Improving Instantaneous Capacity and Outage Probability in DF-Relaying

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Abstract— **Consider a Rayleigh flat fading relay channel under two different power constraints: sum power constraint and individual power constraints. In this paper a simple method for resource allocation based on partial channel state information at transmitter and full channel state information at receiver is introduced. It is shown that by using different time portions for the source and the relay in relaying, when the distance from relay to the source is very different from its distance to the destination, achievable rate can be enhanced and outage probability can be reduced under sum power constraint. Also in the case of individual power constraints, time allocation scheme can improve the achievable rate and outage probability.**

I. INTRODUCTION

Cooperative communication was introduced by Van Der Meulen in [1], as a technique for providing diversity and coverage enhancement in wireless systems. Some information theoretic issues like the evaluation of Gaussian relay channel capacity were discussed by Cover and El Gamal in [2]. One of the important benefits of cooperative techniques is providing space diversity so that destination receives uncorrelated signals from source and relay [3], [4].

In [3] two protocols based on fixed relaying schemes were defined: Amplify-and-Forward (AF), where the relay just amplifies the received signal including its receiver noise, without decoding its content and Decode-and-Forward (DF) where the relay decodes the message completely, encodes it again and transmits it to the destination.

Resource allocation for the source and the relay under different constraints and for various purposes has been studied in recent years. Capacity maximization under sum power constraints were discussed in [5]-[6]. Outage probability minimization under sum power constraints have also been studied in [7]. Also some literature has studied resource allocation for frequency selective channels. Power allocation under individual power constraints for OFDM transmission was addressed in [8]-[9].

In conventional relaying, the source and the relay have same amount of time for sending and forwarding information in one timeslot. However, in Decode-and-Forward (DF) mode, time allocation is not necessarily equal. Instantaneous capacity depends on the minimum of the rates that can be transferred through Source-Relay (S-R) and Relay-Destination (R-D) links. So if one of these links is worse than the other one, achievable rate is affected by the disadvantaged link. It is clear that average achievable rate reaches its maximum when relay is located in the middle of the distance between source and destination and decreases at boundaries (when relay is closer to the source or destination). This is because S-R and R-D links have close path losses in the middle of the S-D distance but their difference becomes larger as the relay reaches near boundaries. We can compensate this asymmetry by allocating different portions of timeslot to the source and relay based on their relative distances.

In [10] an opportunistic protocol is introduced in which the source can use channel state information at transmitter (CSIT) to choose between DF relaying or direct transmission. Also unlike conventional relaying in this protocol different time portions can be allocated to the source and the relay. Authors in [10] showed that their proposed protocol significantly improves the delay-limited capacity of the system and performs very close to the cut-set bound. Also they minimized average total power for a given delay limited capacity. However, we present a simple and practical method for resource allocation when there is a constraint on total average power or individual constraints on instantaneous powers of the source and the relay. This work is different from previous literature like [10] since they have not considered power and time allocation for maximizing instantaneous capacity under sum power constraint. It is shown that our approach improves performance relative to opportunistic decode and forward (choosing among direct transmission or decode and forward) especially when relay is located closer to the boundaries. However, our proposed method is not optimal for all channel states but by dividing these states into three categories based on the relative distances, an enhancement can be achieved when S-R and R-D link power gains have different means and it changes the problem to a simple practical problem.

The rest of this paper is organized as follows. In section II our system model is introduced. In section III we formulate resource allocation subject to total power constraint. We analyze resource allocation where there is a constraint on total

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average power. It is shown that time allocation by the proposed simple method can improve the performance at the boundaries. In section IV, resource allocation under individual power constraints is discussed. In this case there are constraints on the instantaneous powers of the source and the relay. As will be shown, time allocation can significantly improve the performance.

Finally section V, contains simulation results which confirm the improvements obtained by the proposed protocol.

II. SYSTEM MODEL

Let's consider three nodes for our model, a source (S), a destination (D) and a relay (R) as shown in Fig. 1. It is assumed that the links between the nodes have independent, quasi-static Rayleigh fading and of course path loss. Channel gains among terminals are represented by h_i, i ∈ {1, 2, 3}.

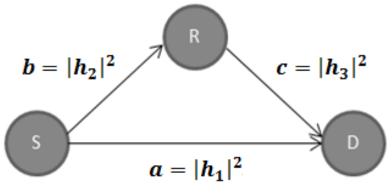

Fig. 1: Illustration of the cooperative relay system model

Also, independent, zero-mean additive white Gaussian noise with normalized variance is assumed at each receiver. We choose block fading model in which channel coefficients are assumed to be constant over each block (the time during which one codeword is transmitted), and are independent from one block to the other one. Power coefficients of the channel are represented by $a = |h_1|^2$, $b = |h_2|^2$ and $c = |h_3|^2$ as in Fig. 1. Also they are exponentially distributed random variables with mean values of μ_a, μ_b, and μ_c, respectively. In order to study the effect of the relay location on the performance of the network, we consider the model used in [10] which is shown in Fig. 2.

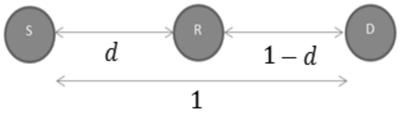

Fig. 2: The model for the source, the relay and the destination locations

By normalizing the distance between source and destination, and assuming that the relay is located between them, We can denote the S-R distance as d and the R-D distance as $1 - d$, where $0 < d < 1$. The overall channel state can be represented by independent exponential random variables a, b and c with mean values of $\mu_a = 1$, $\mu_b = \frac{1}{d^\alpha}$, and $\mu_c = \frac{1}{(1-d)^\alpha}$ respectively, where α is the path loss exponent. For the simulation we choose α equal to 4. Now let's assume that the channel coefficient amplitudes are known at the source, the relay and the destination, while the phase information is only available at the receivers (destination and relay). Our cooperation protocol is based on the Opportunistic Decode-and-Forward (ODF) which uses CSIT for deciding between relaying or direct transmission. It is easy to show that using relay in the cases that S-R or R-D links are worse than S-D link, decreases capacity and performance.

In ODF protocol, transmitter decides whether to use the relay or not. If $a < b$ and $a < c$, relay is used otherwise the source transmits throughout the whole time slot. Now we explain the phases of relaying. In the first phase, the source transmits to both the relay and the destination, and in the second phase the relay forwards the message that it has received in the previous phase, to the destination. It is necessary to note that the relay is assumed to be half-duplex, so it is not able to receive and transfer at the same time. When relay is used, destination receives two copies of the same message from two independent fading channels. Because of our assumption of having channel state information at receiver (CSIR), destination can use Maximal Ratio Combining.

III. RESOURCE ALLOCATION FOR SUM POWER CONSTRAINT (SPC)

$P(s) = (P_s(s), P_r(s), t(s))$ is a resource allocation rule defined over the set of all possible network states $s = (a, b, c)$, where $P_s(s)$ and $P_r(s)$ are the transmission powers of the source and the relay, respectively, and $t(s)$ is the ratio of the time slot allocated for the source transmission with $0 < t(s) \leq 1$. If $t(s) = \frac{1}{2}$ then instantaneous capacity can be written as:

$$I_{ODF_{t(s)=\frac{1}{2}}} = \frac{1}{2}\min\{\log(1+p_s b), \log(1+p_r c+p_s a)\} \tag{1}$$

Where p_s and p_r are the source and the relay instantaneous powers respectively (we express $P_s(s)$ by p_s and $P_r(s)$by p_r for simplicity). In the case of $t(s) = t \neq 0.5$, we consider instantaneous capacity without use of direct link. So the lower bound for instantaneous capacity in the case of $t(s) = t$ can be stated as:

$$I_{ODF-lower\ bound_{t(s)=t}} = \min\{t.\log(1+p_s b), (1-\mathrm{t})\log(1+p_r c)\} \tag{2}$$

Now it can be shown that even the mentioned lower bound has a better performance for both ends of Source-Destination (S-D) link. First of all let's express this problem as bellow:

$$\text{maximize}_{P_{total} \leq P_T}\{I_{ODF-lower\ bound_{t(s)=t}}\} \tag{3}$$

Where $P_{total}$is the average power of both the source and relay and can be expressed as:

$$P_{total} = t(s)p_s + \big(1 - t(s)\big)p_r \tag{4}$$

Explicitly, capacity is an increasing function of power, so its maximum can be achieved by maximum allowed power (P_T). So (3) can be rewritten as:

$$\text{maximize}_{\,P_{total}=P_T}\{I_{ODF-lower\ bound_{t(s)=t}}\} \tag{5}$$

Also it is clear that we have optimum power and time allocation if phrases in (2) are equal otherwise one of them limits the other one (because of *min* operator) and some of our resources are wasted. So we have to maximize $I_{ODF-lower\ bound_{t(s)=t}}$ under the conditions:

$$\begin{cases} p_r = \frac{(1+p_s b)^{\frac{t}{1-t}}-1}{c} \\ P_T = tp_s + (1-t)p_r \end{cases} \tag{6}$$

But allowing t to be anything between 0 and 1 may make practical and calculation difficulties. So we consider an easier and more practical case of having just three possible choices for *t* ($0.25, 0.5$ and 0.75). Our rule for deciding which one to use is the ratio of S-R path length to R-D:

$$t = \begin{cases} 0.25 & \frac{d_{SR}}{d_{RD}} < 0.5 \\ 0.5 & 0.5 \leq \frac{d_{SR}}{d_{RD}} \leq 1.5 \\ 0.75 & 1.5 < \frac{d_{SR}}{d_{RD}} \end{cases} \tag{7}$$

In which d_{SR} and d_{RD} show the distance between S-R and R-D respectively. Intuitively we can explain it as allocating more time to the relay for sending when it is in the vicinity of the source and less portion to the source because S-R link has greater capacity than R-D link. So we can summarize the procedure for resource allocation as bellow:

1-Finding $t(s)$ using (7)

2-Finding p_s and p_r by solving (6) for the given *t* in step1

3-Finally instantaneous capacity can be found by:

$$I_{ODF-lower\ bound_{t(s)=t}} = t.\log(1+p_s b) \tag{8}$$

IV. INDIVIDUAL POWER CONSTRAINT (IPC)

In this case we assume individual constraints for instantaneous powers of the source and the relay. Our goal is to maximize instantaneous capacity under these constraints. As in previous section using equal ratios for the source and the relay can cause resource wasting. So we can allocate different ratios to each one depending on S-R and R-D distances. Without assuming diversity this problem can be expressed as (9). Obviously we can achieve optimal resource allocation if S-R and R-D links can support same rates.

$$\begin{cases} Maximize\{min(t.\log(1+p_s b), (1-\mathrm{t})\log(1+p_r c))\} \\ \qquad p_s \leq p_{s_{th}} \\ \qquad p_r \leq p_{r_{th}} \end{cases} \tag{9}$$

Also, since the achievable rate is an increasing function of the source and the relay power, we can achieve maximum rate by using $p_s = p_{s_{th}}\ and\ p_r = p_{r_{th}}$. So we can easily find optimum value for *t* (shown by t^*) by the following equation:

$$t^* = \frac{\log_2(1+p_{r_{th}}c)}{\log_2(1+p_{r_{th}}c)+\log_2(1+p_{s_{th}}b)} \tag{10}$$

Finally maximum achievable rate can be obtained by substituting 10 in 9:

$$I_{ODF_{t(s)=t^*}} = \frac{\log_2(1+p_{r_{th}}c)\log_2(1+p_{s_{th}}b)}{\log_2(1+p_{r_{th}}c)+\log_2(1+p_{s_{th}}b)}. \tag{11}$$

Equation 10 shows that like the sum power constraint case, optimum time allocation can be achieved by allocating more time to the source when S-R link has less capacity than R-D link and vice versa.

Simulation results are shown in the next section. As it can be seen, time allocation can improve the outage probability and the average capacity at the boundaries especially when instantaneous powers of the source and the relay have to be less than a threshold.

V. SIMULATION RESULTS

It is important to note that because of assuming a slow fading channel, outage probability is a better tool for comparison. However, average capacity versus distance is also shown in the simulation results.

A. Sum power constraint

In this part numerical results of DF relaying with time allocation are presented. Fig. 3 plots average capacity for both ODF protocol and lower bound of ODF with time allocation. As we discussed earlier, in conventional DF relaying average capacity falls at two ends of S-D link because of asymmetry of S-D and R-D links. As it can be seen in Fig. 3 using different time slot partitioning based on distance, achieves higher average rates at two ends of S-D link.

In Fig. 4 percentage of increase in average capacity is shown. Fig. 5 shows outage probability versus distance and it confirms the improvement by this protocol relative to ODF. Time allocation can decrease outage probability at boundaries in the case of SPC.

B. Individual power constraint

In Fig. 6 average capacities with and without time allocation are shown. As can be seen time allocation can cause dominant improvement when there are constraints for instantaneous powers of the source and the relay.

Fig. 7 illustrates percentage of improvement in average capacity.

Finally Fig. 8 shows outage probability versus distance. It shows decrease in outage probability in the case of using time allocation.

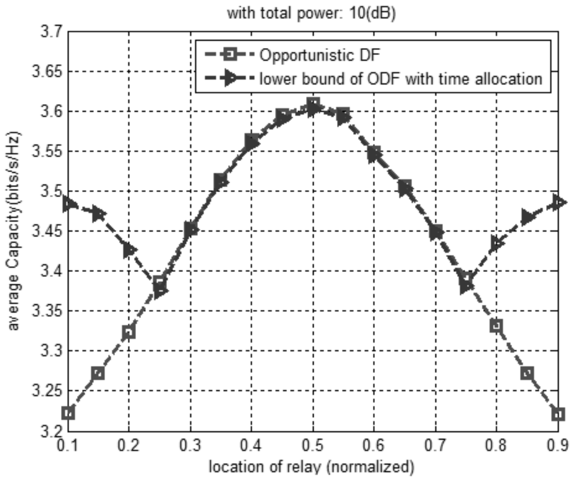

Fig. 3: Average achievable rate versus distance for SPC

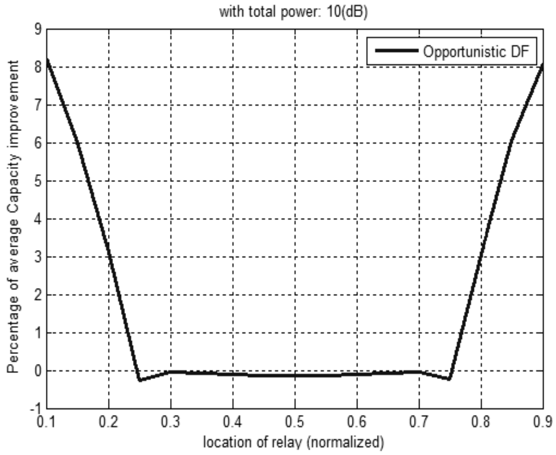

Fig. 4: Percentage of improvement for proposed method versus fixed time DF-relaying for SPC

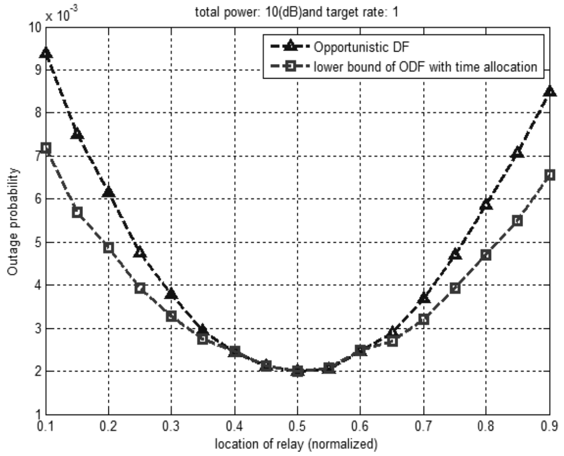

Fig. 5: Outage probability versus distance for SPC

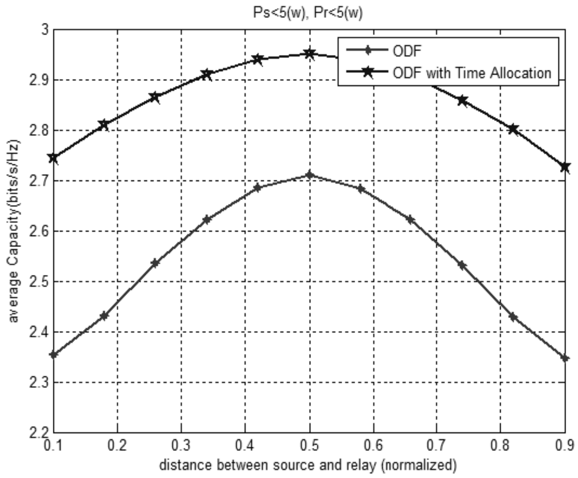

Fig. 6: Average capacity versus distance for IPC

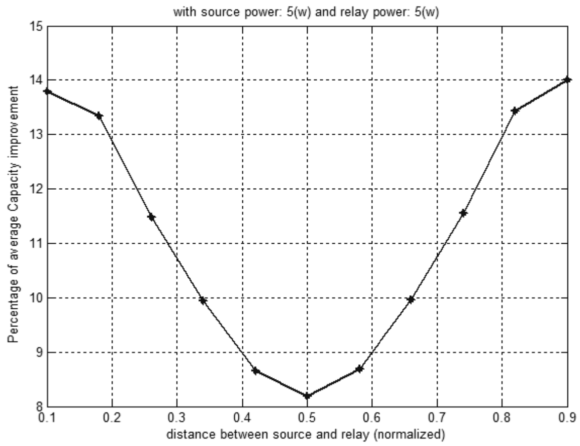

Fig. 7: Percentage of increase in average capacity versus distance for IPC

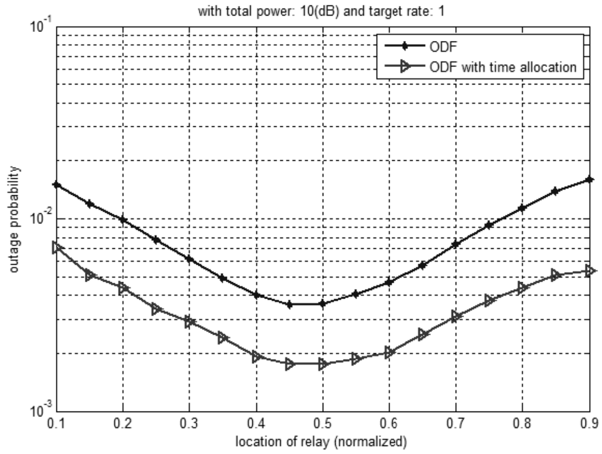

Fig. 8: Outage probability versus distance for IPC

VI. CONCLUSION

In this paper, we have proposed a simple protocol for gaining the advantages of time allocation in DF relaying for two cases of SPC and IPC. For simplicity in the equations we neglected the effect of direct path (diversity) on the instantaneous capacity, however, we showed that time allocation can cause better performance even without using diversity. So it is possible to improve the rate or decrease the outage probability by using diversity.

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