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# Low-complexity CFO compensation technique for interleaved OFDMA system uplink

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ABSTRACT

Carrier frequency offset (CFO) is a challenging problem in the uplink of orthogonal frequency division multiple access (OFDMA). In this paper, we present a new CFO compensation technique for interleaved OFMA uplink. The received signal for each user is corrected by the CFO composed of CFO values from two adjacent users and then the residual interference is cancelled by the parallel interference canceller (PIC) algorithm. In the following we analyze the performance of the proposed method and derive a signal-to-interference (SIR) expression. Finally, simulation results show acceptable performance of the proposed method in comparison with some conventional techniques with lower complexity.



## 1. Introduction

Orthogonal frequency division multiple access (OFDMA) is widely used for wireless communication networks, e.g., IEEE 802.11, IEEE 802.16, etc. A precoded version of OFDMA, called SC-FDMA, is used in long term evolution (LTE). This is because of its high spectral efficiency and immunity against multipath fading in wireless channels [1]. In addition, as an important advantage, OFDMA systems enjoy simple frequency domain equalization techniques against many wireless systems.

Subcarriers in OFDMA are partitioned into several mutually exclusive subsets. Each subset of subcarriers is assigned to a distinct user for simultaneous transmissions. There are different schemes for subcarrier assignment. Two main categories are block and interleaved schemes [1]. In the block allocation all the assigned subcarriers to each user are adjacent, where they are distributed uniformly for the whole available bandwidth in the interleaved allocation.

Similar to orthogonal frequency division multiplexing (OFDM), OFDMA is sensitive to carrier frequency offset (CFO) caused by the Doppler effect and oscillator instabilities [1,2]. In OFDMA, CFO disrupts orthogonality among subcarriers. It results in inter-carrier interference (ICI), which is the interference between the subcarriers of each user, and multiple access interference (MAI), which is the interference between subcarriers of different users [1].

Frequency synchronization is a more challenging problem in uplink transmission than that of downlink, where the received signal is the summation of incoming signals from different users through various wireless channels with distinct CFO values [1]. It is not possible to compensate different CFOs at the base station (BS) for different users simultaneously [1]. Therefore, the performance of OFDMA system in uplink is limited by interference that is caused by each user for other users, called MAI. The interleaved allocation is more sensitive to CFO compared to other assignment schemes [3]. Also subcarriers of a certain user in interleaved allocation are far apart. Therefore ICI in the interleaved allocation is negligible and the major part of interference is MAI.

There are many proposed schemes to correct CFO values in the uplink of OFDMA systems [1]. These schemes are classified into two categories, feedback adjustment and user detection [1]. Feedback schemes need to send feedback information that reduces data rate and spectral efficiency. Also, the performance gain cannot be obtained in time-variant channels. In the user detection techniques, synchronization errors are compensated directly in base station using advanced signal processing techniques.

Most of the user detection techniques can be divided into two categories. The first group uses a straightforward approach. The CFO of each user is compensated in time domain directly. This technique is called single user detection (SUD) [4]. The computational complexity of SUD increases by increasing the number of users. To reduce the complexity, a modified version has been proposed by Choi-Lee-Jung-Lee's (CLJL) in frequency domain [5]. Although the CLJL technique cannot improve the performance in comparison with SUD, but it has a lower complexity than SUD. Both CLJL and

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SUD will omit ICI but MAI remains. Since residual MAI limits the performance of SUD and CLJL, some interference cancellation (IC) algorithms have been used to remove MAI [1]. Interference cancellation is performed in both successive and parallel modes [1]. Successive interference cancellation (SIC) removes MAI from subcarriers user by user, where parallel interference cancellation (PIC) removes MAI on all subcarriers simultaneously. PIC schemes are more common because of their faster computation due to parallel processing. Conventional linear PIC (CLPIC) algorithm performs interference cancellation in the frequency domain to improve the performance of SUD technique [6]. A modified version of the CLPIC that uses optimum weights for MAI constellation improves the performance while it increases the complexity [6]. Huang–Letaief circular convolution (HLCC) removes MAI in frequency domain to improve the performance of CLJL technique [7]. A comparison between HLCC and CLPIC reveals that the HLCC scheme performs better than CLPIC when the individual CFO values are small, where the later case performs better than HLCC scheme for the small difference between CFO values(even if the CFO values are large) [6]. Both HLCC and CLPIC schemes improve the performance at the cost of high complexity. Furthermore, the above mentioned techniques are sensitive to power control imperfection [8].

Another class of user detection techniques is multi user detection (MUD), where CFO compensation and equalization are performed simultaneously. MUD is based on the fact that the received signal at BS is combination of transmitted symbols by different users. This linear combination is not reversible. However the minimum mean square error (MMSE) detection can be used to obtain the transmitted symbols. Although the accuracy of MMSE solution is acceptable, but it requires high computational complexity due to $N \times N$ matrix inversion, where $N$ is the total number of subcarriers. Some researches are focused on complexity reduction of MMSE [9–15]. Recently, a low complexity CFO compensation technique based on the least squares (LS) and MMSE criteria applicable to interleaved and block interleaved carrier assignment schemes has been proposed [13]. This technique utilizes the special block circulant property of the interference matrix that has the same performance as MUD. Also a low complexity CFO compensation method with receiver windowing has been proposed [14]. It is worth noting that this complexity reduction is archived in expense of some spectral efficiency loss.

Briefly, SUD-based methods maximize desired signal power and mitigate ICI in CFO compensation stage, and also remove MAI using interference cancellation stages, where MUD-based methods extract transmitted symbols directly. Although MUD-based methods improve the performance of CFO compensation significantly, but SUD-based methods have lower computational complexity [1]. This paper proposes a low-complexity SUD-based method for the interleaved OFDMA system. Our proposed method performs compensation with the goal of signal-to-interference (SIR) maximization. The main idea is upon the feature that each subcarrier suffers from high MAI and negligible ICI in the interleaved OFDMA system [16]. Conventional schemes use estimated CFO value to compensate CFO for each user directly, whereas the proposed scheme uses a combination of CFO values from two adjacent users to compensate CFO. The complexity of our approach is comparable to other SUD-based methods. Simulation results show satisfying performance with reasonable complexity. Also, the proposed method shows notable stability against imperfect power control due to MAI minimization. To further improve the performance, a PIC algorithm is proposed. According to simulation results, the proposed algorithm has a comparable performance with CLPIC and HLCC but with a much lower complexity.

The rest of this paper is organized as follows. In Section 2 OFDMA system uplink in the presence of CFO is modelled. The proposed CFO compensation technique is presented in Section 3. Analysis of the proposed PIC scheme is reported in Section 4. Section 5 provides simulation results and finally this paper is concluded in Section 6.

## 2. System model

Consider an uplink OFDMA system with $M$ active users. Each user communicates with BS through an independent multi-path channel (Fig. 1). There are $N$ subcarriers on each OFDMA symbol assigned to active users with interleaved allocation. The information symbol of the $m$th user on the $k$th subcarrier is denoted by $X_m(k)$, $k \in \mathcal{I}_m$, where $\mathcal{I}_m$ is the set of subcarrier indices that are assigned to the $m$th user. Also $\mathcal{I}_i = \{i + Mq | q = 0, \ldots, (N/M)\}$ satisfies $\mathcal{I}_i \bigcap \mathcal{I}_j = \emptyset$, for $i \neq j$ and $\bigcup_{i=1}^{M} \mathcal{I}_i = \{1, 2, \ldots, N\}$. After IDFT processing and adding guard period, the time domain transmitted signal of the $m$th user is [1]

$$x_m(n) = \sum_{k \in \mathcal{I}_m} X_m(k) e^{\frac{j2\pi kn}{N}}, \quad -N_g \leqslant n \leqslant N-1 \tag{1}$$

where $N_g$ is the length of guard interval. The received signal at the BS from the $m$th user is

$$y_m(n) = x_m(n) * c_m(n) \tag{2}$$

where $c_m(n)$ is the channel impulse response (CIR) between the $m$th user and BS. Channel coefficients $c_m(n)$, $m = 1, \ldots, M$, $n = 0, \ldots, L-1$ are statistically independent and i.i.d complex Gaussian random variables with zero mean and

$$\mathrm{E}\{|c_m(n)|^2\} = \beta_m e^{\frac{-n}{L}}, \quad n = 0, 1, \ldots, L-1, \tag{3}$$

where $L$ is the maximum delay spread of the channel and $\beta_m$ is a scaling factor for the average energy of CIR, as

$$\sum_{n=0}^{L-1} \mathrm{E}\{|c_m(n)|^2\} = \gamma_m. \tag{4}$$

The received signal at the BS after coarse frequency synchronization is [1]

$$r(n) = \sum_{i=1}^{M} y_i(n) \cdot e^{j2\pi n \varepsilon_i / N} + w(n), \quad -N_g \leqslant n \leqslant N-1. \tag{5}$$

where $w(n)$ is AWGN with zero mean and variance $\sigma^2$ and $\varepsilon_m$ is the normalized CFO (to subcarrier spacing) between the $m$th user and BS. According to (5), the CFO effect is a frequency shift on the transmitted signal of each user. After removing guard interval, the received signal on the $k$th subcarrier is [1]

$$\mathbf{R}(k) = \mathrm{DFT}\{r(n)\} = \sum_{i=1}^{M} \sum_{u \in \mathcal{I}_i} X_i(u) C_i(u) D(k, u, \varepsilon_i) + W(k),$$
$$k = 1, \ldots, N, \tag{6}$$

where $X_i(u)$, $C_i(u)$ and $W(k)$ are information symbol, CIR from the $i$th user to BS and noise in the frequency domain respectively. Also $D(k, u, \varepsilon_i)$ is the leakage power of the $u$th subcarrier on the $k$th subcarrier, where $u \in \mathcal{I}_i$. The leakage power $D(k, u, \varepsilon_i) = f_N(u - k + \varepsilon_i)$, where $f_N(x)$ is [1]

$$f_N(x) = \sin(\pi x) \cdot e^{j\pi x (1 - \frac{1}{N})} / N \sin\left(\frac{\pi x}{N}\right). \tag{7}$$

It is clear that the absolute value of $f_N(x)$ is a periodic quasi Sinc function with period $N$. Interestingly, it has a large main lobe and the power of sidelobes are dropped very fast by keeping out from the main lobe. For each subcarrier, we can categorize $\mathbf{R}(k)$ into four

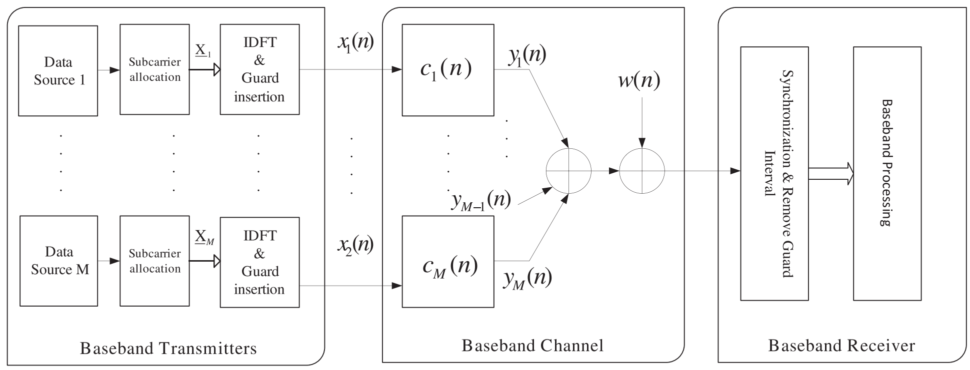


**Fig. 1.** OFDMA system uplink.

parts as

$$\mathbf{R}(k)=\underbrace{X_m(k)C_m(k)D(k,k,\varepsilon_m)}_{\text{desired symbol}}+\underbrace{\sum_{u\in\mathcal{I}_m,u\neq k}X_m(u)C_m(u)D(k,u,\varepsilon_m)}_{\text{ICI}}$$

$$+\underbrace{\sum_{i=1,i\neq m}^{M}\underbrace{\sum_{u\in\mathcal{I}_i}X_i(u)C_i(u)D(k,u,\varepsilon_i)}_{\text{MAI}^i}}_{\text{MAI}}+W(k),\quad k\in\mathcal{I}_m. \tag{8}$$

The first part is the desired signal on the $k$th subcarrier of the $m$th user, the second is the interference caused by other subcarriers of the $m$th user (ICI), the third is the interference caused by other users known as MAI, and finally the forth is noise. The average SIR on the $k$th subcarrier, $k\in\mathcal{I}_m$, is

$$\bar{\text{SIR}}(m,k)=\frac{\text{SP}(m,k)}{\text{IP}(m,k)+\sum_{\substack{n=1\\n\neq m}}^{M}\text{MP}(n,m,k)}, \tag{9}$$

where $\text{SP}(m,k)$, $\text{IP}(m,k)$ and $\text{MP}(n,m,k)$ are the desired signal power on the $k$th subcarrier, ICI power and MAI power respectively. In the interleaved OFDMA system uplink, we have [16]

$$\text{SP}(m,k)=|f_N(\varepsilon_m)|^2, \tag{10}$$

$$\text{IP}(m,k)=|f_M(\varepsilon_m)|^2-|f_N(\varepsilon_m)|^2, \tag{11}$$

$$\text{MP}(n,m,k)=|f_M(n-m+\varepsilon_n)|^2, \tag{12}$$

A simple mathematical description for SIR in the interleaved OFDMA system is obtained by substituting (10), (11) and (12) in (9). The CFO correction technique SUD is achieved by multiplying $r(n)$ to $e^{-(j2\pi n\hat{\varepsilon}_m/N)}$, where $\hat{\varepsilon}_m$ is the estimated CFO for the $m$th user. Accordingly, the average SIR on the $k$th subcarrier of the $m$th user is

$$\overline{\text{SIR}}(m,k)=\frac{|f_N(\varepsilon_m-\hat{\varepsilon}_m)|^2}{\sum_{\substack{n=1\\n\neq m}}^{M}|f_M(n-m+\varepsilon_n-\hat{\varepsilon}_m)|^2+|f_M(\varepsilon_m-\hat{\varepsilon}_m)|^2-|f_N(\varepsilon_m-\hat{\varepsilon}_m)|^2}, \tag{13}$$

## 3. Proposed CFO compensation technique

In the conventional SUD method, the received signal from the $m$th user is compensated by $\hat{\varepsilon}_m$ (an estimation of the $m$th user's CFO). For simplicity, we assume that $\hat{\varepsilon}_m=\varepsilon_m$, $m=1,\ldots,M$, thus,

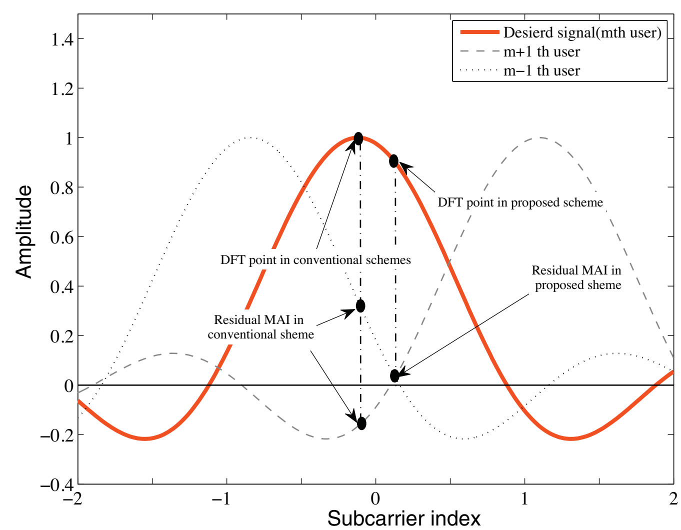


**Fig. 2.** Proposed scheme minimize MAI in CFO compensation stage.

the FFT points would be aligned on the peak power of the desierd carrier spectrum as shown in Fig. 2. However, there is a considerable amount of MAI from neighboring carriers. Our method propose another compensation value so that the DFT points are moved, where the amount of MAI power is kept very low.

As shown in Fig. 2, the conventional SUD scheme maximizes signal power but it will suffer from MAI. The proposed scheme maximizes SIR of the $m$th user to find a new compensation value for the $m$th user called $\varepsilon'_m$ that suppresses MAI and improves system performance. We assume that $\varepsilon'_m$ exists and SIR is an analytical function of $\varepsilon_m$ in the vicinity of $\varepsilon'_m$. Then by calling (13) and substituting $\hat{\varepsilon}_m$ by $\varepsilon_c$ we can find $\varepsilon'_m$ as

$$\left.\frac{\partial\bar{\text{SIR}}(m,k)}{\partial\varepsilon_c}\right|_{\varepsilon_c=\varepsilon'_m}=0. \tag{14}$$

### 3.1. Calculation of $\varepsilon'_m$

According to (13), (14) leads to a complicated expression. However, it should be noted that a major part of the interference is caused by the two adjacent subcarriers and contribution of other subcarriers is negligible. Also, according to (7) and (11), ICI is very small in comparison with MAI and it can be ignored, too.

If we assume that the difference between the normalized CFOs of the two adjacent users is less than 0.2, the correction value for a desired user is approximately [17] (see Appendix A)

$$\varepsilon'_m \approx \frac{(\varepsilon_{m-1} + \varepsilon_{m+1})}{2}. \tag{15}$$

Interestingly, $\varepsilon'_m$ is not dependent to $\varepsilon_m$ of the desired user. Also it should be noted that BS has to use an estimation of $\varepsilon_{m-1}$ and $\varepsilon_{m+1}$. According to (8), the received symbol on the $k$th subcarrier after CFO compensation by $\varepsilon'_m$ is

$$R_m(k) = \underbrace{X_m(k)C_m(k)D(k,k,\varepsilon_m - \varepsilon'_m)}_{\text{desired symbol}} + \underbrace{\sum_{u\in\mathcal{I}_m, u\neq k} X_m(u)C_m(u)D(k,u,\varepsilon_m - \varepsilon'_m)}_{\text{ICI}} + \underbrace{\sum_{i=1,i\neq m}^{M} \underbrace{\sum_{u\in\mathcal{I}_i} X_i(u)C_i(u)D(k,u,\varepsilon_i - \varepsilon'_m)}_{\text{MAI}^i}}_{\text{MAI}} + W^{(1)}(k). \tag{16}$$

Accordingly the desired part, may be written as $X_m(k)C_m(k)f_N(\varepsilon_m - \varepsilon'_m)$ $k \in \mathcal{I}_m$, is affected by $f_N(\varepsilon_m - \varepsilon'_m)\cdot C_m(k)$ and should be equalized to restore the constellation points. Thus the desired section of the received symbol is corrected by setting the single tap equalizer coefficient as

$$\alpha_m^k = \frac{\xi_m}{C_m(k)}, \tag{17}$$

where $\xi_m = 1/f_N(\varepsilon_m - \varepsilon'_m)$. Therefore, the required steps for the proposed scheme are listed in Table 1.

### 3.2. Proposed PIC scheme

To improve the performance of the proposed CFO compensation in [17], a PIC scheme is added to remove the residual interference. The MAI on each subcarrier is calculated and cancelled iteratively. As mentioned before, ICI can be neglected in the interleaved allocation.

According to (8), the MAI on the $k$th subcarrier of the $m$th user after CFO compensation by the proposed scheme is

$$\text{MAI}_m(k) = \underbrace{\sum_{\substack{i=1\\ i\neq m}}^{M} \underbrace{\sum_{u\in\mathcal{I}_i} X_i(u)C_i(u)D(k,u,\varepsilon_i - \varepsilon'_m)}_{\text{MAI}^i}}_{\text{MAI}}, \quad k \in \mathcal{I}_m. \tag{18}$$

The proposed PIC scheme can be described as an algorithm, where $\hat{Y}_m^{(j)}(k)$ denoting the restored signal at the $j$th step.

- Initialization: set $j=1$ and

$$\hat{Y}_m^{(1)}(k) = R_m(k), \quad m = 1,\ldots,M \tag{19}$$

**Table 1**
Proposed CFO compensation scheme.

| |
|---|
| 1. Calculate $\varepsilon'_m$ for each user using (15). |
| 2. Compensate the received signal $r(n)$ by $\varepsilon'_m$ |
| 3. Setting the single tap equalizer coefficient by (17) |

- Loop: $j=j+1$ and

$$\text{MAI}_m^{(j)}(k) = \sum_{i=1,i\neq m}^{M} \xi_i \sum_{u\in\mathcal{I}_i} \rho_{m,i}^{k,u} \hat{Y}_i^{(j-1)}(u), \; m = 1,\ldots,M, \tag{20}$$

$$\rho_{m,i}^{k,u} \triangleq D(k,u,\varepsilon_i - \varepsilon'_m) = \frac{\sin(\pi(u-k+\varepsilon_i - \varepsilon'_m))}{N\sin(\frac{\pi}{N}(u-k+\varepsilon_i-\varepsilon'_m))} e^{j\pi(1-\frac{1}{N})(u-k+\varepsilon_i-\varepsilon'_m)}, \tag{21}$$

$$\hat{Y}_m^{(j)}(k) = R_m(k) - \text{MAI}_m^{(j)}(k), \quad m = 1,\ldots,M \tag{22}$$

- Finally amplitude and phase correction and also channel equalization will be done by multiplying symbols to $\alpha_m^k$, to extract data symbols as

$$\hat{X}_m(k) = \alpha_m^k \hat{Y}_m^{(j)}(k). \tag{23}$$

## 4. Analysis and discussion

### 4.1. SIR analysis for the proposed PIC scheme

The average SIR on the $k$th subcarrier of the $m$th user after CFO compensation was presented in (9). When the PIC is applied, signal and interference powers should be calculated. Intuitively average SIR depends on the number of subcarriers, number of users, channel impulse response (CIR), CFO values and subcarrier allocation scheme.

The received symbol on the $k$th subcarrier of the $m$th user for $j=2$ in the PIC algorithm is

$$\hat{Y}_m^{(2)}(k) = S_m^{(2)}(k) + \text{ICI}_m^{(2)}(k) + \text{MAI}_m^{(2)}(k) + W^{(2)}(k), \tag{24}$$

where the signal power, remaining ICI, and MAI on the $k$th subcarrier can be written as (see Appendix B for more details)

$$S_m^{(2)}(k) = Y_m(k)\left(\frac{1}{\xi_m} - \sum_{n=1,n\neq m}^{M} \xi_n \sum_{u\in\mathcal{I}_n} \rho_{m,n}^{k,u}\rho_{n,m}^{u,k}\right), \tag{25}$$

$$\text{MAI}_m^{(2)}(k) = -\sum_{n=1,n\neq m}^{M} \sum_{u\in\mathcal{I}_n} Y_n(u) \sum_{l=1,l\neq m,n}^{M} \xi_l \sum_{v\in\mathcal{I}_l} \rho_{l,n}^{v,u}\rho_{m,l}^{k,v}, \tag{26}$$

$$\text{ICI}_m^{(2)}(k) = \sum_{u\in\mathcal{I}_m, u\neq k} \left(Y_m(u)(\rho_{m,m}^{k,u} - \sum_{n=1,n\neq m}^{M} \xi_n \sum_{v\in\mathcal{I}_n} \rho_{n,m}^{v,u}\rho_{m,n}^{k,v})\right), \tag{27}$$

where $Y_m(k) = X_m(k)C_m(k)$ and $W^{(2)}(k)$ is the noise power after two iterations.

By system model considerations, the received power from all users are equal, so signal and interference powers can be written as follow:

$$\text{E}\{|S_m^{(2)}(k)|^2\} = \left(\frac{1}{\xi_m} - \sum_{\substack{n=1\\ n\neq m}}^{M} \xi_n \sum_{u\in\mathcal{I}_n} \rho_{m,n}^{k,u}\rho_{n,m}^{u,k}\right)^2, \tag{28}$$

$$\text{E}\{|\text{MAI}_m^{(2)}(k)|^2\} = \sum_{n=1,n\neq m}^{M} \left(\sum_{u\in\mathcal{I}_n} \sum_{l=1,l\neq m,n}^{M} \xi_l \sum_{v\in\mathcal{I}_l} \rho_{l,n}^{v,u}\rho_{m,l}^{k,v}\right)^2, \tag{29}$$

$$\mathrm{E}\{|\mathrm{ICI}_m^{(2)}(k)|^2\} = \left( \sum_{u \in \mathcal{I}_m, u \neq k} \{\rho_{m,m}^{k,u} - \sum_{n=1, n \neq m}^{M} \xi_n \sum_{v \in \mathcal{I}_n} \rho_{n,m}^{v,u} \rho_{m,n}^{k,v}\} \right)^2, \tag{30}$$

Thus SIR at the output of PIC with $j = 2$ on the $k$th subcarrier of the $m$th user is

$$\bar{\mathrm{SIR}}^{(2)}(m,k) = \frac{\mathrm{E}\{|S_m^{(2)}(k)|^2\}}{\mathrm{E}\{|\mathrm{MAI}_m^{(2)}(k)|^2\} + \mathrm{E}\{|\mathrm{ICI}_m^{(2)}(k)|^2\}}. \tag{31}$$

### 4.2. Performance analysis in AWGN

As seen in Fig. 2, the power of desired signal drops after CFO compensation. Subsequently, the signal-to-noise ratio (SNR) at DFT output will drop. Since the desired signal is multiplied by $\alpha_m^k$, then both signal and noise powers are amplified resulting in no SNR improvement. Thus the proposed algorithm will outperform SUD-based schemes as long as the reduced SNR is less than the improvement in MAI. Finally, it should be noted that, the performance of the proposed method outperforms SUD when $\mathrm{SINR}_{\mathrm{SUD}} < \mathrm{SINR}_p$, where signal-to-interference-plus-noise ratio (SINR) is defined as the average power of the signal to interference plus noise ratio and $p$ denotes the proposed method. This condition leads to a threshold ($\mathrm{SNR}_t$) for SNR before CFO compensation ($\mathrm{SNR}_0$) (see Appendix C)

$$\mathrm{SNR}_t \triangleq \frac{\xi_m^2 - 1}{\frac{1}{\bar{\mathrm{SIR}}_{\mathrm{SUD}}(m,p)} - \frac{1}{\bar{\mathrm{SIR}}_p(m,p)}}. \tag{32}$$

Accordingly when $\mathrm{SNR}_0 > \mathrm{SNR}_t$, the proposed scheme is better than SUD and vice versa. Our simulation shows that the threshold value is practical and approximately is $\mathrm{SNR}_t \approx 10dB$ for desired CFO vector (see Section 5 for more details).

### 4.3. Complexity comparison

In this part, a comparison between the computational complexity of different detectors is performed. The complexity of various detectors in terms of number of complex multiplications are listed in Table 2. Proposed scheme, compared to SUD has no additional complexity. The complexity of the proposed method with PIC is equal to that for CLPIC. The required number of the complex multiplications for each subcarrier is $(M - 1) \times Q$, where $M$ is the number of users and $Q$ is the number of assigned subcarriers to each user, $Q = N/M$. Thus the total complexity for each iteration is $N \times Q \times (M - 1) = (N^2 - N^2/M)$.

The complexity of the proposed method with PIC can be reduced. According to (21) and (26), the MAI from the $u$th subcarrier on the $k$th subcarrier is proportional to their distance. When $|k - u|$ increases, the denominator of (21) increases and $\rho_{m,n}^{k,u}$ decreases. If $|k - u|$ is sufficiently large, MAI can be neglected in PIC algorithm. Complexity reduction parameter, $P$, is a threshold for distance between subcarriers. Clearly the number of complex multiplications for each subcarrier decreases to $2P$ and the total number of complex multiplications for each iteration is $2NP$.

Simulation reveals that the performance of the proposed PIC scheme does not decrease significantly by choosing $P = 10$ (see the next section for more details). Additionally, to achieve an acceptable performance, the proposed scheme requires a lower number of iterations compared to CLPIC and HLCC schemes. For a more clear insight, complexity of different detectors have been calculated in Table 3. As shown in this table the proposed scheme provides a CFO compensation technique with lower complexity than that of CLPIC and HLCC schemes.

**Table 2**
Complexity comparison.

| Detector | Complexity (Number of complex multiplication) |
|---|---|
| SUD [4] | $\frac{MN}{2}\log N - \left[\frac{MN}{2}\log M - \frac{3}{2}(M-1)N\right]$ |
| CLJL [5] | $\frac{N^2}{M} + \frac{N}{2}\log N$ |
| HLCC [7] | $\frac{N^2}{M} + \frac{N}{2}\log N + (s-1)\left[N^2 + \frac{N^2}{M}\right]$ |
| CLPIC [6] | $\frac{MN}{2}\log N - \left[\frac{MN}{2}\log M - \frac{3}{2}(M-1)N\right] + (s-1)\left(N^2 - \frac{N^2}{M}\right)$ |
| Proposed scheme | $\frac{MN}{2}\log N - \left[\frac{MN}{2}\log M - \frac{3}{2}(M-1)N\right]$ |
| Proposed scheme + PIC | $\frac{MN}{2}\log N - \left[\frac{MN}{2}\log M - \frac{3}{2}(M-1)N\right] + (s-1)\left(N^2 - \frac{N^2}{M}\right)$ |
| Proposed scheme + low complexity PIC | $\frac{MN}{2}\log N - \left[\frac{MN}{2}\log M - \frac{3}{2}(M-1)N\right] + 2(s-1)NP$ |

$N$: number of subcarriers, $M$: number of subscribers, $s$: stage index, $P$: complexity reduction parameter.

**Table 3**
Numerical complexity comparison.

| Detector | $N = 64$ | $N = 256$ | $N = 1024$ |
|---|---|---|---|
| HLCC 2 iter. | 11,456 | 181,248 | 2,888,704 |
| CLPIC 2 iter. | 6944 | 102,528 | 1,593,856 |
| Proposed scheme with PIC 1 iter. | 3872 | 53,367 | 807, 424 |
| Proposed scheme with PIC, $P = 10$ | 2080 | 9344 | 41, 472 |
| Proposed scheme with PIC, $P = 5$ | 1440 | 6784 | 31, 232 |

$N$: number of subcarriers.

It is further noted that complexity reduction techniques similar to mentioned above can be done for CLPIC and HLCC schemes. HLCC reduces its complexity in two ways. first, HLCC reduces complexity as well as CLJL [5]. Secondly, HLCC uses a modified vector for circular convolution as reported in [7]. In CLPIC, complexity may be reduced by way of ignoring weak subcarriers or other user subcarriers far-off from desired user's subcarriers as well [6]. However as shown in our simulation (See Section 5 for more details), the performance of CLPIC is worse in comparison with HLCC and the proposed method and then complexity reduction technique impairs its performance to some extend.

## 5. Simulation results

Computer simulations compare the performance of different CFO compensation techniques. Consider an interleaved OFDMA uplink system with $N = 64$ subcarrier, quadrature phase shift keying (QPSK) modulation and $M = 4$ active users that communicate with BS through a multipath Rayleigh channel that has exponential power delay profile and maximum delay spread is $L = 4$. For each user, the channel coding scheme is a rate $-1/2$ convolution code with constraint length 5. Guard period is $N_g = 8 > L$.

### 5.1. SIR

Figs. 3 and 4 illustrates SIR on each subcarrier for different compensation schemes. Compensation techniques are compared for two sets of CFO values $\mathbf{CFO}_1 = [0.22, 0.27, 0.19, 0.15]$, $\mathbf{CFO}_2 = [-0.1, 0.3, 0.25, -0.15]$. The first set indicates large individual CFO values with small CFO differences, where the second set presents small individual CFO values with large CFO difference. As shown in Figs. 3 and 4, the proposed scheme shows considerable improvement over SUD and CLJL in both cases. Adding the PIC algorithm

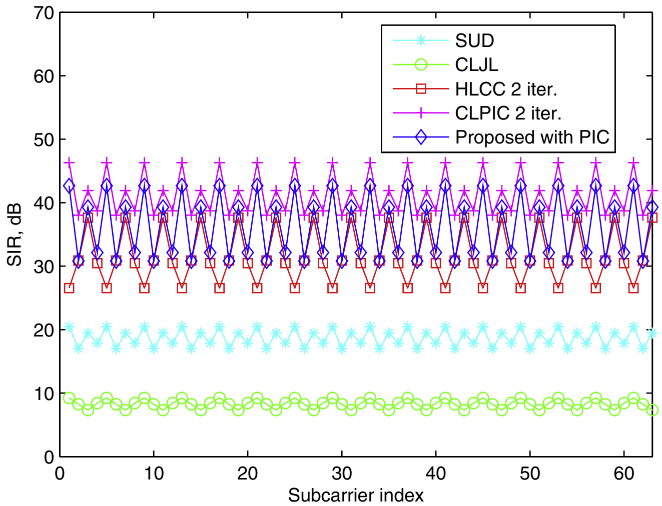


**Fig. 3.** Comparison of the SIR performance of different CFO compensation schemes with $\mathbf{CFO}_1 = [0.22, 0.27, 0.19, 0.15]$.

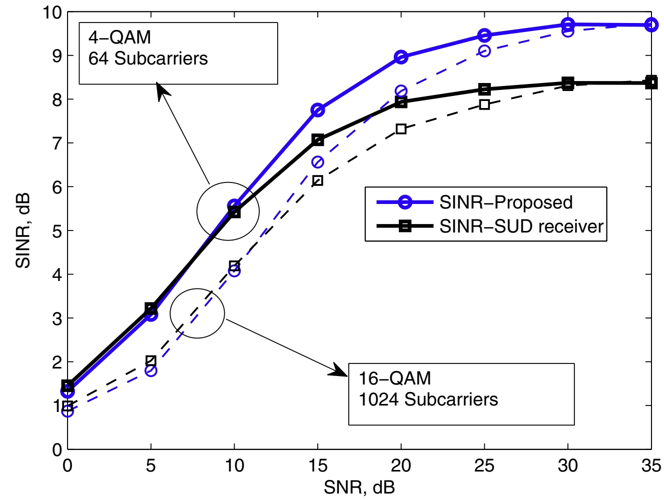


**Fig. 5.** Comparison of the SINR performance of different CFO compensation schemes with $\mathbf{CFO}_2 = [-0.1, 0.3, 0.25, -0.15]$.

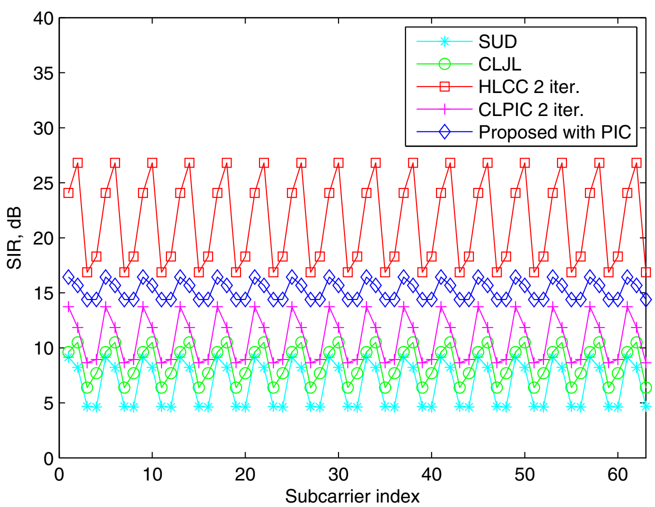


**Fig. 4.** Comparison of the SIR performance of different CFO compensation schemes with $\mathbf{CFO}_2 = [-0.1, 0.3, 0.25, -0.15]$.

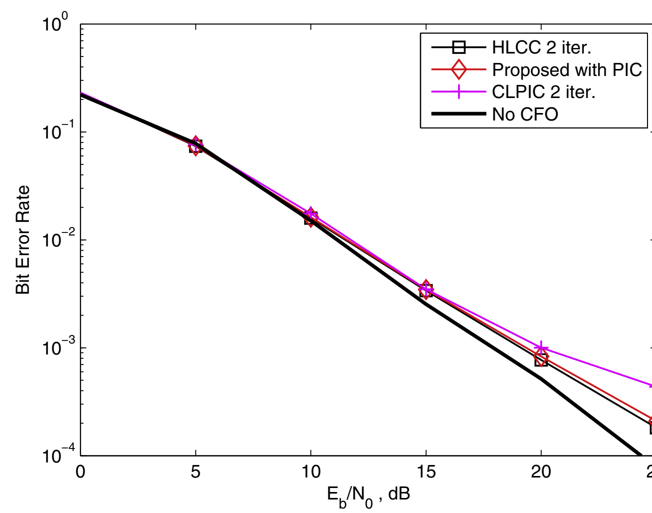


**Fig. 6.** Comparison of the BER performance of different CFO compensation techniques for CFO values uniformly distributed between $[-0.1, 0.1]$, QPSK.

to the proposed scheme achieves significant improvement in term of SIR. Also, for $\mathbf{CFO}_1$ the CLPIC scheme has the best performance [6]. Clearly, the proposed method with only one stage PIC algorithm has higher SIR than HLCC with two stages and also it is very close to CLPIC with two stages. As shown in Fig. 4, HLCC scheme has the best performance. Also the proposed method with one stage PIC shows acceptable performance to HLCC with two iterations. Totally, the proposed method provides acceptable SIR with lower complexity than that of CLPIC and HLCC as shown in Table 3.

### 5.2. SINR

To verify (32), Fig. 5 presents SINR versus SNR in the presence of CFO vector $\mathbf{CFO}_2 = [-0.1, 0.3, 0.25, -0.15]$ and AWGN for two different setting for the second user. As shown in this figure, the proposed method outperforms SUD-based method for $\mathrm{SNR}_t \approx 10$ dB.

### 5.3. BER performance

The bit error rate (BER) performance of various methods are illustrated in Figs. 6 and 7. The CFO values considered to be random values uniformly distributed in intervals $[-0.1, 0.1]$. This intervals is 5 times greater than the accepted range for the CFO values at the output of the synchronization stage of IEEE802.16 standard. As shown in Fig. 6, the proposed scheme with PIC is close to the performance of HLCC and CLPIC schemes. But this performance is obtained by only one PIC stage, so the complexity of the proposed scheme is almost two times less than that of HLCC and CLPIC. The performance of the proposed scheme with different values of complexity reduction parameter $P$ is presented in Fig. 7. As shown in this figure, the performance of the proposed method with $P = 5$, 10 are approximately equal to HLCC at SNR = 20 dB with a much less complexity. Also we have provided BER curve for higher order modulation 16-QAM, and larger number of subcarriers 1024 in Fig. 8. As shown in this figure, the proposed method has reasonable performance in comparison with two other methods.

### 5.4. BER performance without power control

In a wireless OFDMA system, users are located on different distances from the BS. Thus, to equalize the power of their received signals at the BS, BS uses power control technique. Perfect power

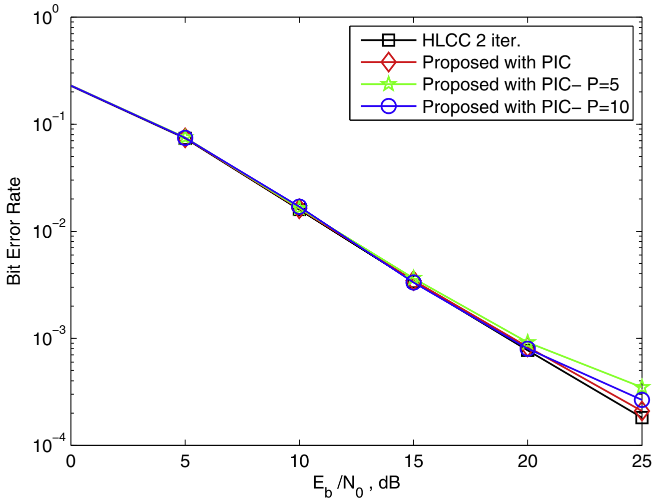


**Fig. 7.** Effect of complexity reduction on the performance of proposed scheme for CFO values uniformly distributed between [−0.1, 0.1], QPSK.

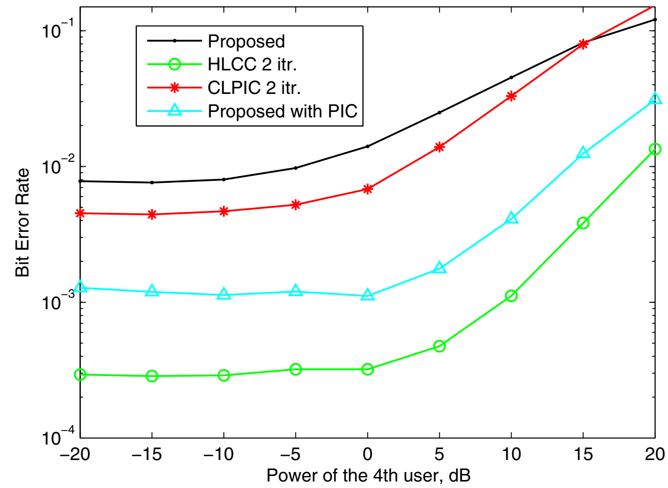


**Fig. 9.** Comparison of the BER performance of the different CFO compensation schemes without power control for CFO values uniformly distributed between [−0.25, 0.25] and SNR = 25 dB.

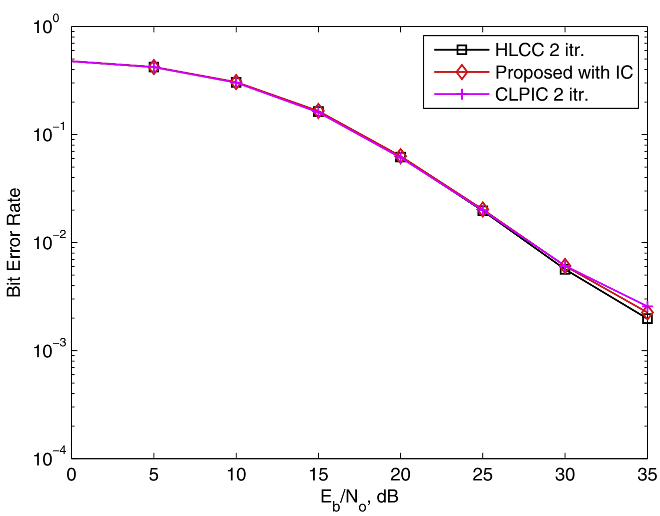


**Fig. 8.** Comparison of the BER performance of different CFO compensation techniques for CFO values uniformly distributed between [−0.1, 0.1], 16-QAM.

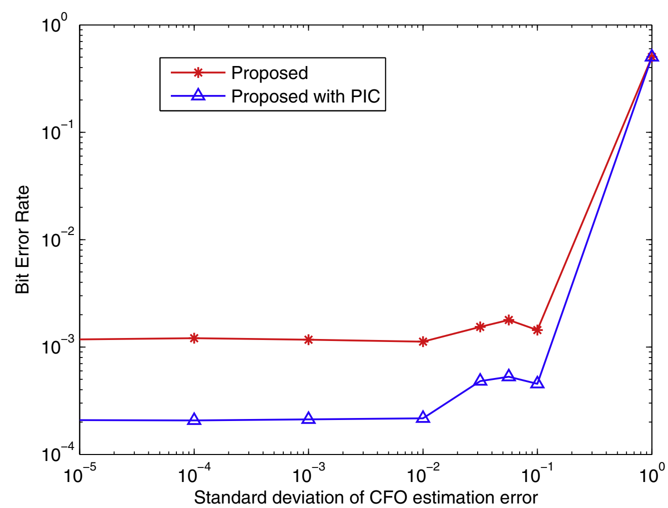


**Fig. 10.** BER performance of the proposed scheme with CFO estimation error for CFO values uniformly distributed between [−0.1, 0.1] and SNR = 25 dB.

control requires high computational complexity [8]. However the performance of all CFO compensation techniques will be examined in the presence of imperfect power control. Thus the power of three users assume to be fixed to the same level and power of the 4th user is changed from −20 dB to 20 dB. To study the effect of imperfect power control, the average BER of three users are depicted versus power of the 4th user in different detectors in Fig. 9. The CFO values are random and uniformly distributed in [−0.25, 0.25]. As seen from Fig. 9, the proposed method with and without PIC show good robustness against imperfect power control effect.

### *5.5. BER performance with estimation error*

Before any compensation, CFO values are estimated. Fig. 10 shows the BER performance of the proposed scheme with CFO estimation error. It can be seen that the performance of the proposed scheme is robust to CFO error until the standard deviation is increased to 0.1. This requirement is provided by estimators [1].

## 6. Conclusion

In this paper, a low complexity CFO compensation scheme was presented for the interleaved OFDMA uplink. The proposed method works for small CFO interval, e.g. [−0.1, 0.1]. The CFO for each user was compensated by the average value of CFOs from two adjacent users. Also we proposed a new PIC algorithm to eliminate MAI. Simulation results presented an acceptable performance of the proposed scheme over conventional schemes with lower complexity.

## Appendix A. CFO compensation value calculation

In the interleaved allocation, the SIR on the $m$th subcarrier after CFO correction by $\varepsilon_c$ may be approximated by

$$\bar{\mathrm{SIR}}_{\varepsilon_c} \approx \frac{\dfrac{\sin^2(\pi(\varepsilon_m-\varepsilon_c))}{M^2\sin^2\left(\frac{\pi(\varepsilon_m-\varepsilon_c)}{M}\right)}}{\dfrac{\sin^2(\pi(-1+\varepsilon_{m-1}-\varepsilon_c))}{M^2\sin^2\left(\frac{\pi(-1+\varepsilon_{m-1}-\varepsilon_c)}{M}\right)}+\dfrac{\sin^2(\pi(1+\varepsilon_{m+1}-\varepsilon_c))}{M^2\sin^2\left(\frac{\pi(1+\varepsilon_{m+1}-\varepsilon_c)}{M}\right)}}, \tag{A.1}$$

where the dominant interference comes from two adjacent subcarriers $m+1$ and $m-1$. By assuming $|\varepsilon_i - \varepsilon_c| \le 0.2, \forall i \in \{0, 1, \ldots, M-1\}$, and replacing of each sinusoidal function by Taylor extension, we have

$$\sin(\pi(\pm 1 + \varepsilon_i - \varepsilon_c)) = \pm[\underbrace{(0.2\times\pi)}_{0.63} - \underbrace{\frac{1}{6}(0.2\times\pi)^3}_{0.04} + \underbrace{\frac{1}{120}(0.2\times\pi)^5}_{8.4\times10^{-4}} - \ldots]. \tag{A.2}$$

The second and the third terms in (A.2) are very small and may be neglected. Therefore (A.1) will be simplified to

$$\bar{\mathrm{SIR}}_{\varepsilon_c} \approx \frac{1}{\frac{(\varepsilon_{m-1}-\varepsilon_c)^2}{(-1+\varepsilon_{m-1}-\varepsilon_c)^2} + \frac{(\varepsilon_{m+1}-\varepsilon_c)^2}{(1+\varepsilon_{m+1}-\varepsilon_c)^2}}. \tag{A.3}$$

To Maximize (A.3) according to $\varepsilon_c$, we have

$$\frac{(\varepsilon_c-\varepsilon_{m-1})}{(\varepsilon_c-\varepsilon_{m-1}+1)^2} + \frac{(\varepsilon_c-\varepsilon_{m-1})^2}{(\varepsilon_c-\varepsilon_{m-1}+1)^3} + \frac{(\varepsilon_c-\varepsilon_{m+1})}{(\varepsilon_c-\varepsilon_{m+1}+1)^2} + \frac{(\varepsilon_c-\varepsilon_{m+1})^2}{(\varepsilon_c-\varepsilon_{m+1}+1)^3} = 0. \tag{A.4}$$

After solving this equation and some manipulations, the $m$th user can be compensated by

$$\varepsilon'_m \approx \frac{\varepsilon_{m-1}+\varepsilon_{m+1}}{2}. \tag{A.5}$$

The correction value for each user is the average of CFO for two adjacent users approximately.

## Appendix B. SIR in PIC algorithm

In this appendix the SIR on each subcarrier after the PIC algorithm is calculated analytically. By calling (16) and defining $Y_m(k) \triangleq X_m(k)C_m(k)$, according to (13) and (19), we have

$$R_m(k) = Y_m(k)\frac{1}{\xi_m} + \sum_{\substack{u\in\mathcal{I}_m\\u\neq k}} Y_m(u)\rho_{k,u}^{m,m} + \sum_{\substack{n=1\\n\neq m}}^{M}\sum_{u\in\mathcal{I}_n} Y_n(u)\rho_{m,n}^{k,u} + W^{(1)}(k), \tag{B.1}$$

the MAI on the $k$th subcarrier of the $m$th user is

$$\mathrm{MAI}_m^{(2)}(k) = \sum_{\substack{n=1\\n\neq m}}^{M}\sum_{u\in\mathcal{I}_n} Y_n(u)\rho_{m,n}^{k,u}, \tag{B.2}$$

Clearly, we have no access to the $Y_n(u), n=1,\ldots,N, u\in\mathcal{I}_n$ and we must use its estimated value that is $\hat{Y}_n^{(1)}(u) = R_n(u)$. $R_n(u)$ depends on MAI so the calculated MAI is not accurate. Subsequently, the interference cancellation cannot be done perfectly and it results in new interference on the signal. To find desired signal power, MAI and ICI after interference cancellation we have

$$\hat{Y}_m^{(2)}(k) = Y_m(k)\frac{1}{\xi_m} + \sum_{\substack{u\in\mathcal{I}_m\\u\neq k}} Y_m(u)\rho_{m,m}^{k,u} - \sum_{\substack{n=1\\n\neq m}}^{M}\sum_{u\in\mathcal{I}_n}\xi_n\rho_{m,n}^{k,u}\sum_{\substack{v\in\mathcal{I}_n\\v\neq u}} Y_n(v)\rho_{n,n}^{u,v} - \sum_{\substack{l=1\\l\neq m}}^{M}\sum_{v\in\mathcal{I}_l}\xi_l\rho_{m,l}^{k,v}\sum_{\substack{n=1\\n\neq l,m}}^{M}\sum_{u\in\mathcal{I}_n} Y_n(u)\rho_{l,n}^{v,u} + W^{(2)}(k), \tag{B.3}$$

where $R_m^{(2)}(k)$ is the received symbol on the $k$th subcarrier after interference cancellation. Now the desired signal is the summation of all terms that involve $Y_m(k)$

$$S_m^{(2)}(k) = \frac{1}{\xi_m}Y_m(k) - \sum_{\substack{n=1\\n\neq m}}^{M}\sum_{u\in\mathcal{I}_n}\xi_n\rho_{m,n}^{k,u}Y_m(k)\rho_{n,m}^{u,v} = Y_m(k)\left(\frac{1}{\xi_m} - \sum_{\substack{n=1\\n\neq m}}^{M}\xi_n\sum_{u\in\mathcal{I}_n}\rho_{m,n}^{k,u}\rho_{n,m}^{u,k}\right) \tag{B.4}$$

The ICI is the summation of all terms involve $Y_m(u), u\neq k$

$$\mathrm{ICI}_m^{(2)}(k) = \sum_{\substack{u\in\mathcal{I}_m\\u\neq k}} Y_m(u)\rho_{m,m}^{k,u} - \sum_{\substack{n=1\\n\neq m}}^{M}\xi_n\sum_{v\in\mathcal{I}_n}\rho_{k,v}^{m,n}\sum_{\substack{u\in\mathcal{I}_m\\u\neq k}} Y_m(u)\rho_{n,m}^{v,u} = \sum_{\substack{u\in\mathcal{I}_m\\u\neq k}}\left(Y_m(u)(\rho_{m,m}^{k,u} - \sum_{\substack{n=1\\n\neq m}}^{M}\xi_n\sum_{v\in\mathcal{I}_n}\rho_{n,m}^{v,u}\rho_{m,n}^{k,v})\right), \tag{B.5}$$

and MAI can be written as the summation of remaining terms that involve $Y_n(u), n\neq m$

$$\mathrm{MAI}_m^{(2)}(k) = -\sum_{\substack{l=1\\l\neq m}}^{M}\sum_{v\in\mathcal{I}_l}\xi_l\rho_{m,l}^{k,v}\sum_{\substack{n=1\\n\neq l,m}}^{M}\sum_{u\in\mathcal{I}_n} Y_n(u)\rho_{l,n}^{v,u} = -\sum_{\substack{n=1\\n\neq m}}^{M}\sum_{u\in\mathcal{I}_n} Y_n(u)\sum_{\substack{l=1\\l\neq n,m}}^{M}\xi_l\sum_{v\in\mathcal{I}_l}\rho_{l,n}^{v,u}\rho_{m,l}^{k,v}. \tag{B.6}$$

Finally SIR can be obtained easily.

## Appendix C. AWGN effect on the performance

Clearly, for $\mathrm{SINR}_{\mathrm{SUD}}(m,k) < \mathrm{SINR}_p(m,k), m = 1,\ldots,M, \quad k \in \mathcal{I}_m$, the proposed algorithm will outperform SUD scheme. This condition can be written as

$$\frac{\mathrm{SP}_{\mathrm{SUD}}(m,k)}{\mathrm{IP}_{\mathrm{SUD}}(m,k)+\mathrm{MP}_{\mathrm{SUD}}(m,k)+W_0} < \frac{\mathrm{SP}_p(m,k)}{\mathrm{IP}_p(m,k)+\mathrm{MP}_p(m,k)+W_0}, \tag{C.1}$$

where $W_0$ is the noise power before CFO compensation. In the following, we assume $\hat{\varepsilon}_m = \varepsilon_m, \quad m = 1,\ldots,M$. Accordingly, by calling (11), we have

$$\mathrm{SP}_{\mathrm{SUD}}(m,k) = \xi_m^2 \mathrm{SP}_p(m,k) = 1. \tag{C.2}$$

Thus the condition (C.1) is

$$\frac{1}{\mathrm{IP}_{\mathrm{SUD}}(m,k)+\mathrm{MP}_{\mathrm{SUD}}(m,k)+W_0} < \frac{\frac{1}{\xi_m}}{\mathrm{IP}_p(m,k)+\mathrm{MP}_p(m,k)+W_0}, \tag{C.3}$$

After manipulation we have

$$\frac{1}{\bar{\mathrm{SIR}}_{\mathrm{SUD}}(m,k)} - \frac{1}{\bar{\mathrm{SIR}}_p(m,k)} > \frac{\xi_m^2-1}{\bar{\mathrm{SNR}}_0}, \tag{C.4}$$

where $\mathrm{SNR}_0$ is the SNR before DFT block. Finally, the proposed scheme outperforms SUD when

$$\mathrm{SNR}_0 > \mathrm{SNR}_t \triangleq \left\{ \frac{\xi_m^2-1}{\frac{1}{\bar{\mathrm{SIR}}_{\mathrm{SUD}}(m,k)} - \frac{1}{\bar{\mathrm{SIR}}_p(m,k)}} \right\}. \tag{C.5}$$

## References


[1] Morelli BM, Kuo CCJ, Pun MN. Synchronization techniques for orthogonal frequency division multiple access (OFDMA): a tutorial review. Proc of the IEEE 2007;95(July(7)):1394–427.

[2] Moose PH. A technique for orthogonal frequency-division multiplexing frequency offset correction. IEEE Trans Commun 1994;42(October (10)):2908–14.

[3] Song SH, Chen GL, Letaief KB. Localized or interleaved? A tradeoff between diversity and CFO interference in multipath channels. IEEE Trans Wireless Commun 2011;10(September(9)):2829–34.

[4] Tonello A, Pupolin S. Performance of single user detectors in multitone multiple access asynchronous communications. In: In: IEEE vehicular technology conference 2002(VTC 2002). 2002 May. p. 199–203.

[5] Choi J, Lee C, Jung HW, Lee YH. Carrier frequency offset compensation for uplink of OFDM-FDMA systems. IEEE commun Lett 2000;4(December (12)):414–6.

[6] Manohar S, Tikiya V, Sreedhar D, Chockalingam A. Cancellation of multiuser interference due to carrier frequency offsets in uplink OFDMA. IEEE Trans Wireless Commun 2007;6(July(7)):2560–71.

[7] Huang D, Letaief KB. An interference cancellation scheme for carrier frequency offsets correction in OFDMA systems. IEEE Trans Commun 2005;53(July(7)):1155–65.

[8] Detti P, Nicosia G, Pacifici A, Servilio M. Optimal power control in OFDMA cellular networks. Seventh Cologne-Twente workshop 2011;57(3):200–11.

[9] Cao Z, Tureli U, Yao YD. Low-complexity orthogonal spectral signal construction for generalized OFDMA uplink with frequency synchronization errors. IEEE Trans Veh Technol 2007;56(May(3)):1143–54.

[10] Fa R, Zhang L. A generalized subcarrier-grouped MMSE based multi-stage interference cancellation scheme for OFDMA uplink systems with CFOs. In: IEEE international symposium on wireless communication systems 2010 (ISWCS 2010). 2010 April. p. 912–6.

[11] Cao Z, Tureli U, Yao YD, Honan P. Frequency sinchronization for generalized OFDMA uplink. In: IEEE global telecommunications conference 2004 (GLOBECOM 2004). 2004 December. p. 1071–5.

[12] Lee K, Lee SR, Moon SH, Lee I. MMSE-Based CFO compensation for uplink OFDMA systems with conjugate gradient. IEEE Trans Wireless Commun 2012;11(August(8)):2767–75.

[13] Farhang A, Marchetti N, Doyle L. Low complexity LS and MMSE based CFO compensation techniques for the uplink of OFDMA systems. In: In: IEEE international conference on communications 2013 (ICC 2013). 2013 June. p. 5748–53.

[14] Farhang A, Marchetti N, Doyle L, Farhang-Boroujeny B. Low complexity CFO compensation in uplink OFDMA systems with receiver windowing. IEEE Trans Signal Process 2015;63(April(10)):2546–58.

[15] Al-kamali FS, Dessouky MI, Sallam BM, Shawki F, Abd El-Samie FE. Equalization and carrier frequency offsets compensation for the SC-FDMA system. Springer Wireless Personal Commun 2012;67(November(2)):113–38.

[16] Hashemizadeh SK, Saeedi-Sourck H, Omidi MJ. Sensitivity analysis of interleaved OFDMA system uplink to carrier frequency offset. In: IEEE personal, indoor and mobile radio communications 2011 (PIMRC 2011). 2011 November. p. 1631–5.

[17] Mohammadi-Siahboomi J, Omidi MJ, Saeedi-Sourck H. A low-complexity CFO compensation technique for interleaved OFDMA system uplink. In: In: IEEE Sixth international symposium on telecommunication 2012(IST 2012). 2012 June. p. 221–5.