in block-interleaved subcarrier allocation for Υ_1 CFO vector

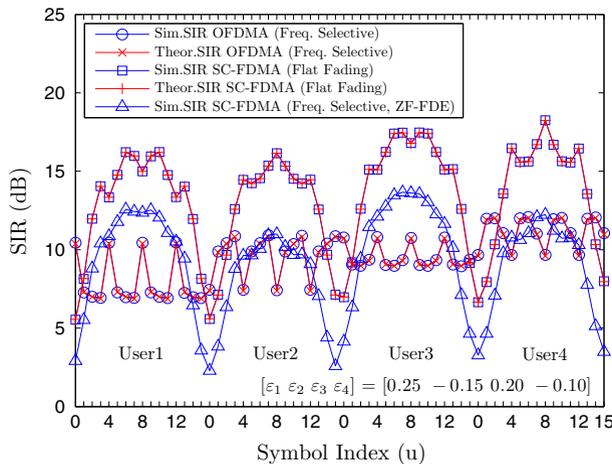


Fig. 9 Received SIR as a function of symbols index in block-interleaved subcarrier allocation for Υ_2 CFO vector

block-interleaved allocations by calculating desired signal and interferences terms. It was shown that in interleaved allocation, the derived expressions for these terms were reduced to very simple forms for both OFDMA and SC-FDMA systems. Finally, the theoretical results were verified using simulations and the two systems were compared upon a set of results.

Appendix 1: Normalized ICI and MAI Powers Calculation for OFDMA System

Using Parseval's theorem we have the following identity

$$\begin{aligned}
 \sum_{k=0}^{N-1} |f_N(\varepsilon_m - k)|^2 &= \sum_{k=0}^{N-1} |C_m[k]|^2 \\
 &= N \sum_{n=0}^{N-1} |c_m[n]|^2 \\
 &= 1.
 \end{aligned} \tag{39}$$

For OFDMA system with interleaved allocation, the normalized ICI power over the k th subcarrier for $k \in \mathcal{I}_m$ is equal to

$$\begin{aligned}
 P_{ICI}^n(k) &= \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} |f_N(\varepsilon_m - k + r)|^2 \\
 &\stackrel{(a)}{=} \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} |f_N(\varepsilon_m - i_m - qM + i_m + q'M)|^2 \\
 &\stackrel{(b)}{=} \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \frac{\sin^2(\pi \varepsilon_m)}{N^2 \sin^2\left(\frac{\pi}{N}(\varepsilon_m + (q' - q)M)\right)} \\
 &\stackrel{(c)}{=} \frac{\sin^2(\pi \varepsilon_m)}{M^2 \sin^2\left(\frac{\pi \varepsilon_m}{M}\right)} \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \frac{\sin^2(\pi \varepsilon_m / M)}{Q^2 \sin^2\left(\frac{\pi}{Q}(\varepsilon_m / M + q' - q)\right)} \\
 &\stackrel{(d)}{=} |f_M(\varepsilon_m)|^2 \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \left| f_Q\left(\frac{\varepsilon_m}{M} + q' - q\right) \right|^2 \\
 &\stackrel{(e)}{=} |f_M(\varepsilon_m)|^2 \left(1 - \left| f_Q\left(\frac{\varepsilon_m}{M}\right) \right|^2 \right) \\
 &\stackrel{(f)}{=} |f_M(\varepsilon_m)|^2 - |f_N(\varepsilon_m)|^2,
 \end{aligned} \tag{40}$$

where (a) follows from substituting the index set of interleaved subcarrier allocation; (b) and (d) follow from definition of $f_N(\cdot)$; (c) and (f) are based on a mathematical dissection, and (e) follows from (39) identity. By following the same line of derivations as (40), we can show that the normalized MAI power caused by the m th user over the k th subcarrier for $k \in \mathcal{I}_{m'}$, $m' \neq m$, is given by

$$\begin{aligned}
 P_{MAI}^n(m, k) &= \sum_{r \in \mathcal{I}_m} |f_N(\varepsilon_m - k + r)|^2 \\
 &= \sum_{q'=0}^{Q-1} |f_N(\varepsilon_m - i_{m'} - qM + i_m + q'M)|^2 \\
 &= |f_M(i_m - i_{m'} + \varepsilon_m)|^2 \sum_{q'=0}^{Q-1} \left| f_Q\left(\frac{i_m - i_{m'} + \varepsilon_m}{M} + q' - q\right) \right|^2 \\
 &= |f_M(i_m - i_{m'} + \varepsilon_m)|^2.
 \end{aligned} \tag{41}$$

For block allocation we calculate the normalized ICI power. Calculation of the normalized MAI power can be performed in a similar way and we leave it for interested readers. The normalized ICI power for $k \in \mathcal{I}_m$ is given by

$$\begin{aligned}
 P_{\text{ICI}}^n(k) &= \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} |f_N(\varepsilon_m - k + r)|^2 \\
 &\stackrel{(a)}{=} \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} |f_N(\varepsilon_m - i_m Q - q + i_m Q + q')|^2 \\
 &\stackrel{(b)}{=} \sum_{\substack{q'=0 \\ q' \neq q}}^{N-1} |f_N(\varepsilon_m - d + q')|^2 - \sum_{q'=Q}^{N-1} |f_N(\varepsilon_m - d + q')|^2 \\
 &\stackrel{(c)}{=} 1 - |f_N(\varepsilon_m)|^2 - \sum_{q'=Q}^{N-1} \frac{\sin^2(\pi \varepsilon_m)}{N^2 \sin^2\left(\frac{\pi}{N}(\varepsilon_m + q' - d)\right)} \\
 &\stackrel{(d)}{\approx} 1 - |f_N(\varepsilon_m)|^2 - \frac{\sin^2(\pi \varepsilon_m)}{N^2} \int_{Q-0.5}^{N-0.5} \frac{d\xi}{\sin^2\left(\frac{\pi}{N}(\varepsilon_m + \xi - d)\right)} \\
 &\stackrel{(e)}{=} 1 - |f_N(\varepsilon_m)|^2 - \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi Q}{N}\right)}{N\pi \left(\cos\left(\frac{\pi}{N}(Q - 2d - 1 + 2\varepsilon_m)\right) - \cos\left(\frac{\pi Q}{N}\right)\right)}, \quad (42)
 \end{aligned}$$

where (a) follows from substituting the index set of block subcarrier allocation; (b) follows from definition of $d \triangleq k - i_m Q$ and breaking the summation; (c) follows from the identity (39) and definition of $f_N(\cdot)$; (d) follows from the approximation of the summation by an integration, and (e) follows from the calculation of the integral. When $d = 0$ or $d = Q - 1$, for some values of ε_m the integrand in (42) tends to its singular points, and the approximation lose its validity. Thus, it must be corrected for $d = 0$ and $d = Q - 1$. For $d = 0$, we have

$$\begin{aligned}
 P_{\text{ICI}}^n(k) &= \sum_{q'=1}^{Q-1} \frac{\sin^2(\pi \varepsilon_m)}{N^2 \sin^2\left(\frac{\pi}{N}(\varepsilon_m + q')\right)} \\
 &\approx |f_N(1 + \varepsilon_m)|^2 + \frac{\sin^2(\pi \varepsilon_m)}{N^2} \int_{1.5}^{Q-0.5} \frac{d\xi}{\sin^2\left(\frac{\pi}{N}(\varepsilon_m + \xi)\right)} \\
 &= |f_N(1 + \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi(Q-2)}{N}\right)}{N\pi \left(\cos\left(\frac{\pi(Q-2)}{N}\right) - \cos\left(\frac{\pi}{N}(Q + 1 + 2\varepsilon_m)\right)\right)}, \quad (43)
 \end{aligned}$$

and for $d = Q - 1$ we have

$$\begin{aligned}
 P_{\text{ICI}}^n(k) &= \sum_{q'=0}^{Q-2} \frac{\sin^2(\pi \varepsilon_m)}{N^2 \sin^2\left(\frac{\pi}{N}(\varepsilon_m + q' - Q + 1)\right)} \\
 &\approx |f_N(1 - \varepsilon_m)|^2 + \frac{\sin^2(\pi \varepsilon_m)}{N^2} \int_{-0.5}^{Q-2.5} \frac{d\xi}{\sin^2\left(\frac{\pi}{N}(\varepsilon_m + \xi - Q + 1)\right)}
 \end{aligned}$$

$$= |f_N(1 - \varepsilon_m)|^2 + \frac{2 \sin^2(\pi \varepsilon_m) \sin\left(\frac{\pi(Q-2)}{N}\right)}{N\pi \left(\cos\left(\frac{\pi(Q-2)}{N}\right) - \cos\left(\frac{\pi}{N}(Q + 1 - 2\varepsilon_m)\right)\right)}. \quad (44)$$

Appendix 2: Normalized Desired Signal, ISI and MAI Powers Calculation for IFDMA System

For an IFDMA system, the normalized desired signal power on the u th received symbol of the m th user is given by

$$\begin{aligned} P_S^n(m, u) &= \frac{1}{Q^2} \left| \sum_{k \in \mathcal{I}_m} \sum_{r \in \mathcal{I}_m} f_N(\varepsilon_m - k + r) e^{j2\pi u(g_m(k) - g_m(r))/Q} \right|^2 \\ &\stackrel{(a)}{=} \frac{1}{Q^2} \left| \sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} f_N(\varepsilon_m + (q' - q)M) e^{j2\pi u(q - q')/Q} \right|^2 \\ &\stackrel{(b)}{=} \frac{1}{Q^2} \left| \sum_{q=0}^{Q-1} \sum_{q'=0}^{Q-1} f_M(\varepsilon_m) f_Q(\varepsilon_m/M + q' - q) e^{j2\pi u(q - q')/Q} \right|^2 \\ &\stackrel{(c)}{=} \frac{|f_M(\varepsilon_m)|^2}{Q^2} \left| \sum_{q'=0}^{Q-1} e^{-\frac{j2\pi u q'}{Q}} \sum_{q=0}^{Q-1} f_Q(\varepsilon_m/M + q' - q) e^{\frac{j2\pi u q}{Q}} \right|^2 \\ &\stackrel{(d)}{=} \frac{|f_M(\varepsilon_m)|^2}{Q^2} \left| \sum_{q'=0}^{Q-1} e^{-\frac{j2\pi u q'}{Q}} e^{\frac{j2\pi u(\varepsilon_m/M + q')}{Q}} \right|^2 \\ &\stackrel{(e)}{=} |f_M(\varepsilon_m)|^2, \end{aligned} \quad (45)$$

where (a) follows from substituting the index set of interleaved subcarrier allocation and corresponding subcarriers mapping rule; (b) is based on a mathematical dissection; (c) and (e) follow from some straightforward manipulations; and (e) follows from the IDFT of sequence $\{f_Q(\varepsilon_m/M + q' - q)\}_{q=0}^Q$. In a similar way, for the normalized ISI power on the u th received symbol of the m th user, we have

$$\begin{aligned} P_{ISI}^n(m, u) &= \frac{1}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{k \in \mathcal{I}_m} \sum_{\substack{r \in \mathcal{I}_m \\ r \neq k}} f_N(\varepsilon_m - k + r) e^{j2\pi(u g_m(k) - v g_m(r))/Q} \right|^2 \\ &= \frac{|f_M(\varepsilon_m)|^2}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{q=0}^{Q-1} \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} f_Q(\varepsilon_m/M + q' - q) e^{j2\pi(u q - v q')/Q} \right|^2 \\ &= \frac{|f_M(\varepsilon_m)|^2}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{q'=0}^{Q-1} e^{-\frac{j2\pi v q'}{Q}} \sum_{\substack{q=0 \\ q \neq q'}}^{Q-1} f_Q(\varepsilon_m/M + q' - q) e^{\frac{j2\pi u q}{Q}} \right|^2 \\ &= \frac{|f_M(\varepsilon_m)|^2}{Q^2} \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{q'=0}^{Q-1} e^{-\frac{j2\pi v q'}{Q}} \left(e^{\frac{j2\pi u(\varepsilon_m/M + q')}{Q}} - f_Q\left(\frac{\varepsilon_m}{M}\right) e^{\frac{j2\pi u q'}{Q}} \right) \right|^2 \end{aligned}$$

$$\begin{aligned}
 &= \frac{|f_M(\varepsilon_m)|^2}{Q^2} \left| e^{\frac{j2\pi u \varepsilon_m}{N}} - f_Q\left(\frac{\varepsilon_m}{M}\right) \right|^2 \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} \left| \sum_{q'=0}^{Q-1} e^{-\frac{j2\pi(v-u)q'}{Q}} \right|^2 \\
 &= \frac{|f_M(\varepsilon_m)|^2}{Q^2} \left| e^{\frac{j2\pi u \varepsilon_m}{N}} - f_Q\left(\frac{\varepsilon_m}{M}\right) \right|^2 \sum_{\substack{v=0 \\ v \neq u}}^{Q-1} |Q\delta[v-u]|^2 \\
 &= 0.
 \end{aligned} \tag{46}$$

Finally, by following a similar approach in derivation of (45) and (46), the normalized MAI power caused by the m th user over the u th received symbol of the m' th user ($m' \neq m$) is equal to

$$\begin{aligned}
 P_{\text{MAI}}^n(m, m', u) &= \frac{1}{Q} \sum_{r \in \mathcal{I}_m} \left| \sum_{k \in \mathcal{I}_{m'}} f_N(\varepsilon_m - k + r) e^{j2\pi u g_{m'}(k)/Q} \right|^2 \\
 &= \frac{|f_M(i_m - i_{m'} + \varepsilon_m)|^2}{Q} \sum_{q'=0}^{Q-1} \left| \sum_{q=0}^{Q-1} f_Q\left(\frac{i_m - i_{m'} + \varepsilon_m}{M} + q' - q\right) e^{\frac{j2\pi u q}{Q}} \right|^2 \\
 &= \frac{|f_M(i_m - i_{m'} + \varepsilon_m)|^2}{Q} \sum_{q'=0}^{Q-1} \left| e^{\frac{j2\pi u (i_m - i_{m'} + \varepsilon_m)/M + q'}{Q}} \right|^2 \\
 &= |f_M(i_m - i_{m'} + \varepsilon_m)|^2.
 \end{aligned} \tag{47}$$

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