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Optimal Training Sequence Design for Channel Estimation in MIMO systems

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***Abstract-* in this paper we try to review recent works on optimal training sequence design for MIMO and MIMO OFDM systems. We also classify these works, according to assuming MIMO channel model, into two main categories. First, works that assume MIMO channel are independent identically distributed. Second group assumes MIMO channel are correlated. Simulation results are presented. It has been illustrated that it is helpful to make full use of the knowledge of channel correlation at the transmitter to realize optimal training.**

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) system has recently received a significant amount of attention, for it has prominent capacity potential to meet the growing demand for high data rates in wireless communication systems without additional power or bandwidth consumption [1, 2]. However, to exploit the capacity advantage of MIMO systems, the Channel State Information (CSI) is required at the receiver (and transmitter in some cases). Therefore, accurate and efficient channel estimation plays a key role in MIMO communication systems.

CSI can be obtained in different ways; one is based on training symbols that are *a priori* known at the receiver, whereas the other is blind, i.e., relies only on the received symbols, and acquires CSI by, e.g., exploiting statistical information and/or transmitted symbol properties (like finite alphabet, constant modulus, etc.) [3], [4]. However, compared with training, blind channel estimation generally requires a long data record. Hence, it is limited to slowly time-varying channels and entails high complexity. For these reasons, we restrict our attention to training-based channel estimation in this paper.

Finding optimum training sequences so that channel estimation is done well is of great importance. Optimal training sequence designs proposed in literature can be categorized into two groups. In the first one, the impact of MIMO channel correlation on the design of both estimator and training sequences in an explicit way have not been dealt with. In this group channel coefficients are assumed to be independent identically distributed. How ever in certain propagation environments, MIMO channel bears various correlations, especially spatial fading correlation and multipath tap gain correlation when the channel is frequency selective. Both correlations must be taken into account for designing optimal training sequences.

In [5], optimal estimation for correlated MIMO channels with flat fading using pilot signals have been addressed, assuming knowledge of the second-order channel statistics at the transmitter. Assuming block fading channel model and minimum mean square error (MMSE) estimation at the receiver, the transmitted signal (training sequence) was designed to optimize two criteria: MMSE and the conditional mutual information between the MIMO channel and the received signal.

For the frequency-selective case, In [6] optimal design of training sequences for MIMO channel estimation is considered. The channel is assumed to be frequency selective and obey block fading law with both spatial fading correlation and multipath tap gain correlation known at both transmitter and receiver. To minimize the channel estimation error, optimal training sequences are designed to exploit full information of channel correlation under the criterion of Minimum Mean Square Error (MMSE). It is investigated that channel correlation is helpful to decrease the estimation error and the proposed training sequences have good performance via simulations.

There is similar discussion for MIMO OFDM systems. Optimal placement and energy allocation of training symbols or pilot tones for both single-carrier and OFDM systems were considered in [7] for frequency-selective block-fading channel estimation. The training signal placement design is based on maximizing a lower bound on the training-based capacity with the assumption that all training symbols or pilot tones have the same energy. For OFDM systems, the optimal placement of pilot tones is equal spacing in the frequency domain. In [8], optimal design and placement of pilot symbols for frequency selective block-fading channel estimation are addressed for single-input single-output (SISO) as well as multiple-input multiple-output (MIMO) single-carrier systems by minimizing the Cramer-Rao bound. The same problem was addressed in [9] by maximizing a lower bound on the average capacity.

In [10], optimal training signal design and power allocation for frequency-selective block-fading channel estimation in linearly-precoded OFDM systems (which include OFDM systems as well as single-carrier systems with cyclic prefix) were presented where it was shown that the L pilot tones (which is the minimum required to estimate an L tap channel) are equi-powered and equi-spaced. For doubly selective fading channels characterized by the basic expansion model, an optimal training structure was presented in [11] for SISO single-carrier systems by maximizing a lower bound on the average channel capacity (equivalently minimizing the minimum mean square error). In [12], MIMO training signal design for single-carrier systems was reduced into a SISO design with a longer training sequence using the space-time code structure. Furthermore, some sample training sequence constructions were presented.

In [13], optimal training signal design for frequency selective block-fading channel estimation in MIMO OFDM systems was analyzed based on minimizing channel estimation mean square error (MSE). The optimal pilot tones for channel estimation based on one OFDM symbol were shown to be equi-powered and equi-spaced. Furthermore, pilot tones from different antennas must be phase-shift orthogonal. For channel estimation based on Q OFDM symbols, the conditions on pilot tones for the case of one OFDM symbol are just spread out over the Q symbols. Note that [14] also presented an optimal training signal design for MIMO OFDM systems where all sub-carriers are used as pilot tones with equal power and pilot tones from different antennas are phase-shift orthogonal. A similar design with BPSK pilot symbols (a phase-shift of $\pm\pi$ among pilot tones of different antennas) was used in [15] for two transmit antennas and later extended to more transmit antennas in [16]. Reference [17] presents general classes of optimal training signals for the estimation of frequency-selective channels in MIMO OFDM systems. Basic properties of the discrete Fourier transform are used to derive the optimal training signals which minimize the channel estimation mean square error. Both single and multiple OFDM training symbols are considered. Several optimal pilot tone allocations across the transmit antennas are presented and classified as frequency division multiplexing, time-division multiplexing, code-division multiplexing in the frequency-domain, code-division multiplexing in the time-domain, and combinations thereof. All existing optimal training signals in the literature are special cases of the presented optimal training signals and their designs can be applied to pilot-only schemes as well as pilot-data-multiplexed schemes.

Works on channel estimation in MIMO OFDM that were introduced above assumes that MIMO channels are *independent and identically distributed* (*i.i.d.*). In certain propagation environments, there exists spatial correlation among channels corresponding to different pairs of transmit and receive antennas. The spatial correlations can be exploited to improve channel estimation. In [18] a *minimum mean-square-error* (MMSE) channel estimator for MIMO-OFDM systems have been developed that can make full use of the spatial correlation. They have designed optimum training sequences that minimize the channel estimation error. When MIMO channels are *i.i.d.*, the training sequences for different transmit antennas are orthogonal and with equal power. However, when MIMO channels are spatially correlated, the power allocation for training sequences can be further optimized. The proposed MMSE estimator can exploit spatial and frequency correlations of MIMO channels in OFDM systems and therefore has good performance.

The rest of the paper is organized as follows. In Section II, we will overview two mentioned categories in design of optimal training in detail. So we have next three subsections:

A. Optimal training sequences for i.i.d MIMO OFDM channels
B. Optimal training sequences in correlated MIMO channels

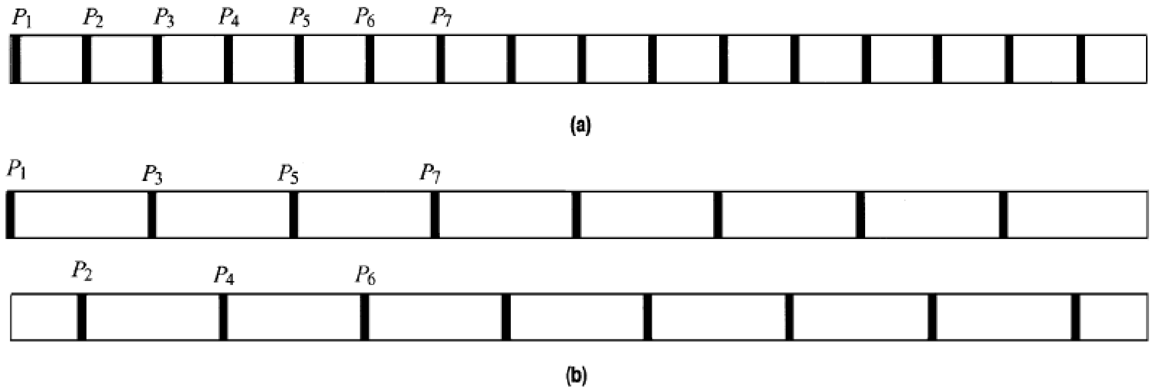

Fig. 1. (a) Training over one OFDM symbol. (b) Training over two OFDM symbols

C. Optimal training sequences in correlated MIMO OFDM channels

Finally, conclusions are presented in Section III.

II. OVERVIEW

A. Optimal training sequences for i.i.d MIMO OFDM channels

HIGH-DATA rate techniques in communication systems have gained considerable interest in recent years. A technique that has attracted a lot of attention is orthogonal frequency division multiplexing (OFDM), which is a multicarrier modulation technique. This is due to its simple implementation, and robustness against frequency-selective fading channels, which is obtained by converting the channel into flat fading subchannels. OFDM has been standardized for a variety of applications, such as digital audio broadcasting (DAB), digital television broadcasting, wireless local area networks (WLANs), and asymmetric digital subscriber lines (ADSLs). Combining OFDM with multiple antennas has been shown to provide a significant increase in capacity through the use of transmitter and receiver diversity [19]. However such systems rely upon the knowledge of channel state information (CSI) at the receiver.

In this section we try to explain one of the most cited articles in the field of optimal training design for MIMO OFDM. In [13] a LS channel estimation scheme for MIMO OFDM systems based on pilot tones is described. First, the MSE of the LS channel estimate is computed. Then, optimal pilot sequences and optimal placement of the pilot tones with regard to this MSE are derived. To reduce the training overhead, an LS channel estimation scheme over multiple OFDM symbols is also discussed. Moreover, to enhance channel estimation, a recursive LS (RLS) algorithm is proposed.

Table I lists various scenarios and the constraints they impose on the optimal pilot sequences. L is the maximum length of all channels. Phase shift orthogonality in the frequency domain corresponds to circular shift orthogonality in the time domain. In other words, the pilot sequence of one antenna must not only be orthogonal to the pilot sequences of other antennas but to circularly shifted copies of these sequences as well.

TABLE I
CONSTRAINTS ON OPTIMAL PILOT SEQUENCES FOR VARIOUS SCENARIOS

Configuration	PS Requirement
Single TX	Equipowered + Equispaced [7]
Multiple TX Flat Fading: $L = 1$	Equipowered + Equispaced + Orthogonal
Multiple TX Frequency-Selective Fading: $L > 1$	Equipowered + Equispaced + Phase Shift Orthogonal $\phi \in \{-L+1, \ldots, L-1\}$

we can design optimal pilot sequences over multiple OFDM symbols (g symbols, g>1), arbitrarily split each sequence of length P into g subsequences of length P/g , and arbitrarily assign each subsequence to a different OFDM symbol. see, for example, Fig. 1 represents training over two consecutive OFDM symbols. P is the number of pilot tones dedicated for training.

SIMULATION RESULTS

channels are assumed with $L = 8$ taps. These taps are simulated as i.i.d. and correlated in time with a correlation function according to Jakes' model $r_{hh}(\tau) = \sigma_h^2 J_0(2\pi f_d \tau)$. We consider $K = 128$ sub carriers and a cyclic prefix of length $\nu = 8$. The number of pilot tones dedicated for training is $P = 16$, which satisfies the minimum number of training and maximum spacing. Hence, when training is performed over g consecutive OFDM symbols, $P/g = 16/g$ pilot tones are used for training in each OFDM symbol. The OFDM symbol duration is $T_f = 1.13$ ms. QPSK signaling is applied. Finally, 2 transmit and 4 receive antennas are assumed. The performance of the system is measured in terms of the MSE of the channel estimate, and the bit error rate (BER) versus SNR for a zero-forcing equalizer based on the channel estimate. The SNR is defined as $\text{SNR} = LN_t \sigma_h^2 E_s / \sigma_n^2$, where is the QPSK symbol power (the power dedicated for training is $\rho = (P/(K+L))\rho_{tot}$, where is ρ_{tot} the total power used to transmit a single OFDM symbol). We run the simulations for different Doppler spreads $f_d = 5, 20, 40$, and 100 Hz.

In our simulations, we evaluate a variety of choices for the pilot sequences:

i) equipowered, equispaced random pilot tones;

ii) equipowered, equispaced, orthogonal pilot tones;

iii) equipowered, equispaced, phase shift orthogonal pilot tones.

As shown in Figs. 2 and 3, using phase shift orthogonal pilot sequences outperforms the use of random or orthogonal pilot sequences in terms of MSE of the channel estimate and BER. We can see a 2-dB gain in SNR for phase shift orthogonal over or orthogonal pilot sequences at a BER of 10^{-2} and Doppler spread $f_d = 5$ Hz and a 3.5-dB gain in SNR at a BER of 10^{-2} and Doppler spread $f_d = 100$ Hz. Random pilot sequences are clearly useless. Similar results hold when training over two and four consecutive OFDM symbols is considered (see Figs. 4–7). It is found that training over multiple OFDM symbols pays off especially for slowly time-varying channels. For example, for channels with a Doppler spread $f_d = 5$ Hz, training can be performed over two or four consecutive OFDM symbols without any performance loss, whereas for fast time-varying channels, this scheme will experience an increased BER and becomes even prohibitive for very fast time-varying channels, as shown in Figs. 4–7.

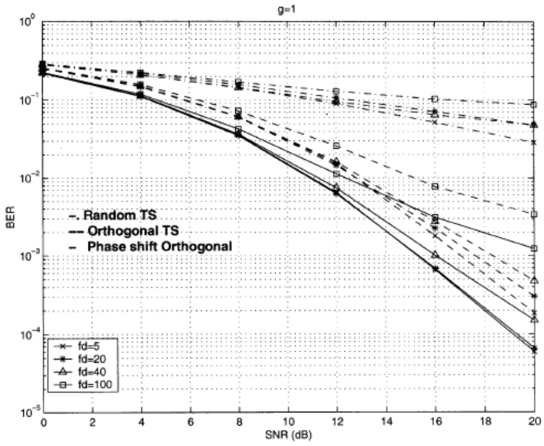

Fig. 2. BER versus SNR for training over one OFDM symbol.

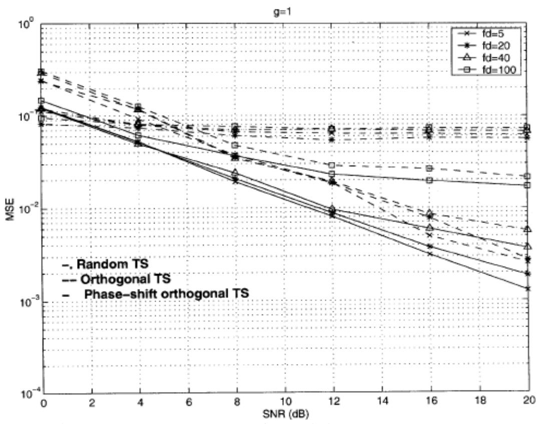

Fig. 3. MSE versus SNR for training over one OFDM symbol.

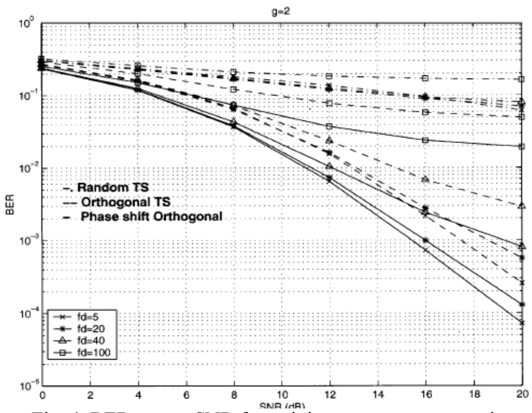

Fig. 4. BER versus SNR for training over two consecutive OFDM symbols.

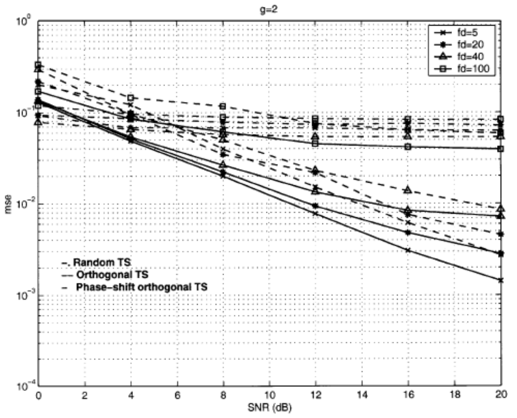

Fig. 5. MSE versus SNR for training over two consecutive OFDM symbol.

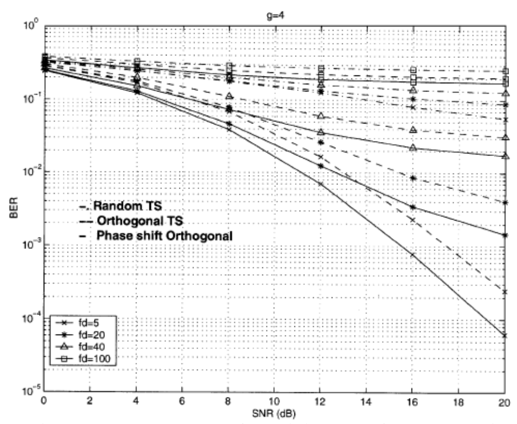

Fig. 6. BER versus SNR for training over four consecutive OFDM symbols.

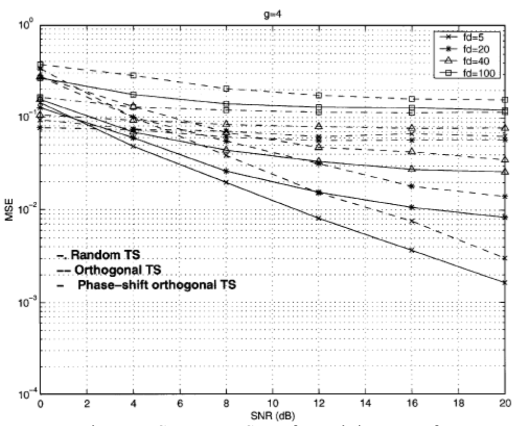

Fig. 7. MSE versus SNR for training over four consecutive OFDM symbol.

In [13], an LS channel estimation scheme for MIMO OFDM systems based on pilot tones has been proposed. To obtain the minimum MSE of the LS channel estimate, the pilot sequences must be equipowered, equispaced, and phase shift orthogonal. Increasing the number of transmit antennas requires more pilot tones for training and, hence, decreases the efficiency. This effect can be mitigated by estimating the channel parameters over multiple OFDM symbols when the channel is slowly time-varying.

B. Optimal training sequences in correlated MIMO channel

Accurate and efficient channel estimation is of critical importance in the design of coherent communication systems. In multiple-input multiple-output (MIMO) channels, this problem is particularly challenging, due to the large number of channel parameters to be estimated in general. Under the idealized assumption of independent and identically distributed (i.i.d.) channel coefficients, the solution is relatively straightforward due to the i.i.d nature of the coefficients [20], [21]. However, this idealized assumption does not generally hold in practice, and hence, a study of correlated channels is of interest. Indoor environments are examples of such channels.

In [4], the optimal training has been addressed for correlated MIMO channels with flat fading. A MIMO channel with *P* transmit and *Q* receive antennas has a maximum of *PQ* unknowns to be estimated. However, correlated MIMO channels possess fewer degrees of freedom, and hence, fewer than *PQ* parameters need to be estimated. In view of the large number of channel coefficients to be estimated, this important fact has been exploited to design efficient signaling schemes for optimal channel estimation. They considered a general model for correlated MIMO channels that exposes the true degrees of freedom of the channel.

In [6], the channel is assumed to be frequency selective Rayleigh fading with tap gain correlation and spatial correlations at both ends. The channel correlation matrix is modeled by Kronecker product of three correlation matrices, i.e., tap gain, transmit and receive correlation matrices, which is the most popular and widely used way to represent MIMO channel correlation [22], [23]. Based on these assumptions, the optimal training sequences are designed using MMSE (Minimum Mean Square Error) estimator.

In [6] training design for correlated frequency-selective MIMO channels. Assuming the channel is block Rayleigh fading and obeys Kronecker correlation model, they have deduced the optimal structure of training sequences which has general applicability. Design algorithms have been presented in the form of a theorem and some special cases verified via numerical computations.

NUMERICAL RESULT

In this section, we present two numerical examples to illustrate the impacts of channel correlation on the MSE and the performance achieved by the optimal training sequences.

For the sake of simplicity and without loss of generality, we assume the numbers of transmit antennas and receive antennas are 4 and the number of resolvable paths is 3. The channel correlation is generated by the exponential correlation function [24].

1) Assume there is no spatial correlation at the receiver, $\boldsymbol{R}_t(i,j) = \alpha^{|i-j|}, i, j = 0,1,2,3$, where α is defined as the spatial correlation coefficient with $|\alpha| \le 1$, and $\boldsymbol{R}_L(i,j) = \beta^{|i-j|}, i, j = 0,1,2$, where β is defined as the tap gain correlation coefficient with $|\beta| \le 1$. Fig. 8 demonstrates the MSE (normalized by the total number of channel coefficients) for different channel correlations. From Fig. 8, we can know that channel correlation is helpful to decrease the estimation error. The stronger the channel correlation is, the smaller the MSE is, and spatial correlation has a greater effect on MSE than tap gain correlation does.

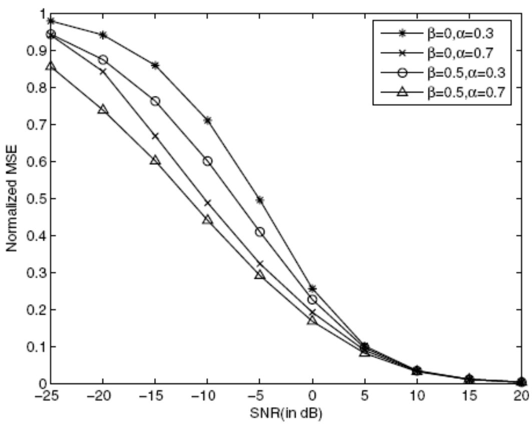

Figure 8. MSE for optimal training sequences without spatial correlation at the receiver.

2) Assume there is no tap gain correlation, $\boldsymbol{R}_t(i,j) = \boldsymbol{R}_r(i,j) = \alpha^{|i-j|}, i, j = 0,1,2,3$. Fig. 9 shows the MSE comparison explicitly between orthogonal and asymptotically optimal training sequences, where 'opt' stands for the case of the optimal training and 'ort' for the orthogonal case. It can be seen that the MSEs corresponding to 'opt' case are smaller than those in the 'ort' case, especially in the strong correlation and low SNR region. So that the performance of channel estimation can be improved by exploiting the knowledge of channel correlation.

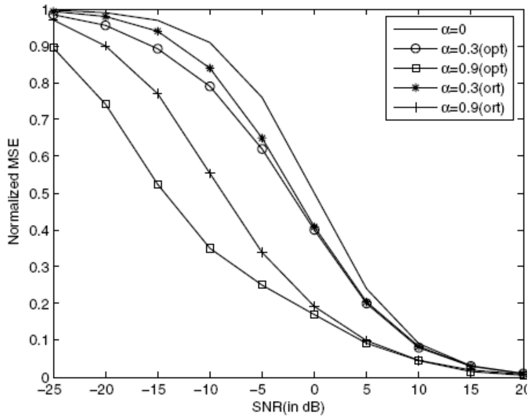

Figure 9. MSE comparison for optimal / orthogonal training sequences without tap gain correlation.

It has been illustrated that the MSE is smaller in the correlated MIMO channel than that in the uncorrelated one, and it is helpful to make full use of the knowledge of channel.

C. Optimal training sequences in correlated MIMO OFDM channels:

For MIMO channels with independent channel imuplse responses corresponding to different pairs of transmit and receive antennas, channel estimation approaches have been developed in [15] and [13] and optimum training sequences have been designed to minimize *mean-square error* (MSE) of channel estimation. However, in practical environments, there are spatial correlations between MIMO channels, which can be used to improve channel estimation.

In [18] channel estimation for spatially correlated broadband MIMO channels in OFDM systems are addressed. Since statistics of MIMO channels vary very slowly with time [25], assuming that the correlation matrices are known at the transmitter and receiver. With the knowledge of correlation matrices, *minimum mean-square-error* (MMSE) channel estimation can be developed. We also investigate optimum training sequences that minimize the estimation error.

It can be easily shown that the training sequences for independent fading channels in [14] and in [13] satisfy the derived conditions in [18]. These optimum training conditions are certainly applicable to flat fading MIMO channels. In this case, they are equivalent to those developed in [5].

Note that estimation in [18] is based on one OFDM block. If multiple training blocks or decision-directed estimation are used, more accurate estimation can be obtained by exploiting the time correlation of channel parameters using the method developed in [26].

SIMULATION RESULTS

In this section, we present simulation results to demonstrate the performance of the proposed channel estimation and training sequence design approaches.

A. Example 1

To study the impact of the propagation parameters, we use a simple channel model in Examples 1 and 2. In the simulated system, two transmit antennas and two receive antennas are used, that is, $M_T = M_R = 2$. The channel's bandwidth is divided into 32 subchannels, and the cyclic prefix is longer than the delay spread. The channel is assumed to have two taps with powers $\sigma_0^2 = 0.8$ and $\sigma_1^2 = 0.2$, respectively. The relative antenna space $\Delta = 1$.

In this example, we study the impact of the angle spread on the performance of the estimation. The average angles of departure are $\bar{\theta}_{t0} = 13.5°$, and $\bar{\theta}_{t1} = 26.4°$ and the average angles of arrival are $\bar{\theta}_{r0} = 290.3°$, and $\bar{\theta}_{r1} = 332.3°$. The angle spread for all taps at both transmitter and receiver are same. Figure 10 demonstrates the MSE for different angle spreads. For comparison, the figure also shows the MSE of the LS estimation. From the figure, the performance of the MMSE estimator is always better than the LS estimator. However, the improvement reduces when SNR increases if angle spread is nonzero. We also observe that the MSE is decreasing when the angle spread decreases. That shows that the MMSE estimator can capture the spatial correlation of MIMO channels while the performance of the LS estimator is independent of the angle spread.

B. Example 2

In this example, we set the angle of departure for two taps to be same. The angle spread is 8.6°. Other parameters are the same as in Example 1. In this case, we can use the training sequences derived in the special cases. Figure 11 demonstrates the performance of LS, MMSE estimation with equal and optimum power allocation training sequences. From the figure, the training sequence with optimum power allocation achieves the best performance, especially at low SNR regime.

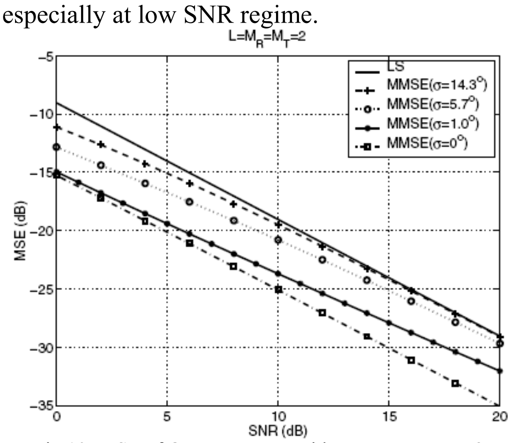

Fig.10. MSE of OFDM system with $L = MT = MR = 2$.

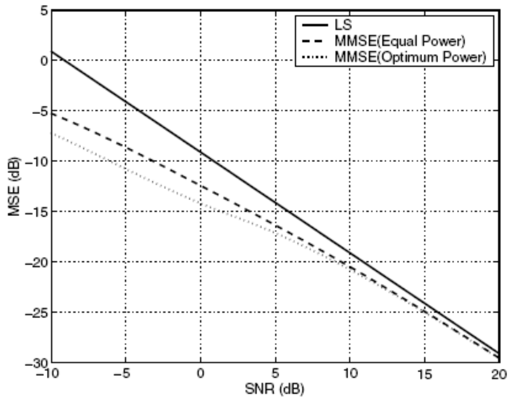

Fig.11. MSE of OFDM system with the same angle of departure.

In this paper, we address channel estimation for MIMOOFDM systems in spatially correlated fading channels. Exploiting the spatial correlation, the proposed channel estimator can achieve better performance than the LS estimator. Based on the MMSE estimation, In [18] the conditions are derived and developed the approaches for optimum training sequences. The proposed approaches can be used in wireless LAN where MIMO channels are correlated.

III. CONCLUSIONS

In this article we have presented an overview of training sequence design in MIMO and MIMO OFDM systems. They have designed optimum training sequences that minimize the channel estimation error. When MIMO channels are *i.i.d.*, the training sequences for different transmit antennas are orthogonal and with equal power. However, when MIMO channels are spatially correlated, the power allocation for training sequences can be further optimized. The proposed MMSE estimator can exploit spatial and frequency correlations of MIMO channels in OFDM systems and therefore has good performance.

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