Oversampled Legendre Basis Expansion Model for Doubly-Selective Channels

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Abstract—In broadband wireless communication, high rate data transmission and multipath propagation results in frequency selectivity. On the other hand, high mobility contributes to time selectivity in fading channels. A finite parameter model which can be used for modeling doubly-selective channels is the basis expansion model (BEM). In this model, channel is estimated based on some basis functions which are weighted by time-invariant coefficients during a time block. In this paper, Legendre basis expansion model (LBEM) is discussed for doubly-selective fading channels and oversampled Legendre basis expansion model (OLBEM) is proposed. Simulation results show improvement in the mean square error (MSE) of our channel modeling scheme compared to that of other oversampled basis expansion models (i.e. OBEM, and oversampled polynomial basis expansion model (OPM)). In our approach the BEM coefficients are calculated using the least-squares approximation.

Keywords-basis expansion model (BEM); Legendre polynomials; oversampling; doubly-selectivity; fading channels

I. INTRODUCTION

Frequency selective fading occurs when the bandwidth of the transmitted signal is greater than the coherence bandwidth of the fading channel. Time selectivity is induced by the Doppler shift and the relative motion between transmitter and receiver. This happens when the period of the transmitted symbol is greater than the coherence time of the channel.

Recently basis expansion model (BEM) has been used for modeling fading channels due to its finite parameters, low complexity and acceptable tracking of the actual channel. The channel's time variant impulse response is expanded on some basis functions and the coefficients are obtainable by several different methods (e.g. LS, RLS, LMS, Kalman filter, etc.). Some of the frequently used basis expansion models are: complex exponential basis expansion model (CE-BEM), oversampled basis expansion model (OBEM), Slepian basis expansion model, polynomial basis expansion model (PBM), oversampled polynomial basis expansion model (OPBM), and Karhunen-Loeve basis expansion model (KL-BEM).

In CE-BEM, the basis functions used are complex exponentials [1] [2] [3], the advantage of this model over other BEMs is its simplicity but it fails to track the channel in the edges of the windowed channel. OBEM was proposed to alleviate this problem [2]. Slepian basis expansion model is based on the discrete spheroidal sequences (DPS) which are proposed by Slepian in [4]. The DPS sequences used as Kernel functions are orthogonal over a finite and infinite set, this property avoids the spectral leakage and hence the error performance is better than CE-BEM and OBEM [5].

Polynomial basis expansion model uses Taylor polynomials as basis functions [6]. Its error performance is better than that of CE-BEM, OBEM and Slepian BEM. Oversampled polynomial basis expansion model (OPM) was proposed to gain better channel estimation compared to CE-BEM and OBEM [7]. In Karhunen-Loeve basis expansion model (KL-BEM) [8] [9] the basis functions are obtained from second-order statistics of the channel. For a Rayleigh fading channel that follows the Jakes' model [10] [11], the channel correlation is a zero order Bessel function of first kind. A considerable disadvantage of KL-BEM is that in real situations the channel correlation does not follow the Jakes' model [12], and obtaining the channel correlation due to environmental factors is a difficult task. The basis expansion model can be used in channel estimation. Channel estimation and equalization methods have been proposed in the literature based on BEM [1] [3] [13] [14].

In this paper, two basis expansion models based on Legendre polynomials are discussed. The BEM coefficients are obtained directly using the least-squares method by the assumption that the real channel is a doubly-selective Rayleigh fading channel which follows the Jakes' model. In this paper the error performances of these models are compared to that of other basis expansion models.

The first model discussed is the Legendre basis expansion model (LBEM) [15]. LBEM is based on Legendre polynomials and the coefficients are obtained using the least-squares approximation. Simulation results show that LBEM has better error performance than CE-BEM, OBEM and Slepian and is comparable to PBM. In this article we propose the oversampled Legendre basis expansion model (OLBEM). This model is defined and the basis functions are obtained by oversampling the basis of LBEM. The idea was obtained from OPM [7], as it was proposed as a new BEM model based on oversampling the Taylor polynomials and the results proved to have better error performance than CE-BEM, OBEM and Slepian BEM [7]. It will be shown that OLBEM has lower modeling error compared to the other oversampled basis expansion models (i.e. OBEM, OPM).

The paper is organized as follows: Section II explains the system model. In sections III LBEM, and in section IV OLBEM are explained. Section V presents the simulation results. The paper is concluded in section VI.

Notations: Throughout this paper $\|.\|$ is the Euclidian Norm, $(.)^T$ is the Transpose, $(.)^H$ denotes the Hermitian, Upper Bold case letters denotes matrices and lower case letters is used for vectors.

II. SYSTEM MODEL

Consider a wireless doubly-selective fading channel. A general form of expressing the input-output equation for time index n is expressed as follows [8]:

$$y(n) = \sum_{l=0}^{L-1} h(n;l)u(n-l) + w(n), \quad (1)$$

where L is the number of taps, $u(n)$ and $y(n)$ are the nth transmitted and received symbols. $h(n;l)$ is the lth tap of channel's impulse response at time index n, and $w(n)$ denotes the additive white Gaussian noise.

A general form of expressing the channel by its basis function and coefficients is given by [1]:

$$\hat{h}(n;l) = \sum_{q=0}^{Q-1} g(q,l) f_q(n), \quad (3)$$

where Q is the number of basis functions, $f_q(n)$ denotes the qth basis function at time n and $g(q,l)$ is the qth coefficient of the lth tap. Equation (3) can be expressed in the matrix form as:

$$\widehat{\boldsymbol{H}} = \boldsymbol{F}\boldsymbol{G}, \quad (4)$$

where $\widehat{\boldsymbol{H}} = \left[\hat{h}(0;l), \hat{h}(1;l), \dots, \hat{h}(N-1;l)\right]^T$ is the $N \times L$ matrix form of the channel expressed by BEM for $l = [0, \dots, L-1]$, and $\boldsymbol{F} = \left[f_q(0), f_q(2), \dots, f_q(N-1)\right]^T, q = [0, \dots, Q-1]$ is a $N \times Q$ matrix of the kernel functions and $\boldsymbol{G} = [g(0,l), g(1,l), \dots, g(Q-1,l)]^T, l = [1, \dots, L-1]$ is the $Q \times L$ matrix of coefficients. For expressing the channel using BEM, the coefficient matrix needs to be found. By choosing $Q \times L < N$, channel can be modeled with fewer parameters than the block size N, this is an important advantage of using BEM for modeling fading channels.

In the two following sections, Legendre basis expansion model (LBEM) and oversampled Legendre basis expansion model (OLBEM) are presented.

III. LEGENDRE BASIS EXPANSION MODEL

This section discusses Legendre basis expansion model (LBEM) [15] [16]. These polynomials have been used in modeling the fading channels [16]. In [17] Legendre polynomials are used in predicting the fading envelope. In this paper, Legendre polynomials are used as basis expansion model for doubly-selective fading channels, and QR-decomposition is applied to obtain the Legendre kernel functions which are orthogonal on an infinite set.

The Legendre polynomials are the solution to the following differential equation [15]:

$$\frac{d}{dx}\left[(1-x^2)\frac{d}{dx}P_n(x)\right] + n(n+1)P_n(x) = 0, \quad (5)$$

The solution to (5) is given by

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right] \quad for \ n = \{0,1,2,\dots\}, \quad (6)$$

where $P_n(x)$ is the Legendre polynomial of order n. By having $P_0(x) = 1$, $P_1(x) = x$ and using the Bonnet recursion formula, the Legendre polynomial of order $n+1$ is given by:

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x), \quad (7)$$

Figure (1) shows the first five Legendre polynomials for $-1 \le x \le 1$.

One of the properties of Legendre Polynomials is that they are orthogonal on $-1 \le x \le 1$ interval as bellow:

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{nm}, \quad (8)$$

where δ_{nm} is the Kronecker delta.

For expanding the channel by its basis functions and coefficients using the Legendre polynomials, the following can be stated:

$$\begin{bmatrix} h_L(0;1) & \dots & h_L(0;L) \\ \vdots & & \vdots \\ h_L(N-1;1) & \dots & h_L(N-1;L) \end{bmatrix} = \widetilde{\boldsymbol{F}}_L \begin{bmatrix} \tilde{g}_{0_L}(1) & \dots & \tilde{g}_{0_L}(L) \\ \vdots & & \vdots \\ \tilde{g}_{Q-1_L}(1) & \dots & \tilde{g}_{Q-1_L}(L) \end{bmatrix}, \quad (9)$$

The subscript L denotes LBEM. $\widetilde{\boldsymbol{F}}_L$ is the non-orthogonal matrix of Legendre polynomials basis functions, which is

$$\widetilde{\boldsymbol{F}}_L = \begin{bmatrix} 1|_{x=\frac{0}{N}} & x|_{x=\frac{1}{N}} \dots & P_Q(x)|_{x=\frac{N-1}{N}} \\ \vdots & \vdots & \vdots \\ 1|_{x=\frac{0}{N}} & x|_{x=\frac{1}{N}} \dots & P_Q(x)|_{x=\frac{N-1}{N}} \end{bmatrix}, \quad (10)$$

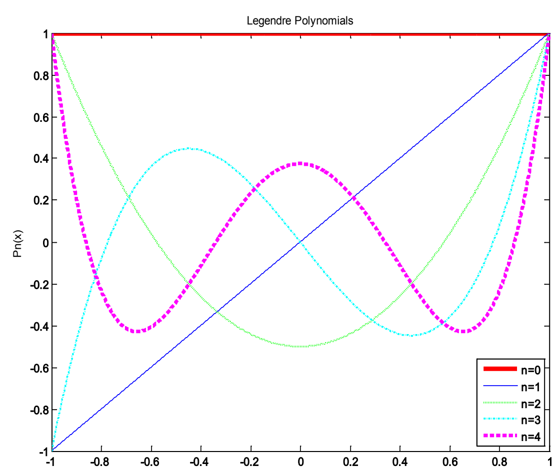

Figure 1 Legendre polynomials

where $P_Q(x)$ is the Qth order of Legendre polynomials in the normalized time x. Since these basis do not hold the orthogonally characteristic in an infinite interval, we use the QR decomposition to reach this goal and also to make the calculations simple. $\boldsymbol{F}_L$ is the orthogonal Kernel matrix after the QR decomposition is applied.

For expressing our channel using BEM, the following should be minimized:

$$\| \boldsymbol{H}_L - \boldsymbol{F}_L \boldsymbol{G}_L \|^2, \tag{11}$$

where $\boldsymbol{H}_L$is the real channel, which in this case is obtained from the Jakes' model and $\boldsymbol{G}_L$ is the coefficient matrix that should be approximated using the least-squares method.

Using the least-squares approximation the coefficient matrix can be approximated as follows:

$$\widehat{\boldsymbol{G}}_L = (\boldsymbol{F}^T{}_L \boldsymbol{F}_L)^{-1} \boldsymbol{F}^T{}_L \boldsymbol{H}_L, \tag{12}$$

And the least-squares channel approximate would be:

$$\widehat{\boldsymbol{H}}_L = \boldsymbol{F}_L \widehat{\boldsymbol{G}}_L. \tag{13}$$

IV. OVERSAMPLED LEGENDER BASIS EXPANSION MODEL

Oversampled Legendre basis expansion model (OLBEM) is based on oversampling the basis functions of LBEM. In [7] oversampling Taylor polynomials (OPM) were proposed for obtaining a better error performance than CE-BEM and OBEM. By oversampling the Legendre polynomials the modeling error would be less than that of OPM.

For the oversampled Legendre basis expansion model (OLBEM) the matrix of Kernel functions can be expressed as follows:

$$\widetilde{\boldsymbol{F}}_{OL} = \begin{bmatrix} 1|_{x=\frac{-(P*N)/2}{P*N}} & x|_{x=\frac{-(P*N)/2+1}{P*N}} \cdots & P_Q(x)|_{x=\frac{(P*N)/2-1}{P*N}} \\ \vdots & \vdots & \vdots \\ 1|_{x=\frac{-(P*N)/2}{P*N}} & x|_{x=\frac{-(P*N)/2+1}{P*N}} \cdots & P_Q(x)|_{x=\frac{(P*N)/2-1}{P*N}} \end{bmatrix}, \tag{14}$$

where P is the oversampling rate and x denotes the normalized time. The basis functions are not orthogonal and QR-decomposition is applied to $\widetilde{\boldsymbol{F}}_{OL}$ and the N middle columns are extracted to form the orthogonal Kernel matrix denoted as $\boldsymbol{F}_{OL}$. Hence, the following equation can be written:

$$\begin{bmatrix} h_{OL}(0;1) & \dots & h_{OL}(0;L) \\ \vdots & & \vdots \\ h_{OL}(N-1;1) & \dots & h_{OL}(N-1;L) \end{bmatrix} = \boldsymbol{F}_{OL} \begin{bmatrix} \tilde{g}_{0_{OL}}(1) & \dots & \tilde{g}_{0_{OL}}(L) \\ \vdots & & \vdots \\ \tilde{g}_{Q-1_{OL}}(1) & \dots & \tilde{g}_{Q-1_{OL}}(L) \end{bmatrix}, \tag{15}$$

For obtaining the coefficient matrix, we should minimize the following:

$$\| \boldsymbol{H}_{OL} - \boldsymbol{F}_{OL} \boldsymbol{G}_{OL} \|^2. \tag{16}$$

The least-squares channel approximation can be written as:

$$\widehat{\boldsymbol{G}}_{OL} = (\boldsymbol{F}^T{}_{OL} \boldsymbol{F}_{OL})^{-1} \boldsymbol{F}^T{}_{OL} \boldsymbol{H}_{OL}. \tag{17}$$

The proposed model has similar complexity compared to PBEM, OPM and LBEM, but it is more complex than CE-BEM and OBEM. Since the proposed method is based on polynomials, it is easier to achieve its basis functions compared to KL-BEM and Slepian BEM, having in mind that the former's basis functions are obtained through extracting the columns from the correlation matrix and the latter's basis functions are the DPS functions.

V. SIMULATION RESULTS

In this section, simulation results are represented for LBEM and OLBEM. The window size used is 512. The maximum Doppler frequency shift is 250 Hz, the number of taps is 7 and the sampling period is 10 µsec. As an assumption, the channel follows Jakes' spectrum and consequently its correlation is the zero order Bessel function of first kind as $R_h(t) = \sigma_h^2 J_0(2\pi f_{max} tTs)$, where f_{max} is the maximum Doppler frequency shift, Ts is the sampling period and σ_h^2 is the channel variance. In OBEM, OPM and OLBEM the oversampling rate was chosen equal to 2 for a fair comparison.

Figure 2 shows the MSE versus maximum Doppler frequency shift for LBEM. The results from LBEM are compared to that of CE-BEM, OBEM, Slepian and KL-BEM. As it can be seen, for Doppler frequency lower than 250 Hz, LBEM has the least MSE compared to the other BEM mentioned earlier. As the maximum Doppler frequency shift increases, LBEM shows higher MSE error, although for higher

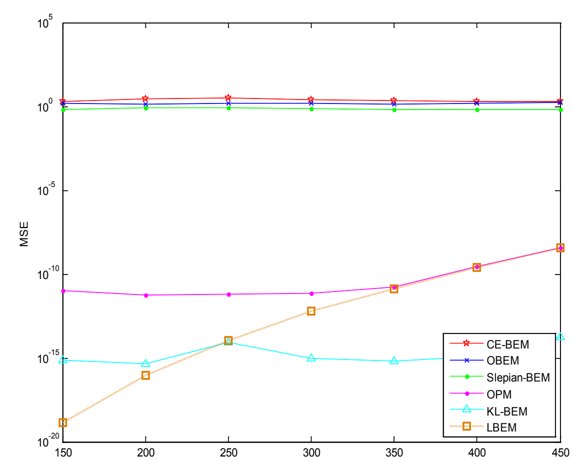

Figure 2 MSE versus Doppler frequency for LBEM

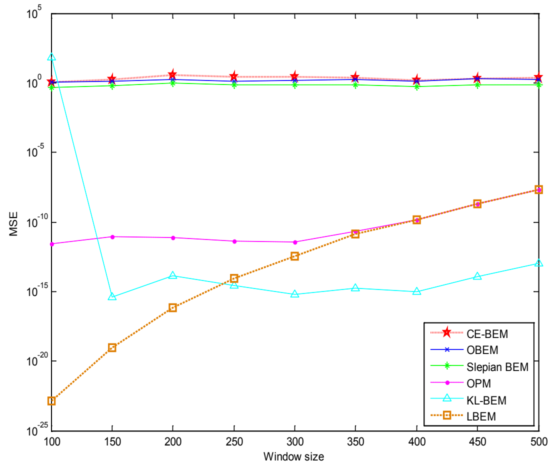

Figure 3 MSE versus window size for LBEM

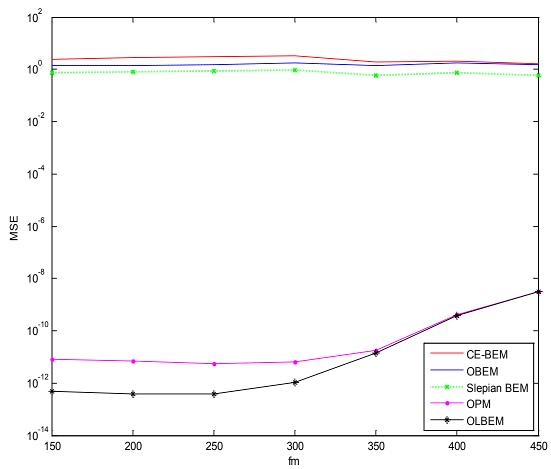

Figure 4 MSE versus Doppler frequency for OLBEM

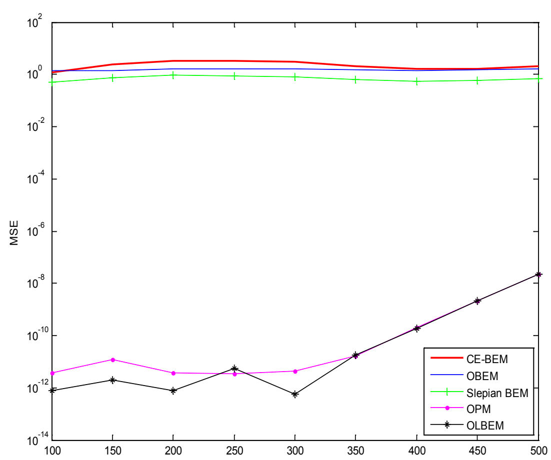

Figure 5 MSE versus window size for OLBEM

maximum Doppler frequency shift, KL-BEM has better error performance, but the optimum KL-BEM is used and obtaining the channel correlation in practical cases is not an easy task.

Figure 3 shows the MSE versus different window size for CE-BEM, OBEM, Slepian and KL-BEM. By analyzing Fig. 3, it can be concluded that for lower window sizes, LBEM presents a better LS channel modeling. As the window size increases the MSE for LBEM gets higher and more similar to OPM.

Figure 4 shows MSE versus maximum Doppler frequency for OLBEM. The results are compared to that of OBEM and OPM. By analyzing Fig. 4 it is concluded that OLBEM has an acceptable MSE for different Doppler frequencies. For smaller Doppler frequencies, OLBEM has better modeling performance. It also has less least-squares channel modeling MSE compared with OPM.

Figure 5 plots MSE versus different window sizes for OLBEM, the results are compared to that of OBEM and OPM. It is concluded that for smaller window sizes, OLBEM models the channel more accurately.

VI. COCLUSION

In this paper, Legendre basis expansion model was discussed and a new basis expansion model based on Legendre polynomials is proposed. The proposed BEM uses Legendre Kernel functions and oversamples them to form the oversampled Legendre basis expansion model (OLBEM). The proposed model has an acceptable channel modeling MSE which is better than the other oversampled BEMs.

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