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RESEARCH ARTICLE

A novel low complexity energy-efficient resource allocation for OFDM systems

Jafar Kazemi[1]*, Mohammad Javad Omidi[1] and Keivan Navaie[2]

[1] Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran 84156-83111
[2] School of Computing and Communications, Lancaster University, Lancaster, UK, LA1 4WY

ABSTRACT

In this paper, we study energy efficiency of an orthogonal frequency division multiplexing based system under power constraint and minimum rate requirement with system circuit power consideration. The optimal radio resource allocation is then formulated as an energy-efficient maximisation where we propose a novel lowcomplexity solution for obtaining the optimal solution. The solution consists of the optimal source transmission power and its distribution among subcarriers. In spite of the iterative solution with high computational complexity in previous works, we propose a quickly convergent low complexity scheme based on solving a nonlinear logarithmic equation. To evaluate the accuracy of the proposed method, we compared its accuracy through simulations with the optimal solution in systems with and without circuit power consideration. Simulation studies indicate that the proposed method provide accurate solutions in both low and high signal to noise ratio regimes. The simulation results also indicate using our proposed method results in a significant improvement in the computational complexity. Copyright © 2015 John Wiley & Sons, Ltd.

***Correspondence**
J. Kazemi, Electrical and Computer Engineering Department, Isfahan University of Technology, Isfahan, Iran 84156-83111.
E-mail: j.kazemi@ec.iut.ac.ir

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1. INTRODUCTION

As the share of communication systems in the world's energy consumption increases, the energy efficient (EE) system design becomes an important issue and gains a lot of attention in both industry and academia. In industrial areas, both vendors and operators are expecting higher energy-saving architectures and techniques to reduce the energy consumption and thus the total cost of operation. In the recent years, several research projects, such as Energy Aware Radio and Network Technologies, Optimising Power Efficiency in Mobile Radio Networks and Green Radio, have been introduced to develop energy-efficient wireless communication systems [1].

Conventionally, EE was defined as 'information bits per unit of transmit energy', however, practical concerns cause to take into account circuit energy consumption into the energy consumption model and redefine the EE metric as 'information bits per unit of consumed energy (not only transmit energy but also circuit power energy)' where an additional circuit power (CP) factor needs to be considered [2, 3]. It is shown in [4] that the transceiver power consumed in 802.11x in idle mode is comparable with the transmit mode. This example along with other researches suggests that the circuit energy consumption is not always ignorable compared with the transmit power (e.g. [5–14]). A complete circuit model has been considered in [15]. In [16], authors analysed the best modulation strategy to minimise the total energy needed to transmit a given number of bits. Their power consumption model (PCM) involves transmit power besides constant CP of each node. They provide a clear and thorough explanation to justify the modelling of the CP of the transmitter and receiver as a constant factor in PCM.

Orthogonal Frequency Division Multiplexing (OFDM) has been adopted as a promising transmission technique for broadband wireless networks [17, 18]. It has already been used in wire line communications such as Asynchronous Digital Subscriber Line (ADSL) technology, and wireless communications such as Digital Video Broadcasting (DVB), 3GPP Long Term Evolution (LTE), IEEE 802.1x series (WiFi, WiMAX, …).

Energy efficient OFDM systems considering circuit energy consumption for frequency-selective fading channels, which maximises the energy efficiency (i.e., bits-per-Joule) has been first studied in [11]. In contrast to the conventional trend in researches that maximises throughput under a fixed overall transmit power constraint [19–21], the new scheme in [11] maximises the overall EE by adjusting both the total transmission power and its distribution among subcarriers. Energy efficient OFDM systems have to answer two following questions considering the system constraints in an optimisation problem:

(1) What is the energy-efficient source transmission power?
(2) How can one distribute source transmission power among subcarriers to maximise the end to end energy efficiency?

Authors in [12] developed a model based on non-cooperative games for energy-efficient power optimisation at interference limited communications with OFDM modulation. In [13], power allocation algorithms for energy-efficient multicarrier systems were addressed assuming static CP consumption. Authors in [14] studied energy-efficient subcarrier and power allocation in both downlink and uplink OFDMA network.

Based on strictly quasi-concavity of such optimisation formulation [8–10], there is always a unique global transmit power that maximises energy efficiency [22]. Authors in [9, 10] then decomposed this problem into two layers and solved iteratively by the joint inner-layer and outer-layer optimisation (JIOO) as follows:

(i) Inner layer: distribute nominated transmission power among subcarriers to maximise the energy efficiency.
(ii) Outer layer: using the bisection power search methods such as gradient descend to nominate the next transmission power.

The JIOO is an optimal solution consisting of two iterative algorithms where each algorithm uses the results of the other iteratively to nominate transmission power that maximises energy efficiency. High computational complexity is the main issue in the JIOO-based strategies. In a way that the total computational complexity of the JIOO can be obtained as the product of the corresponding computational complexity of the inner-layer and outer-layer optimisation problems [9, 10]. Authors in [9, 10] proposed a suboptimal scheme to reduce the total computational complexity.

The main objective in this paper is proposing a low complexity scheme to find the solutions in comparison to the JIOO in the problem of energy efficient resource allocation in an OFDM system. To achieve this optimal and low complexity solution, we first formulate the problem as an energy-efficient maximisation by considering the minimum rate requirement and maximum transmission power constraint. We then propose a scheme with very low computational complexity to obtain the solutions.

The main contribution of this paper is presenting a logarithmic equation through which we jointly obtain the optimal transmission power and its distribution among subcarriers in an OFDM system in a frequency selective fading environment. We then compare the complexity of the proposed scheme in this paper with the optimal methods proposed in previous works. We further show that the computational complexity of the proposed method is significantly less than the computational complexity of the joint inner and outer solution.

The rest of this paper is organised as follows. In Section 2, the system model is defined and end to end energy efficiency optimisation problem of an OFDM system is presented. Joint determination of the optimal transmission power and subcarrier power allocation based on a low complexity algorithm is also presented in Section 3. The computational complexity comparison between the proposed and the optimal solution is also presented in Section 3. Using the expressions derived in Section 3 , a fast converging algorithm is proposed to solve energy-efficient optimisation problem. Finally, numerical results and conclusion are presented in Sections 5 and 6.

2. SYSTEM MODEL AND PROBLEM FORMULATION

The considered model in this paper consists of a source (S) and a destination (D). A source transmits to a destination using OFDM modulation scheme, through a wireless frequency-selective fading channel. The total channel bandwidth (B) is divided into N subcarriers. Let us denote $h_{sd,n}$ as the source-destination channel gains for the nth subcarrier. Moreover, σ_d^2 indicates the noise variance at the destination receiver. We assume the source transmits data with power $P_{S,n}$ on the nth subcarrier. The signal to noise ratio (SNR), ρ_n, for subcarrier n at the destination is [23]

$$\rho_n = P_{S,n} a_n \tag{1}$$

where $a_n = \frac{|h_{sd,n}|^2}{\sigma_d^2}$.

We further assume that the source is provided with the perfect channel state information, also noise variance of the destination links, i.e. a_n for all n. Further, P_S is the source transmission power distributed among subcarriers:

$$P_S = \sum_{n=1}^{N} P_{S,n}, \quad P_S \leqslant P_{S,\max} \tag{2}$$

where $P_{S,\max}$ represents the maximum allowable total transmission power at source.

In addition to the required energy for transmission, the energy consumption also includes the circuit energy consumption incurred by active circuit blocks [15]. The overall consumption power similar to [15] and [14] at the source, $P_{S,tot}$, is given as

$$P_{S,tot} = \zeta_S P_S + P_{C,S} \tag{3}$$

where ζ_S is the reciprocal of drain efficiency of power amplifiers in the source and $P_{C,S}$ represents the CP in the source. Let T and $C_{I,n}$ denote the time duration of each timeslot and capacity in the nth subcarrier, respectively. Then, the throughput is $T\sum_{n=1}^{N} C_{I,n}$ and the overall consumed energy for an OFDM is $T\left(P_{C,S}+\sum_{n=1}^{N}\zeta_S P_{S,n}\right)$. We further define the energy efficiency similar to [14] as the ratio of throughput to the total consumed energy:

$$\eta_{EE} @ \frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S}+\sum_{n=1}^{N}\zeta_S P_{S,n}} \tag{4}$$

where,

$$C_{I,n}=\frac{B}{N}\log_2(1+P_{S,n}a_n) \tag{5}$$

Here, the objective is to determine the energy-efficient source transmission power, P_S^*, subject to the maximum total transmission power, $P_S^* \leqslant P_{S,\max}$, as well as the subcarrier power allocation at the source transmitter, $P_{S,n}$, which offers the maximum η_{EE} subject to the minimum rate requirement $\left(C_I=\sum_{i=1}^{N} C_{I,n} \geqslant R_{\min}\right)$. This problem is formulated as follows:

$$\begin{aligned}&\max \eta_{EE},\\ &\text{s.t. } C_I \geqslant R_{\min},\\ &P_S=\sum_{n=1}^{N} P_{S,n} \leqslant P_{S,\max}, \quad P_{S,n}\geqslant 0\end{aligned} \tag{6}$$

3. LOW COMPLEXITY ENERGY-EFFICIENT RESOURCE ALLOCATION

Generally, determining the optimal source transmission power and subcarrier power allocation with the objective of maximising η_{EE} subject to the minimum rate requirement and maximum transmission power is associated with high computational complexity [14]. Furthermore, the source has to rapidly compute the energy-efficient source transmission power and subcarrier power allocation as the wireless channel changes. Hence, low complexity solution is preferred for cost-effective and delay sensitive implementations. To reduce complexity, we decompose this problem into the following two sub problems:

(i) How can one distribute total transmission power among subcarriers to maximise energy efficiency?
(ii) What is the energy-efficient total transmission power with respect to maximum transmission power and minimum rate requirement?

Conventionally, iterative algorithms are used to obtain the solutions based on a joint inner-layer and outer-layer optimisation (JIOO) [5, 14, 24]. The JIOO scheme consisting of two iterative algorithms where each algorithm uses the results of the other iteratively to nominate transmission power that maximises energy efficiency. The first aforementioned question that decides on the distribution of the nominated transmission power (nominated by outer layer) among subcarriers to maximise the total energy efficiency is answered in the inner layer. The second aforementioned question is answered in the outer layer, in which a bisection power search methods, such as gradient descend, are used to nominate the next transmission power.

In this paper, we derive a logarithmic equation that answers, in one shot, the first and second aforementioned questions. Our heuristic solution to derive a logarithmic equation is described in Subsections 3.1, and 3.2. Minimum transmission power that guarantees minimum rate requirement and the comparison between computational complexity of the optimal and our proposed solution are presented in Subsections 3.3 and 3.4, respectively.

The key idea in Subsections 3.1 and 3.2 is sorting the channel gain of all subcarriers in the decreasing order and then achieve the relation between the transmission power of each subcarrier and the same in the first subcarrier (the first subcarrier is the one with the best channel gain). Having the first subcarrier power allocation, this provides power distribution for all subcarrier, and hence, energy-efficient transmission power.

3.1. Subcarrier power allocation to maximise energy efficiency

To determine the subcarrier power allocation, the optimisation problem is formulated as follows:

$$\begin{aligned}&\min -\frac{\sum_{n=1}^{N}\log_2(1+P_{S,n}a_n)}{P_{C,S}+\left(\sum_{n=1}^{N}\zeta_S P_{S,n}\right)},\\ &\text{s.t}\\ &\sum_{n=1}^{N} P_{S,n} \leqslant P_{S,\max},\\ &C_I \geqslant R_{\min}\end{aligned} \tag{7}$$

where $P_{S,\max}$ and $R_{\min}$ are the maximum total transmission power and minimum rate requirement, respectively.

$$L(\mathbf{P}_S,\lambda_1,\lambda_2)=-\frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S}+\left(\zeta_S\sum_{n=1}^{N} P_{S,n}\right)}+\lambda_1(R_{\min}-C_I) + \lambda_2\left(\sum_{n=1}^{N} P_{S,n}-P_{S,\max}\right) \tag{8}$$

where λ_1, λ_2 are Lagrange multipliers. To simplify, we define the total consumed power as follows:

$$M@P_{C,S} + \left(\zeta_S \sum_{n=1}^{N} P_{S,n}\right) \tag{9}$$

As it was mentioned previously, firstly, we sort the channel gain of all subcarriers in the decreasing manner so the first subcarrier is the one with the best channel gain. Using the Karush–Kuhn–Tucker [25] conditions for convex optimisation problem in (7), we have

$$\frac{\partial L(\mathbf{P}_\mathrm{S}, \lambda_1, \lambda_2)}{\partial P_{S,1}} = -\frac{M\frac{\partial C_I}{\partial P_{S,1}} - \zeta_S C_I}{M^2} - \lambda_1 \frac{\partial C_I}{\partial P_{S,1}} + \lambda_2 = 0 \tag{10}$$

and in a similar fashion,

$$\frac{\partial L(\mathbf{P}_\mathrm{S}, \lambda_1, \lambda_2)}{\partial P_{S,n}} = -\frac{M\frac{\partial C_I}{\partial P_{S,n}} - \zeta_S C_I}{M^2} - \lambda_1 \frac{\partial C_I}{\partial P_{S,n}} + \lambda_2 = 0 \tag{11}$$

With comparing (10) and (11), we have

$$P_{S,n} = \max\left(0, P_{S,1} + \frac{1}{a_1} - \frac{1}{a_n}\right) \tag{12}$$

Equation (12), gives the transmission power of the subcarrier based on the transmission power of the first subcarrier. On the other hand, the relation between the total transmission power and the first transmission power is

$$\begin{aligned} P_S = \sum_{n=1}^{N} P_{S,n} &= \sum_{n=1}^{N-m}\left(\frac{1}{a_1} - \frac{1}{a_n} + P_{S,1}\right) \\ &= \frac{N-m}{a_1} + (N-m)P_{S,1} - \sum_{n=1}^{N-m}\frac{1}{a_n} \end{aligned} \tag{13}$$

where m is the number of subcarriers with zero transmission power. We can rewrite (13) as follows:

$$P_{S,1} = \frac{1}{N-m}\left(P_S - \frac{N-m}{a_1} + \sum_{n=1}^{N-m}\frac{1}{a_n}\right) \tag{14}$$

3.2. Nominate energy efficient source transmission power

Within the set provided based on the two constraints in (6), the objective function is a strictly quasi-concave function of P_S . Therefore, there is a unique $\widehat{P}_S$ such that $\frac{\partial \eta_{EE}(\widehat{P}_S)}{\partial P_S} = 0$. To obtain $\hat{P}_S$, we calculate the gradient of $\eta_{EE}(P_S)$ with respect to P_S as follows:

$$\frac{\partial \eta_{EE}}{\partial P_S} = 0 \to M\frac{\partial C_I\left(\hat{P}_S\right)}{\partial P_S} = \zeta_S C_I \tag{15}$$

As it was mentioned previously, we need to derive all variables in (15), including M, $\frac{\partial C_I}{\partial P_S}$ and C_I, based on the first subcarrier transmission power, $P_{S,1}$. In our formulation, the first subcarrier is the one with the highest channel to noise ratio. Finally, we derive the logarithmic equation with one variable, $P_{S,1}$, and using (12) and (13) to calculate each subcarrier transmission power, and hence, energy-efficient total transmission power.

According to (15), we need to calculate $\frac{\partial C_I}{\partial P_S}$ based on the first subcarrier transmission power, $P_{S,1}$. Using the derivation chain rule, we have

$$\frac{\partial C_I}{\partial P_S} = \frac{\partial C_I}{\partial P_{S,1}} \cdot \frac{\partial P_{S,1}}{\partial P_S} \tag{16}$$

According to (12) and (14),

$$\frac{\partial C_I}{\partial P_{S,1}} = \frac{B(N-m)}{N\ln(2)} \cdot \frac{a_1}{1 + P_{S,1}a_1}, \quad \frac{\partial P_{S,1}}{\partial P_S} = \frac{1}{N-m} \tag{17}$$

After substituting (16) in (15) and some straightforward mathematic derivations, we have the following equation:

$$\begin{aligned} &\frac{a_1}{1 + P_{S,1}a_1}\left(P_{C,S} + \zeta_S\left(\frac{N-m}{a_1} + (N-m)\cdot P_{S,1} \right.\right. \\ &\qquad \left.\left. - \sum_{n=1}^{N-m}\frac{1}{a_n}\right)\right) \\ &\quad - \zeta_S\left((N-m)\ln\left(\frac{1}{a_1} + P_{S,1}\right) + \ln\prod_{n=1}^{N-m} a_n\right) = 0 \end{aligned} \tag{18}$$

As it is seen, (18) is a nonlinear logarithmic function of two variables m and $P_{S,1}$. Each variable can be achieved by using the result of the other in an iterative scheme. Using (18) and for an initial value of m, we can obtain $P_{S,1}$ based on numerical methods such as Levenberg-Marquardt, quasi-Newton and Newton-Raphson. Given $P_{S,1}$, then the transmission power of the subcarriers, the total transmission power and the new number of subcarriers with zero transmission power (*mNew*) are obtained through (12) and (13), respectively. If the new number of subcarriers with zero transmission power (*mNew*) in the final step is not equal to the initial value, m, we first replace m with *mNew* and then solve Equation (18) to obtain $P_{S,1}$. This sequence is repeated until the convergence has occurred, i.e. m and *mNew* are equal. We observed in our simulations that usually no more than three iterations are required to converge this algorithm to an optimal m and P_S value so that $\eta_{EE}(P_S)$ is maximised.

Therefore, solving (19) provides us with the optimal transmission power as well as the power allocation to subcarriers as follows:

$$\begin{aligned}\hat{P}_S &= \sum_{n=1}^{N} P_{S,n} = \sum_{n=1}^{N} \max\left(0, \frac{1}{a_1} - \frac{1}{a_n} + P_{S,1}\right) \\ &= \frac{N-m}{a_1} + (N-m)P_{S,1} - \sum_{n=1}^{N-m} \frac{1}{a_n}\end{aligned} \tag{19}$$

where, $\hat{P}_S$ is the energy-efficient source transmission power.

3.3. Minimum source transmission power calculation

One of the constraints in the optimisation problem (7) is the minimum rate requirement ($C_I \geqslant R_{\min}$). In Subsection 3.2, the energy-efficient transmission power $\left(\hat{P}_S\right)$ was calculated, but this source transmission power may not be sufficient to guarantee the minimum rate requirement ($C_I \geqslant R_{\min}$). To calculate minimum transmission power that guarantees minimum rate requirement, the optimisation problem is formulated as follows:

$$\begin{aligned}&\min \sum_{n=1}^{N} P_{S,n}, \\ &\text{s.t} \\ &C_I \geqslant R_{\min}\end{aligned} \tag{20}$$

We start with Lagrangian function as

$$L(\mathbf{P}_\mathrm{S}, \lambda) = \sum_{n=1}^{N} P_{S,n} + \lambda(R_{\min} - C_I) \tag{21}$$

where λ is the Lagrange multiplier. For a fixed λ, the optimal power that minimises the objective function in (20) for subcarrier n can be obtained using Karush–Kuhn–Tucker conditions [25]. Taking the derivative of (21) with respect to $P_{S,n}$, the optimal values of this variable can be obtained as follows:

$$P_{S,n} = \max\left(0, \frac{\lambda}{\ln(2)} - \frac{1}{a_n}\right) \tag{22}$$

The parameter λ is chosen such that the minimum rate requirement ($C_I \geqslant R_{\min}$) is fulfilled.

$$\check{P}_S = \sum_{n=1}^{N} P_{S,n} \tag{23}$$

where $\check{P}_S$ is the minimum required source transmission power to guarantee minimum rate requirement. Finally, P_S^*, as the optimum source transmission power of optimisation problem (7), is calculated as follows:

$$P_S^* = \min\left(P_{S,\max}, \max\left(\check{P}_S, \hat{P}_S\right)\right) \tag{24}$$

3.4. Computational complexity

Traditional solution of the optimisation problem described in (7) consists of an iterative algorithm with the JIOO [5, 14, 24]. In the inner layer of JIOO, the nominated transmission power is distributed among subcarriers to maximise the total throughput, and in the outer layer, a bisection power search method such as gradient descend is used to nominate the next transmission power.

The overall computational complexity of these approaches (based on JIOO) depends on the product of the number of iterations in the inner, N_{IL}, and outer, N_{OL}, layer optimisations [14]. Therefore, the total computational complexity of the JIOO solution is equal to $O\left(N_{OL}N_{IL}\right)$.

According to Mokari *et al.* [26], if we consider δ optimality definition as $R(P_S) - R\left(P_S^*\right) < \delta$ in the inner layer, the computational complexity of water-filling for each inner layer optimisation is $O(1/\delta^2)$. Therefore, the total computational complexity of the conventional solutions is equal to $O\left((1/\delta^2)N_{OL}\right)$, which is in fact the product of the number of iterations in the inner and outer layer optimisation. Authors in [14] represented a low complexity sub-optimal solution to energy efficiency resource allocation in OFDMA systems. The computational complexity of their solution for OFDM system is in order of $O\left(\left(\frac{1}{\delta^2} + N + 1\right)N_{OL}\right)$, which is not suitable to use in OFDM system.

As it was mentioned previously, our proposed method, to jointly answer the energy-efficient source transmission power and subcarriers power allocation, has been based on finding m and $P_{S,1}$ as the solution of nonlinear equation in (18). Assume that N_{NLS} is the number of required iterations in Newton method to find $P_{S,1}$ as the solution of the nonlinear equation in (18) with considering an initial m value. If we consider N_{mValue} as the number of iterations that is required to converge m to the optimal value, then the total computational complexity of the proposed solution is $O\left(N_{mValue}N_{NLS}\right)$. Simulation results show that m value convergences to the optimal value after less than three iterations. Therefore, the maximum total computational complexity is equal to $O\left(3N_{NLS}\right)$, which is significantly less than the same in conventional methods.

4. ENERGY-EFFICIENT ALGORITHM FOR NOMINATION OF THE SOURCE TRANSMISSION POWER AND SUBCARRIER POWER ALLOCATION

We now propose a fast converging algorithm (Table I) using the expressions derived in the previous section

Table I. Energy efficient subcarrier and power allocation.

```
Input: $P_{S,\max}, R_{\min}, \zeta_S, P_{C,S}, N, B, \forall n: a_n$

Output: $\eta_{EE}, C_I, P_S^*, \forall n: P_{S,n}$

1. Calculate $\check{P}_S$ as the minimum source transmission power to guarantee minimum rate requirement, using (23).
2. if $\check{P}_S \leqslant P_{S,\max}$
    2.1 Calculate $P_{S,1}$ and $\hat{P}_S$ according to the following::
        2.1.1 Initialise, $mNew = 0$,
        2.1.2 Do
        2.1.3     $m = mNew$,
        2.1.4     Calculate $P_{S,1}$, $\overset{\frown}{P}_S$ and $mNew$ using (18), (19) and (12), respectively
        2.1.5 While $m\ != mNew$ //convergence is not attained
    2.2 if $\check{P}_S \leqslant \hat{P}_S$
        2.2.1 Distribute source transmission power among subcarriers ($\forall n: P_{S,n}$) using (12),
        2.2.2 Calculate $\eta_{EE}$, $C_I$ and $P_S^*$ using (4), (5) and (24), respectively.
    2.3 else
        2.3.1 Calculate $P_{S,n}$, $\eta_{EE}$ and $P_S^*$ using (22), (4) and (24), respectively.
3. Else
    3.1 Calculate $P_{S,n}$, $\eta_{EE}$ and $P_S^* = P_{S,\max}$ using (22), (4), respectively.
4. Return ($\eta_{EE}, C_I, P_S^*, \forall n: P_{S,n}$).
```

(Subsections 3.1, 3.2 and 3.3) to answer these following two questions:

(1) What is the optimum source transmission power $\left(P_S^*\right)$, as the solution of the optimisation problem (7)?
(2) How can one distribute this optimum transmission power among subcarriers ($\forall n: P_{S,n}$) to maximise energy efficiency?

Table I presents the heuristic solution for an OFDM system with N subcarriers to achieve the energy-efficient source transmission power and subcarrier power allocation, corresponding to the power budget ($P_{S,\max}$), CP ($P_{C,S}$), power amplifier efficiency (ζ_S) and minimum system rate requirement ($R_{\min}$).

5. SIMULATION RESULTS

We evaluate the proposed energy-efficient subcarrier power allocation method using nominated source transmission power budget for OFDM links by means of Monte Carlo simulations. We consider an OFDM system with $N = 16$ subcarriers each with 20 KHz bandwidth and frequency-selective with a complex normal distribution where the complex amplitude of ℓth channel path between two nodes with distance d meters is defined as [27]

$$h_1 \sim CN\left(0, \frac{1}{L(1+d)^{\alpha}}\right) \tag{25}$$

and the power delay profile is defined as

$$PDP(t) = \sum_{1=0}^{L-1} |h_1|^2\, \delta(t - \tau_1) \tag{26}$$

where α is the path loss exponent, L is the number of channel taps and τ_ℓ is the channel propagation delay of path 1. The frequency domain channel power gain is given by Fourier Transformation of the power delay profile with N subcarriers. The minimum rate requirements, $R_{\min}$, are 0.5 and 1 Mbps for low and high SNR regimes, respectively. According to Arnold *et al.* [28], we assume $P_{C,S} = 2.5$, and 0 W in the system with and without CP consideration, and $\zeta_S = 2.5$. Table II indicates our simulation parameters.

5.1. Convergence

Table III illustrates the convergence of the low complexity energy-efficient resource allocation algorithm, described in Subsection 3.2 and shown in Table I of Section 4. This algorithm converges to energy efficiency source transmis-

Table II. Simulation parameters.

$N = 16$	$L = 4$	$\alpha = 3.5$	$\sigma_d^2 = (0.5 \times 10^{-9}, 0.5 \times 10^{-10})$	$d = 1$ Km	$P_S \leqslant 10W$	$P_{C,S} = \{0, 2.5\}$ W	$\zeta_S = 2.5$	$R_{\min} = (0.5, 1)$ Mbps

Table III. Converge ratio of the proposed solution.

Number of required iteration	High SNR regime	Low SNR regime
1 iteration	63%	54%
2 iterations	22%	26%
3 iterations	15%	20%

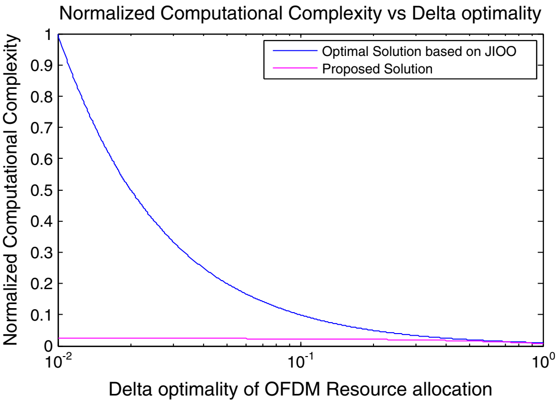

Figure 1. Normalised computational complexity of optimal solution based on JIOO and the proposed solution described in Table I. OFDM, orthogonal frequency division multiplexing; JIOO, joint inner- and outer- layer optimization.

sion power and subcarrier power allocation after maximum three iterations. As it has been depicted in Table III, for both low and high SNR regimes, in more than half of the channel realisations, the proposed solution converges to the optimal value in only one iteration.

5.2. Computational Complexity

Figure 1 illustrates the comparison between the computational complexity of traditional solution based on JIOO optimisation technique and our proposed solution described in Section 4 with variation on δ optimality variable.

The total computational complexity of the solution based on JIOO optimisation solution is equal to $O\left((1/\delta^2)N_{OL}\right)$ as the product of the number of iterations in the inner and outer layer optimisation. As it was mentioned previously, the computational complexity of our proposed solution is equal to the sum of computational complexity of water-filling and nonlinear equation solver methods such as Newton-Raphson. If we consider N_{NLS} as the number of iterations required to solve the nonlinear equation, so $O\left(3N_{NLS}\right)$ indicates the computational complexity of the heuristic described in Table I. In Figure 1, δ optimality value varies between 0.01 to 0.1 and the normalised computational complexity of the solution based on JIOO and our proposed solution is compared.

As it is seen in Figure 1, computational complexity of the proposed method in this paper is lower than that of the JIOO for all values of δ where the difference is much higher for smaller values of δ.

5.3. Energy efficiency

The impact of source transmission power budget variations on the energy efficiency performance in two scenarios (with and without CP consideration) is compared in Figure 2. The impact of P_S variation on the energy efficiency in both low and high SNR regimes is depicted in Figure 2. The comparison between optimal and our heuristic solution in low and high SNR regimes with and without CP consideration is depicted Figure 2. The optimal solution based on JIOO to maximise energy efficiency has been simulated for various transmission powers, but the results of our heuristic solution have been based on the optimisation problem in (6). Simulation results show that the low complexity solution described in Section IV-A directly points (pink and green color) to the optimum values in all scenarios. As it is shown in Figure 2, the energy efficient transmission power has occurred in zero transmission power if the CP not be considered. And in this situation (zero transmit power), the energy efficient value is infinite. Simulation results in Figure 2 confirm this hypothesis; however, we restrict this infinite value to clearly depict the results of the other simulations.

5.4. Energy efficiency versus Spectral Efficiency

According to Chen *et al.* [22], energy efficiency versus spectral efficiency (SE) is one of the four fundamental tradeoffs. The impact of spectral efficiency variations on the energy efficiency performance in two scenarios (with and without CP consideration), in low and high SNR regimes, is compared in Figure 3. To maximise energy efficiency, the optimal solution based on JIOO has been simulated for various spectral efficiency values. But the results of our heuristic solution have been based on the optimisation problem in (6), which nominates the energy-efficient source transmission power, and hence, the related spectral efficiency. It can be seen that our energy-efficient

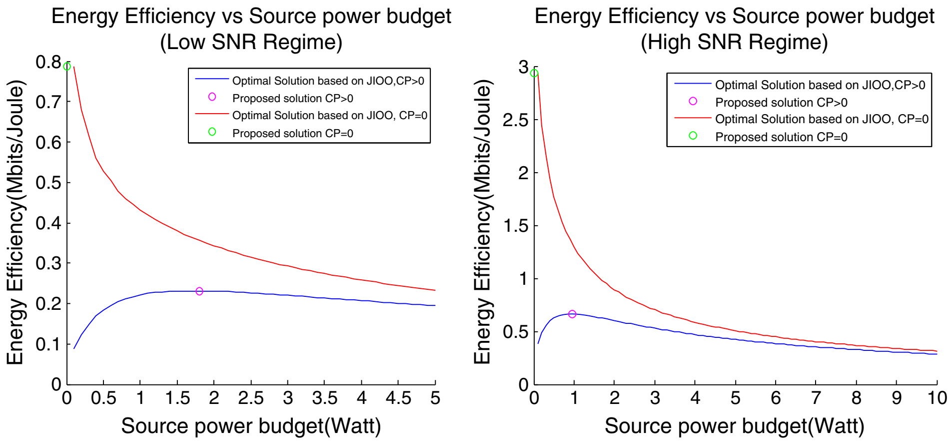

Figure 2. A comparison of energy efficiency versus source power budget variations, where our proposed solution is compared with JIOO for low (left) and high (right) transmission power regimes with and without circuit power (CP) consideration. JIOO, joint inner- and outer- layer optimization; SNR, signal to noise ratio.

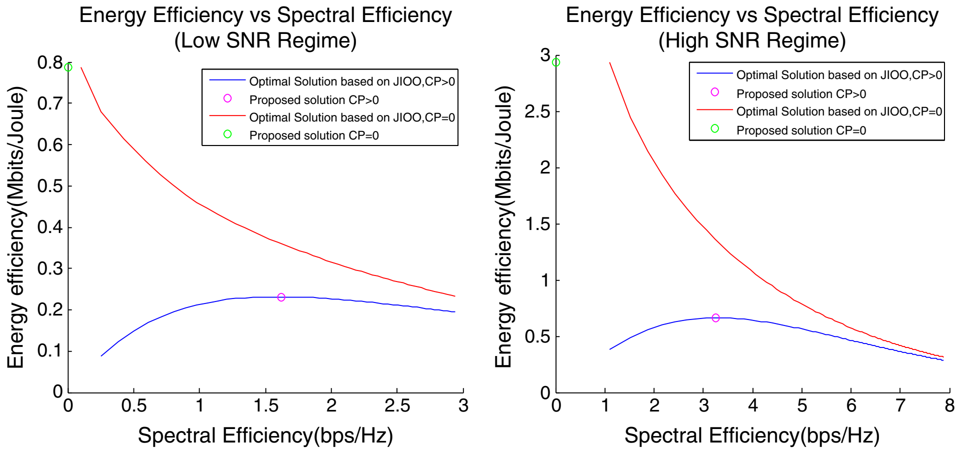

Figure 3. A comparison of energy-efficient variation versus spectral efficiency variations, where our proposed solution is compared with JIOO for low (left side) and high (right side) transmission power regimes with and without circuit power (CP) consideration. JIOO, joint inner- and outer- layer optimization; SNR, signal to noise ratio.

proposed solution directly points to the optimum value in both with (green) and without (pink) CP consideration in low and high SNR regimes.

6. CONCLUSION

In this paper, the end to end energy efficiency of an OFDM system with system CP consideration under maximum source transmission power and minimum rate requirement constraints is studied. It is assumed that the source transmission power can vary in transmission power interval. The problem is first formulated as an energy-efficient optimisation problem and then a low complexity heuristic solution is proposed, which is composed of two sub-problems to answer the following two questions. What is the energy-efficient source transmission power? How can one distribute source transmission power among subcarriers to maximise the energy efficiency?

Conventional solution to answer the aforementioned questions consists of an iterative algorithm with the joint inner and outer layer optimisation. The JIOO scheme consists of two iterative algorithms where each algorithm uses the results of the other algorithm iteratively to nominate transmission power and its distribution among subcarriers to maximise energy efficiency. In the inner layer, the nominated transmission power is distributed among subcarriers to maximise the total throughput. In the outer layer, the bisection power search methods such as gradient descend is used to nominate the next transmit power.

In spite of the iterative solution with high complexity order in JIOO, a quickly convergent low complexity solution is proposed. This solution directly points to the energy-efficient source transmission power and its distribution among subcarriers based on solving a nonlinear logarithmic equation.

To evaluate the accuracy of the proposed method, the results are compared through simulations with the optimal solution in systems with and without CP consideration. The simulation results indicate that our low complexity proposed method results in significant improvement in the complexity order while we get the same results as the optimal solution in both high and low SNR regimes. In general, study of low complexity energy-efficient resource allocation in OFDM systems is just in the initial stage. Solutions for low complexity energy-efficient resource allocation in OFDMA networks and relaying systems need to be investigated more in the future works.

APPENDIX A: ENERGY EFFICIENCY STRICTLY QUASI-CONCAVITY WITH P_S VARIATION AND UNIQUENESS OF TRANSMISSION POWER WHICH MAXIMISES η_{EE}

Using energy efficiency definition according to (4), we have

$$\eta_{EE}\frac{C_I}{P_{C,S}+\zeta_S P_S} \tag{A1}$$

to prove η_{EE} concavity with respect to P_S; first, we prove that C_I is concave with respect to P_S. According to (2) we have

$$\begin{aligned} &P_S = P_{S,1}+P_{S,2}+\ldots P_{S,n},\\ &P_{S,1}=\beta_1 P_S,\\ &P_{S,2}=\beta_2 P_S,\\ &\cdot\quad\cdot\quad\cdot\\ &\cdot\quad\cdot\quad\cdot\\ &\cdot\quad\cdot\quad\cdot\\ &P_{S,n}=\beta_n P_S \end{aligned} \tag{A2}$$

Using the derivation chain rule, we have

$$\begin{aligned} \frac{\partial C_I}{\partial P_S} &= \frac{\partial C_I}{\partial P_{S,1}}.\frac{\partial P_{S,1}}{\partial P_S}+\frac{\partial C_I}{\partial P_{S,2}}.\frac{\partial P_{S,2}}{\partial P_S}+\cdots\frac{\partial C_I}{\partial P_{S,n}}.\frac{\partial P_{S,n}}{\partial P_S}\\ &=\frac{\partial C_I}{\partial P_{S,1}}\beta_1+\frac{\partial C_I}{\partial P_{S,2}}\beta_2+\cdots\frac{\partial C_I}{\partial P_{S,n}}\beta_n \end{aligned} \tag{A3}$$

which can be written as

$$\frac{\partial C_I}{\partial P_S}=\sum_{n=1}^{N}\frac{\partial C_I}{\partial P_{S,n}}\beta_n$$

similarly,

$$\frac{\partial^2 C_I}{\partial P_S^2}=\sum_{n=1}^{N}\frac{\partial^2 C_I}{\partial P_{S,n}^2}\beta_n$$

We note that the sum of concave function is also concave [25]. Furthermore, the relations available in (A6), we prove C_I concavity with respect to P_S. Concavity of C_I with respect to $P_{S,n}$ is proved as follows:

$$\frac{\partial C_I}{\partial P_{S,n}}=\frac{a_n}{\left(1+P_{S,n}a_n\right)\ln(2)} \tag{A4}$$

as it is seen in (A4) by increasing $P_{S,n}$, $\frac{\partial C_I}{\partial P_{S,n}}$ is decreased. Second order derivation of C_I is

$$\frac{\partial^2 C_I}{\partial P_{S,n}^2}=-\frac{a_n^2}{\left(1+P_{S,n}a_n\right)^2\ln(2)} \tag{A5}$$

which is

$$\frac{\partial^2 C_I}{\partial P_{S,n}^2}<0 \tag{A6}$$

This proves C_I strictly concavity with respect to P_S. Using C_I strictly concavity, we then prove that η_{EE} is strictly quasi-concave function of P_S. We know that

$$\eta_{EE}=\frac{C_I}{P_{C,S}+\zeta_S P_S} \tag{A7}$$

We then define super level set of $\eta_{EE}(P_S)$ for any real value of α as

$$S_\alpha=\left\{P_{S,\min}\leqslant P_S\leqslant P_{S,\max}|\eta_{EE}(P_S)\geqslant\alpha\right\} \tag{A8}$$

According to Boyd and Vandenberghe [25], $\eta_{EE}(P_S)$ is strictly quasi-concave in P_S if S_α is convex for any real value of α. Substituting (A7) in (A8), we have

$$\begin{aligned} S_\alpha &= \left\{(P_{S,\min}\leqslant P_S\leqslant P_{S,\max})|\frac{C_I(P_S)}{P_{C,S}+\zeta_S P_S}\geqslant\alpha\right\}\\ &=\{(P_{S,\min}\leqslant P_S\leqslant P_{S,\max})|(P_{C,S}+\zeta_S P_S)\alpha-C_I(P_S)\\ &\quad\leqslant 0\} \end{aligned} \tag{A9}$$

Since $-C_I(P_S)$ is strictly convex, S_α as a super level set for any real value of α is a strictly convex function of P_S, therefore, $\eta_{EE}(P_S)$ is a strictly quasi-concave function of P_S.

The uniqueness of $\hat{P}_S$: we proved that $\eta_{EE}(P_S)$ is strictly quasi-concave function of P_S, and $\hat{P}_S$ is the transmission power that maximises η_{EE}. If we suppose $\hat{P}_S$ is not unique, so there is another transmission power $\left(\tilde{P}_S\right)$, which maximises η_{EE}. $\hat{P}_S\neq\tilde{P}_S$ implies $\eta_{EE}\left(\frac{1}{2}\hat{P}_S+\frac{1}{2}\tilde{P}_S\right)>\min\left(\eta_{EE}\left(\hat{P}_S\right),\eta_{EE}\left(\tilde{P}_S\right)\right)=\eta_{EE}\left(\hat{P}_S\right)$, which implies that the first assumption $\left(\hat{P}_S\neq\tilde{P}_S\right)$ is not correct.

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