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RESEARCH ARTICLE

Energy-efficient resource allocation for amplify-and-forward relaying in OFDM systems

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ABSTRACT

In this paper, we study end-to-end energy efficiency of an orthogonal frequency-division multiplexing relay-based system under power constraints for the base station and relay node with system circuit power considerations. Using a two-phase amplify-and-forward relaying protocol, we assume that relay has a fixed power constraint and the source power can vary in transmission power interval where the source employs a selective relaying mechanism for each individual subcarrier. Therefore, in our model, the source is able to adaptively select some subcarriers to be relayed. The radio resource allocation is then formulated as an energy-efficient mixed binary integer programming, and then we propose a heuristic algorithm to find the suboptimal solution. The solution consists of the suboptimal transmit power of the source, the suboptimal set of subcarriers for relaying and the optimal transmit power of each subcarrier at the source and relay in the first and second time slots. The algorithm obtains energy-efficient transmission power for a given relay transmission power based on a gradient descent method. Then a two-step iterative scheme is proposed to obtain the subcarriers to be relayed as well as the optimal power allocation at the source and relay in the first and second time slots. To evaluate the efficiency of the proposed method, we compare its efficiency through simulations with the cases without using selective subcarrier relaying and without circuit power consideration in low and high relay transmit power regimes. The simulation results indicate that using our proposed method results in a significant improvement in the energy efficiency. Copyright © 2014 John Wiley & Sons, Ltd.

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1. INTRODUCTION

As the share of communication systems in the world's energy consumption increases, an energy-efficient (EE) system design for such systems becomes an important issue and gains much attention in both industry and academia. In industrial areas, both vendors and operators are expecting more energy-saving architectures and techniques to reduce energy consumption and, thus, the total cost of operation. In wireless communication systems, radio access is one of the major energy consumers, which requires more than 70% of the total consumed system energy [1]. In recent years, several research projects, such as Energy Aware Radio and Network Technologies, Optimizing Power Efficiency in Mobile Radio Networks and Green Radio, have been introduced to develop EE wireless communication systems.

Because energy loss in the air interface is directly related to the distance electromagnetic waves travel, a straightforward idea to improve energy efficiency is to decrease the distance between the transmitter and receiver. Relays in wireless networks bring the access point closer to the receiver, thus potentially contributing to the system energy efficiency. For a given energy efficiency, the relays can also significantly improve achievable bit rate for the same reason.

In [2] and [3], relaying is investigated as a candidate solution to increase data rates while reducing energy consumption in wireless communication systems. It is also shown that for a given transmission rate, the total required energy decreases with increasing the number of relay nodes between source and destination. Therefore, by utilizing relaying, the required energy for the transmission of an information bit can be reduced compared with direct transmission.

Orthogonal frequency-division multiplexing (OFDM) has been adopted as a promising transmission technique for broadband wireless networks. It has already been used in wire line communications such as asynchronous digital subscriber line technology and wireless communications such as Digital Video Broadcasting, 3rd Generation Partnership Project Long Term Evolution and IEEE 802.1x series (Wi-fi, WiMax, etc.).

Among other factors in wireless system energy efficiency, circuit energy consumption is considered as an important factor. Besides transmit power, the energy consumption also includes circuit energy consumption incurred by active circuit modules [4]. It is shown in [5] that the transceiver power consumed in 802.11x, in idle mode, is considerable in comparison with that consumed in the transmit mode. This example along with other researches corroborates that the circuit energy consumption is not always ignorable compared with the transmit power (e.g. [6–13]). A complete circuit model has been considered in [4].

Energy-efficient OFDM systems considering circuit energy consumption for frequency-selective fading channels have been first studied in [10]. In contrast to the conventional trend in researches that maximize throughput under a fixed overall transmit power constraint [14], the new scheme in [10] maximizes the overall EE by adjusting both the total transmit power and its distribution among subcarriers. Authors in [11] developed a non-cooperative game for EE power optimization at interference limited communications with OFDM modulation. In [12], power allocation algorithms for EE multicarrier systems were addressed assuming static circuit power consumption. Authors in [13] studied EE subcarrier and power allocation in both downlink and uplink OFDM access network. Optimal power allocation for nonregenerative OFDM relay links was addressed in [15–17]. They provide the joint optimal power allocation at the source and relay to maximize achievable total throughput between source and destination.

This paper investigates end-to-end energy efficiency of an OFDM relay-based system under power constraints for the base station and relay node with system circuit power considerations. Using a two-phase amplify-and-forward relaying protocol [18], we assume that the relay has a fixed power constraint and the source power can vary in transmission power interval. In addition, we suppose that the source uses a selective relaying mechanism for each subcarrier separately. Selective subcarrier relaying is a technique to solve the problem of 'when to relay', for single and multiuser wireless systems. This technique was first addressed for OFDM and OFDM access in [19] and [20] where subcarrier relaying is selectively enabled to maximize overall transmission rate. We first formulate the problem, as an EE mixed binary integer programming problem and then propose a heuristic solution that is composed of several subproblems to answer the following questions. What is the EE source transmit power? Which subcarriers should be selected for relaying? How can one distribute source and relay transmit power between subcarriers in the first and second time slots to maximize the end-to-end energy efficiency?

To answer these questions, we propose a scheme that consists of two fast converging iterative algorithms where each algorithm uses the results of the other iteratively. The first algorithm contains a two-step iterative approach to find necessary and sufficient conditions to select subcarriers for relaying and power allocation at the source and relay in the first and second time slots based on convex optimization solution. The second algorithm contains an iterative solution based on a gradient descent method to choose the next EE source transmit power for a given relay power. This transmission power is fed into the first algorithm, and the sequence repeats until EE convergence is achieved.

Our contributions in this paper include deriving necessary and sufficient conditions for EE subcarrier selection and using convex optimization techniques for EE subcarrier power allocation in the source and relay based on the quasi-concavity of energy efficiency (Appendix B). We also derive an approach based on the gradient descent method to nominate the optimal source transmit power for a given relay power using the gradient of energy efficiency (Appendix C) and present a fast iterative algorithm to cover the pointed contributions. We further investigated the complexity of the proposed heuristic algorithm.

It is shown that by using EE subcarrier selection on which relaying should be performed and by considering circuit power, it is possible to improve both end-to-end system capacity and energy efficiency and hence trade off between energy and spectral efficiency (SE). The results of this method are compared with other scenarios without circuit power consideration that contain only a direct link, with and without subcarrier selection in both low and high relay transmit power regimes. Moreover, we obtain the minimum ratio of (R_S/P_P) that is required to improve energy efficiency in both low and high relay transmit power regimes. Because of the application of circuit power to obtain necessary and sufficient conditions, we can improve end-to-end throughput for each consumed power and hence the total energy efficiency. By improving throughput and energy efficiency, the required energy per transmission of an information bit is thus reduced. The simulation results indicate that using our proposed method results in a significant improvement in the energy efficiency.

In the following, we provide a list of the contribution of this paper:

(1) We derive necessary and sufficient conditions for EE subcarrier selection.
(2) We then adopt convex optimization techniques for EE subcarrier power allocation in the source and relay base on quasi-concavity of energy efficiency.
(3) A novel approach based on the gradient descent method is then proposed to nominate the optimal source transmit power for a given relay power using the gradient of energy efficiency.
(4) Finally, we present a novel, fast and practical iterative algorithm to obtain the solution to the optimal EE resource allocation problem.

Multicarrier communications and relaying are both considered as the main enablers for the next-generation mobile communication systems often referred to as 5G. In such systems, device-to-device communication is an important application that is supported by relaying as well. We believe the proposed algorithm presented in this paper as well as simulation and analytical results shed light on the important parameters in such systems and the corresponding trade-offs.

The rest of this paper is organized as follows. In Section 2, the system model is defined, and end-to-end energy efficiency of an OFDM relay-based system under power constraints for the base station and relay node with system circuit power considerations is formulated. The optimal source power selection for a given relay power, subcarrier selection for relaying and power allocation to maximize energy efficiency is proposed in Section 3. In Section 4, two heuristic and recursive algorithms, using the results of one another, upon the solutions obtained in the previous section, are presented. Finally, numerical results and conclusion are presented in Sections 5 and 6.

2. SYSTEM MODEL AND PROBLEM FORMULATION

The considered model in this paper consists of a source (S), a destination (D) and a relay (R). A source transmits to a destination using an OFDM modulation scheme, either through a relay station or directly. Relaying is conducted in time frames. Each time frame consists of two nonoverlapping time slots. We consider selective subcarrier relaying as in [19] and [20]. In selective subcarrier relaying, for each subcarrier n, in the first time slot, the source transmits data to the destination and relay (Figure 1a). In the second time slot, we use a binary decision parameter, $\mu_n \in \{0,1\}$, to decide between relay or source transmission as depicted in Figure 1b. If $\mu_n = 1$, then the relay station amplifies and forwards the message received in the first time slot corresponding to subcarrier n; otherwise, the direct transmission is conducted.

The channel bandwidth is divided into N subcarriers. Let us denote $h_{sd,n}$, $h_{sr,n}$ and $h_{rd,n}$ as the source–destination, source–relay and relay–destination channel gains for the nth subcarrier, respectively. Moreover, σ_r^2 and σ_d^2 are noise variances at the relay and destination. All these channel gains are assumed to be slow varying. Thus, they are constant for both the first and second time slots. We assume the source transmits data with power $P_{S,n}$ on the nth subcarrier, and the relay amplifies and retransmits the message, using power $P_{R,n}$ on the subcarrier. The signal-to-noise ratio, ρ_n, for subcarrier n in amplify-and-forward relaying at the destination is [21]

$$\rho_n = \frac{P_{S,n}a_n P_{R,n} b_n}{1 + P_{S,n}a_n + P_{R,n}b_n} + P_{S,n}c_n \qquad (1)$$

where $a_n = \frac{|h_{sr,n}|^2}{\sigma_r^2}$, $b_n = \frac{|h_{rd,n}|^2}{\sigma_d^2}$ and $c_n = \frac{|h_{sd,n}|^2}{\sigma_d^2}$.

We further assume that the source has perfect channel state information for all channels and also that the noise variance of the relay and destination links is known, that is, a_n, b_n and c_n for all n. P_S and P_R are the source and relay transmit powers distributed among subcarriers, respectively:

$$P_S = \sum_{n=1}^{N} P_{S,n}, \quad P_{S,\min} \leqslant P_S \leqslant P_{S,\max} \qquad (2)$$

$$P_R = \sum_{n=1}^{N} P_{R,n} \qquad (3)$$

where $P_{S,\min}$ and $P_{S,\max}$ represent the minimum and maximum allowable total transmit power at the source, respectively.

Besides transmit power, the energy consumption also includes circuit energy consumption incurred by active circuit blocks [4]. The overall consumption power similar to [4] and [22] at the source, $P_{S,tot}$, and relay, $P_{R,tot}$, is given as

$$P_{S,tot} = \frac{1}{2}\zeta_S P_S + P_{C,S} \qquad (4)$$

$$P_{R,tot} = \frac{1}{2}\zeta_R P_R + P_{C,R} \qquad (5)$$

where ζ_S and ζ_R are the reciprocal of drain efficiency of power amplifiers in the source and relay and $P_{C,S}$ and $P_{C,R}$ represent the circuit power in the source and relay, respectively. Let T and $C_{I,n}$ denote the time duration of two consecutive time slots and capacity in the nth subcarrier, respectively. Then, the throughput is $T\sum_{n=1}^{N} C_{I,n}$, and the overall consumed energy for an OFDM with nonregenerative relaying is $T(P_{C,S} + P_{C,R} + \frac{1}{2}\sum_{n=1}^{N}(\mu_n \zeta_R P_{R,n} + (1-$

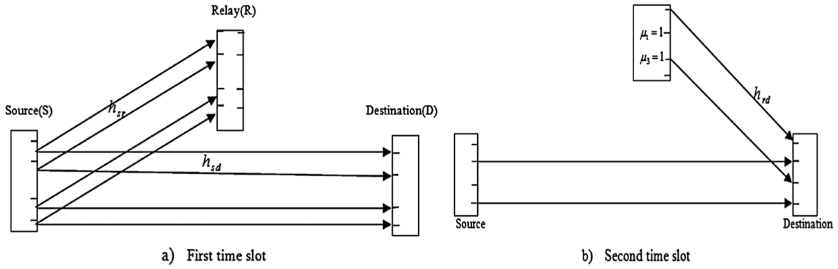

Figure 1. System model. (a) First time slot. (b) Second time slot.

$\mu_n)\zeta_S P_{S,n})$). We further define the energy efficiency similar to [13] and [22] as the ratio of throughput to the total consumed energy:

$$\eta_{EE} \triangleq \frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S} + P_{C,R} + \frac{1}{2}\sum_{n=1}^{N}(\mu_n \zeta_R P_{R,n} + (1-\mu_n)\zeta_S P_{S,n})} \tag{6}$$

where

$$\begin{aligned} C_{I,n} &= \mu_n \frac{1}{2}\log_2(1+\rho_n) + (1-\mu_n)\log_2(1+P_{S,n}c_n) \\ &= \frac{1}{2}\log_2\left(\frac{1+\rho_n}{(1+P_{S,n}c_n)^2}\right)^{\mu_n} + \log_2(1+P_{S,n}c_n) \end{aligned} \tag{7}$$

The 1/2 factor appears here because of the half-duplex relaying process. Our objective is to determine the relay decision, μ_n; the EE source transmit power selection, P_S^*, between $P_{S,\min}$ and $P_{S,\max}$ for a given relay transmit power; and in continuation, the subcarrier power allocation for the source, $P_{S,n}$, and relay, $P_{R,n}$, which offers the maximum η_{EE} while considering the minimum rate requirement $(C_I = \sum_{i=1}^{N} C_{I,n} > R)$.

$$\begin{aligned} &\max \eta_{EE}, \\ &\text{s.t. } C_I > R, \\ &P_{S,\min} \leqslant P_S \leqslant P_{S,\max}, \\ &\mu_n \in \{0,1\} \end{aligned} \tag{8}$$

3. ENERGY-EFFICIENT RESOURCE ALLOCATION

Joint determination of the optimal source transmit power, the set of subcarriers for relaying and the transmit power of each subcarrier at the source and relay in the first and second time slots to maximize η_{EE} is an NP-hard problem [22]. Furthermore, the nonlinear constraint in Equation (8) increases the difficulty in finding the optimal solution, as the feasible set is not convex. The objective function in Equation (8) is quasi-concave in P_S (Appendix A) on the set defined by the first two constraints; however, the nonlinear equality constraint, $\mu_n \in \{0,1\}$, results in a feasible set nonconvexity.

We decompose this problem into two layers and solve it iteratively by the joint inner-layer and outer-layer optimization. In the inner layer, for a given source transmit power ($P_{S,\min} \leqslant P_S \leqslant P_{S,\max}$ as a result of outer-layer optimization), we distribute power among subcarriers at the source and relay to maximize energy efficiency. In the outer layer, in addition to calculating the energy efficiency derivation value, $\frac{\partial \eta_{EE}(P_S)}{\partial P_S}$, we find the next candidate source transmit power based on gradient descent and line search methods [23] to maximize energy efficiency.

The key idea in the inner-layer algorithm is to remove nonlinearity in the constraint, $\mu_n \in \{0,1\}$, using subcarrier selection method. Using the subcarrier selection method, nonlinear constraints will be moved from the constraint function to the objective function. We obtain the necessary and sufficient conditions to select subcarriers for relaying. Selection of subcarriers for relaying is used to disjoint resource allocation for relayed and direct subcarriers. Calculating the gradient of energy efficiency, $\frac{\partial \eta_{EE}(P_S)}{\partial P_S}$, is a key idea in the outer-layer algorithm. The gradient descent and line search methods [23] used in the outer layer are clear and easy to derive.

First, in the inner layer, for a given pair of P_S and P_R, we obtain the necessary and sufficient conditions to select subcarriers for relaying (Section 3.1) and optimal subcarrier power allocation at the source and relay based on a convex optimization solution (Sections 3.2 and 3.3). In the following, we also obtain the optimal source transmit power for a given P_R to maximize the total energy efficiency, which is referred to as η_{EE}^*, utilizing the gradient descent method (Section 3.4) in the outer layer.

3.1. Subcarrier selection based on maximizing energy efficiency

To determine the subcarrier relay decision, μ_n, for a given pair of P_S and P_R, which maximizes energy efficiency, we start from Equations (6) and (7), which indicate the energy efficiency definition, so we have

$$Max \frac{\sum_{n=1}^{N} \frac{1}{2}\log_2\left(\frac{1+\rho_n}{(1+P_{S,n}c_n)^2}\right)^{\mu_n} + \log_2(1+P_{S,n}c_n)}{P_{C,S} + P_{C,R} + \frac{1}{2}\sum_{n=1}^{N}\left(\mu_n \zeta_R P_{R,n} + (1-\mu_n)\zeta_S P_{S,n}\right)} \tag{9}$$

In the inner layer, for given source and relay transmit powers, we distribute powers among subcarriers at the source and relay to maximize energy efficiency. According to this solution and after substituting Equations (2) and (3) in the denominator of Equation (9), the consumed power at the source and relay can be summarized as follows: $P_{C,S} + P_{C,R} + \frac{1}{2}\sum_{n=1}^{N}(\mu_n \zeta_R P_{R,n} + (1-\mu_n)\zeta_S P_{S,n}) = P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_R P_R + \zeta_S P_S)$, so in each iteration, the denominator of Equation (9) is constant; therefore, only the numerator must be considered in the subcarrier relaying decision. Clearly, the first term in the numerator of Equation (9) must be larger than zero for the relaying process to increase the numerator of Equation (9) and consequently the energy efficiency. So we have

$$(1+\rho_n)^{\frac{1}{2}} \geqslant (1+P_{S,n}c_n) \tag{10}$$

We can simplify this inequality as follows:

$$\Gamma_n(P_{R,n}, P_{S,n}) \triangleq \frac{(1+\rho_n)}{S_n} \geqslant 1 \tag{11}$$

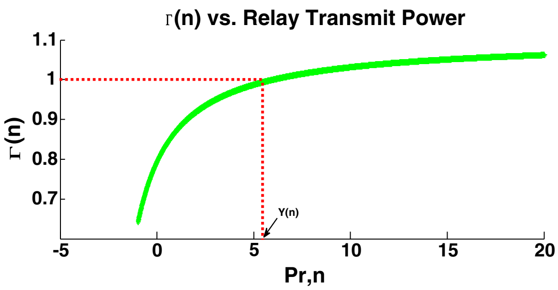

Figure 2. Energy-efficient decision on relay transmit power.

where S_n is defined by the following:

$$S_n \triangleq (1 + P_{S,n}c_n)^2 \tag{12}$$

Therefore, to improve energy efficiency, every subcarrier n that satisfies inequality in Equation (11) can be a candidate for relaying. The left term of Equation (11), Γ_n, depends on the $P_{R,n}$ and $P_{S,n}$ variables. First, to determine the subcarrier relay decision, μ_n, we fix $P_{S,n}$ and achieve necessary and sufficient conditions that $P_{R,n}$ must satisfy to improve energy efficiency. To visualize, $\Gamma_n(P_{R,n}, P_{S,n})$ versus $P_{R,n}$ for a fixed value of $P_{S,n}$ is plotted in Figure 2. According to this figure, we obtain the necessary conditions to determine the subcarrier relay decision, μ_n, as in the following:

Necessary condition: In order to obtain gain in energy efficiency, the horizontal asymptote has to be larger than 1. To obtain the horizontal asymptote, we set $P_{R,n} \rightarrow \infty$ in Equation (11), so we have

$$\frac{P_{S,n}(a_n + c_n) + 1}{S_n} \geqslant 1 \tag{13}$$

In other words, for each subcarrier n that does not satisfy Equation (13), we set $\mu_n = 0$ and $P_{R,n} = 0$. In the following, we obtain the relay power assignment for the remaining subcarriers that satisfy Equation (13).

Sufficient condition: According to Figure 2, in order to guarantee Equation (11), we obtain a threshold for $P_{R,n}, \gamma_n$, as the minimum value of $P_{R,n}$. Using inequality in Equation (11) and after some straightforward mathematical manipulations, we have

$$P_{R,n} \geqslant \gamma_n \tag{14}$$

where

$$\gamma_n = \frac{(P_{S,n}a_n + 1) \times (S_n - P_{S,n}c_n - 1)}{b_n(1 + P_{S,n}(a_n + c_n) - S_n)} \tag{15}$$

In other words, relay decision μ_n is set to 1 if Equation (13) is satisfied as the necessary condition and the minimum value of $P_{R,n}$ is achieved from Equation (15) as the sufficient condition.

3.2. Subcarrier power allocation at the relay to maximize energy efficiency

Convex optimization techniques in [23] are used to achieve $P_{R,n}$ for each subcarrier that satisfies Equation (13) as the necessary condition. In addition, Equation (14) must be also satisfied as the sufficient condition in the optimization problem. The convex optimization problem (problem convexity is proved in Appendix B) is formulated as follows:

$$\begin{aligned}
&\min - \frac{\frac{1}{2} \sum\limits_{\langle \forall n | \mu_n = 1 \rangle}^{N} \log_2(1+\rho_n)}{P_{C,S} + P_{C,R} + \frac{1}{2}\left(\sum\limits_{\langle \forall n | \forall n \mu_n = 1 \rangle}^{N} \zeta_R P_{R,n} + \sum\limits_{n=1}^{N} \zeta_S P_{S,n} \right)}, \\
&\text{s.t} \\
&\gamma_n - P_{R,n} \leqslant 0, \\
&P_R - \sum_{n=1}^{N} P_{R,n} = 0
\end{aligned} \tag{16}$$

Using the Karush–Kuhn–Tucker (KKT) [23] conditions for the convex optimization problem in Equation (16) (Appendix A) and the assumption that relaying is performed for the case where there is a weak direct link, that is,

$$\begin{aligned}
1 + \rho_n &= 1 + \frac{P_{S,n}a_n P_{R,n} b_n}{1 + P_{S,n}a_n + P_{R,n}b_n} + P_{S,n}c_n \\
&\cong 1 + \frac{P_{S,n}a_n P_{R,n} b_n}{1 + P_{S,n}a_n + P_{R,n}b_n}, (\forall n : \mu_n = 1)
\end{aligned} \tag{17}$$

the solution is given by

$$P_{R,n} = \frac{-(2 + P_{S,n}a_n) + \sqrt{(2 + P_{S,n}a_n)^2 - 4\left(P_{S,n}a_n + 1 - \frac{P_{S,n}b_n a_n}{\nu_r(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R))\ln(2)}\right)}}{2b_n} \tag{18}$$

The optimal relay transmission power for each subcarrier is defined as $P^*_{R,n}$ and is obtained as follows:

$$P^*_{R,n} = \max(\gamma_n, P_{R,n}) \tag{19}$$

Parameter ν_r is the Lagrange multiplier and is chosen such that Equation (3) is fulfilled.

3.3. Subcarrier power allocation at the source (first and second time slots) to maximize energy efficiency

In order to maximize energy efficiency, we then need to determine $P_{S,n}$ according to the relay decision, μ_n, and the allocated power in the relay, $P^*_{R,n}$, so that the condition in Equation (13) is also satisfied. After substituting $P^*_{R,n}$ in Equation (13) and some straightforward mathematical manipulations, we obtain

$$\frac{(1+P_{S,n}c_n)(1+P_{S,n}a_n)+P_{S,n}a_nP_{R,n}b_n+(1+P_{S,n}c_n)b_nP_{R,n}}{(1+P_{S,n}a_n+P_{R,n}b_n)S_n} \geqslant 1, \text{ where } P_{R,n}=P^*_{R,n} \tag{20}$$

Any value of $P_{S,n}$ that satisfies Equation (20) can potentially improve total energy efficiency. Assume that when relaying occurs, we can use the approximation in Equation (17), and Equation (20) can be reduced to $P_{S,n} \geqslant 0$. This is also evident based on the simulation result presented in Figure 3 that shows $\Gamma_n(P_{R,n}, P_{S,n})$ versus $P_{S,n}$ for a fixed value of $P_{R,n}$.

The source power allocation is also conducted following the same line of argument as in the previous section. The optimization problem can be formulated as follows:

$$\min -\frac{\frac{1}{2}\sum_{n=1}^{N}\mu_n\log_2(1+\rho_n)+\sum_{n=1}^{N}(1-\mu_n)\log_2(1+P_{S,n}c_n)}{P_{C,S}+P_{C,R}+\frac{1}{2}\sum_{n=1}^{N}(\mu_n\zeta_RP_{R,n}+(1-\mu_n)\zeta_SP_{S,n})},$$

$$\begin{aligned}
&\text{s.t}\\
&R - C_I < 0,\\
&\mu_n = \{0,1\},\\
&P_S - \sum_{n=1}^{N} P_{S,n} = 0
\end{aligned} \tag{21}$$

The nonlinear constraint in Equation (21) increases the difficulty in finding the optimal solution, because the feasible set is not convex. Solving this nonlinearity, we separate the problem into two cases, with $\mu_n = 0$ and $\mu_n = 1$. We then consider the preceding two constraints and then apply the KKT conditions in a similar fashion used in the previous section.

Thus, if $\mu_n = 0$,

$$P_{S,n} = \left(\frac{1}{\nu_s\left(P_{C,S}+P_{C,R}+\frac{1}{2}(\zeta_SP_S+\zeta_RP_R)\right)\ln(2)}\right) - \frac{1}{c_n} \tag{22}$$

and if $\mu_n = 1$,

$$P_{S,n} = \frac{-(2+P_{R,n}b_n)+\sqrt{(2+P_{R,n}b_n)^2-4\left(P_{R,n}b_n+1-\frac{P_{R,n}b_na_n}{\nu_s(P_{C,S}+P_{C,R}+\frac{1}{2}(\zeta_SP_S+\zeta_RP_R))\ln(2)}\right)}}{2a_n} \tag{23}$$

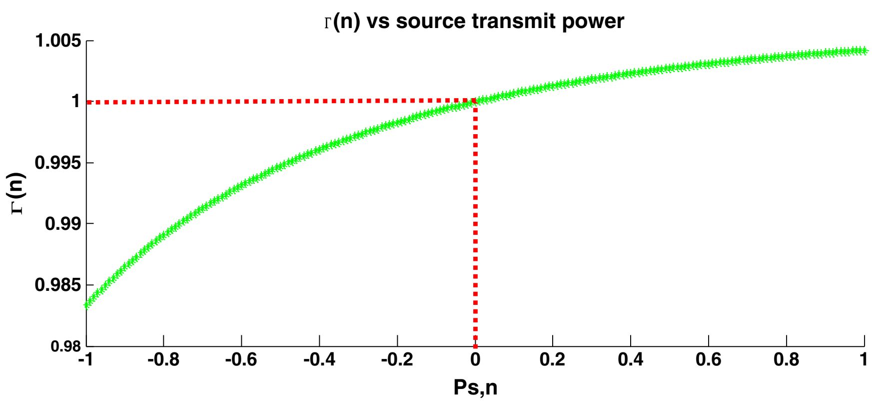

Figure 3. Energy-efficient decision on source transmit power.

we also know that $P_{S,n}$ must be greater than 0; thus,

$$P^*_{S,n} = \max(0, P_{S,n}) \tag{24}$$

Similar to the previous section, we used approximation in Equation (17) when relaying occurs (i.e. weak direct link for relayed subcarriers). Parameter ν_s is the Lagrange multiplier and is chosen such that Equation (2) is fulfilled.

3.4. Nominate energy-efficient source transmit power for a given relay power

Because of $\eta_{EE}(P_S)$ quasi-concavity (as proved in Appendix B), it has a unique P^*_S such that for any $P_S < P^*_S$, $\frac{\partial \eta_{EE}(P_S)}{\partial P_S} > 0$, and for $P_S > P^*_S$, $\frac{\partial \eta_{EE}(P_S)}{\partial P_S} < 0$. To find this optimal source transmit power, P^*_S, we design the gradient descent method to produce a P^i_S sequence, maximizing $\eta_{EE}(P_S)$ and converging it to the optimal energy efficiency named as η^*_{EE}. To improve the convergence rate, the optimum step size, t, is calculated by means of the line search method according to [23]. In the following, based on the previous steps and the optimum step size, the next P_S will be obtained. This iterative process continues until the convergence is obtained (convergence is obtained when $|\eta_{EE}(P^{i+1}_S) - \eta_{EE}(P^i_S)| < \varepsilon$ [23], where ε is a convergence criterion value).

Based on the gradient descent method, the next source transmit power, P^{i+1}_S, is obtained as follows:

$$P^{i+1}_S = P^i_S - t\frac{\partial \eta_{EE}}{\partial P_S} \tag{25}$$

where P^i_S and $\frac{\partial \eta_{EE}}{\partial P_S}$ refer to the previous source transmit power step and energy-efficient gradient with respect to P_S. According to Equation (25), $\frac{\partial \eta_{EE}}{\partial P_S}$ is necessary to compute the next transmitting power, and it is obtained as follows:

$$\begin{aligned}
&\frac{\partial \eta_{EE}}{\partial P_S} = \sum_{n=1}^{N} \frac{\partial \eta_{EE}}{\partial P_{S,n}} \beta_n \\
&\text{where} \\
&\beta_n = \frac{\partial P_{S,n}}{\partial P_S}, \\
&\frac{\partial \eta_{EE}}{\partial P_{S,n}} = \frac{\left(\mu_n \frac{a_n b_n P_{R,n}}{2(1+P_{S,n}a_n+P_{R,n}b_n)(1+P_{S,n}a_n)\ln(2)} + (1-\mu_n)\frac{c_n}{(1+P_{S,n}c_n)\ln 2}\right)}{\left(P_{S,tot}+P_{R,tot}\right)}
\end{aligned} \tag{26}$$

Appendix C describes our method to calculate $\frac{\partial \eta_{EE}}{\partial P_S}$ in Equation (26).

4. ITERATIVE ENERGY-EFFICIENT ALGORITHM FOR NOMINATION OF SOURCE TRANSMIT POWER, SUBCARRIER SELECTION AND POWER ALLOCATION

Based on the previous section, we propose an iterative solution by the joint inner-layer and outer-layer optimization. In the inner layer, an iterative algorithm (Table I) is used to assign the suboptimal energy-efficient subcarrier selection matrix ($\boldsymbol{\mu}$) and the optimal subcarrier power allocation matrix at source and relay ($\forall n : P_{S,n}, P_{R,n}$). In the outer layer, based on quasi-concavity of $\eta_{EE}(P_S)$, we propose a fast converging algorithm with the gradient descent method [23] to nominate the P_S sequence in order to converge $\eta_{EE}(P_S) \to \eta^*_{EE}$ (Table II) for a given relay power.

4.1. Iterative energy-efficient subcarrier selection and power allocation

We now perform the selection of subcarriers and the corresponding power allocation in the source and relay using the expressions derived in the previous section (Sections 3.1, 3.2 and 3.3).

As shown in Appendix B, optimization problems (16) and (21) are both concave in terms of $P_{R,n}$ and $P_{S,n}$. In our algorithm, Equations (19) and (24) as the solutions of optimizations (16) and (21) are iteratively repeated such that the output of the previous optimization is the input to the other. In fact, in this case, there are two dependent variables ($P_{R,n}, P_{S,n}$) forming a bi-convex optimization as described in [24, 25]. Convergence of bi-convex optimization techniques has been analytically shown in [25, 26]. We further confirm the convergence of the proposed algorithm using simulations in Section 5.

Table I presents the iterative algorithm in an OFDM system with N subcarriers to achieve the subcarrier selection matrix, $\boldsymbol{\mu}$, and the subcarrier power allocation in the source and relay, corresponding to the power budget (P_S, P_R), circuit power ($P_{C,S}, P_{C,R}$) and power amplifier efficiency (ζ_S, ζ_R) of the source and relay, respectively.

Table I. Iterative algorithm for energy-efficient subcarrier selection and power allocation.

Input: $P_S, P_R, \zeta_S, \zeta_R, P_{C,S}, P_{C,R}, N$

Output: $\eta_{EE}, C_I, \forall n : (\mu_n, P_{S,n}, P_{R,n})$

1. ***Initialize***: $\forall n : P_{S,n} = \frac{P_S}{N}$
2. ***While*** convergence is not attained do (not decreasing in η_{EE})

   2.1 $\forall n \therefore$ Calculate γ_n, set $\mu_n = 1$

   2.2 $\forall n$ if necessary condition (13) is not satisfied $P_{R,n} = 0, \mu_n = 0$ else continue

   2.3 Sort $\boldsymbol{\gamma}$ in a decreasing manner and repeat while $1^T.\boldsymbol{\gamma} > \boldsymbol{P_R}$, $P_{R,n} = 0, \mu_n = 0 \mathit{for} \langle n | = \max(\boldsymbol{\gamma}) \rangle$

   2.4 ***While*** η_{EE} value is not reduced or ($\boldsymbol{\gamma} = 0$ and $\boldsymbol{\mu} = 0$)

   2.4.1 Calculate $P_{R,n}$ to maximize energy efficiency using Equations (18) and (19) with current $\boldsymbol{\mu}$

   2.4.2 $P_{R,n} = 0, \mu_n = 0 \mathit{for} \langle n | n\gamma_n = \max(\boldsymbol{\gamma}) \rangle$

   2.5 ***End while***

   2.6 Calculate $P_{S,n}$ to maximize energy efficiency according to Equations (22)–(24)
3. ***End while***
4. Calculate $C_I = \left(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_R P_R + \zeta_S P_S)\right) \cdot \eta_{EE}$
5. ***Return*** $(\eta_{EE}, C_I, \forall n : (\mu_n, P_{S,n}, P_{R,n}))$.

Table II. Gradient descent method to find P_S^* and η_{EE}^* for a given relay power.

Input: P_R, $\mathit{initial} P_S, \zeta_S, \zeta_R, P_{C,S}, P_{C,R}, N$

Output: optimal $P_S^*, \eta_{EE}^*, \forall n \therefore (\mu_n, P_{S,n}, P_{R,n})$

1. ***Initialize:*** $P_S^1 = P_{S,\max}, \Delta P_S, \varepsilon$ Calculate $\eta_{EE}\left(P_S^1\right)$ using the proposed algorithm in Table I
2. ***Do***

   2.1 $P_S^0 = P_S^1, \eta_{EE}\left(P_S^0\right) = \eta_{EE}\left(P_S^1\right)$

   2.2 Calculate the optimum step size $(\alpha \in (0, 0.5), \beta \in (0, 1))$

   2.2.1 ***While*** $\eta_{EE}\left(P_S^0 - t\frac{\partial \eta_{EE}}{\partial P_S}\right) > \eta_{EE}\left(P_S^0\right) + \alpha t \left(-\frac{\partial \eta_{EE}}{\partial P_S}\right)^2$

   2.2.2 $t = \beta t$

   2.2.3 ***End while***

   2.3 $\Delta P_S = -\frac{\partial \eta_{EE}}{\partial P_S}$

   2.4 $P_S^1 = P_S^0 + t\Delta P_S$

   2.5 Calculate $\eta_{EE}(P_S^1)$ using the proposed algorithm in Table I
3. ***While*** convergence is not attained do (convergence is obtained when $(|\eta_{EE}(P_S^1) - \eta_{EE}(P_S^0)| < \varepsilon) \mathit{or} (C_I \leqslant R) \mathit{or} (P_S \geqslant P_{S,\max})$
4. ***Return*** $P_S^* = P_S^1, \eta_{EE}^* = \eta_{EE}(P_S^1), \forall n : (\mu_n, P_{S,n}, P_{R,n})$.

As mentioned in Equation (14) for all subcarriers that we want to relay, we must apply $P_{R,n} \geqslant \gamma_n$. First, set $\mu_n = 0$ for the subcarriers with the largest $\mu_n \gamma_n$, until $\sum_{n=1}^{N} \mu_n \gamma_n \leqslant P_R$. We repeat the $P_{R,n}$ calculation for all subcarriers, guarantee necessary and sufficient conditions using Equations (18) and (19) and set $\mu_n = 0$ for the subcarriers with the largest $\mu_n \gamma_n$, until $\sum_{n=1}^{N} \mu_n \gamma_n \leqslant P_R$, η_{EE} is reduced, $\boldsymbol{\gamma} = \mathbf{0}$ or $\boldsymbol{\mu} = \mathbf{0}$.

After $P_{R,n}$ is specified for all subcarriers, $P_{S,n}$ can be calculated using Equations (22)–(24). These stages are repeated until convergence in η_{EE} is obtained.

4.2. Gradient descent search method

We now present an algorithm based on the gradient descent method to obtain the EE source transmit power using the expressions derived in Section 3.4. Table II presents an iterative algorithm in an OFDM system with N subcarriers to achieve the optimal source transmit power, corresponding to the relay power budget, the source and relay circuit power and power amplifier efficiency. We observed in our simulations that usually no more than five iterations are required for this algorithm to converge to an optimal P_S so that $\eta_{EE}(P_S)$ is maximized. The description of the heuristic scheme in Table II is as follows.

Initially, for the random P_S value, the maximum achievable $\eta_{EE}(P_S)$ will be obtained, using iterative algorithm described in Table I. Using an iterative scheme (Table II, 2 and 3), $\eta_{EE}(P_S^1)$, which is the result of substituting the nominated P_S value into the algorithm described in Table I, is used to calculate the next P_S, named as P_S^1 value. Note that the nominated P_S value is obtained by using the previous energy efficiency, $\eta_{EE}(P_S^0)$, and the algorithm described in Section 3.4.

Finally, in addition to obtaining an optimal P_S value, P_S^*, the corresponding $\eta_{EE}(P_S)$ and C_I will be achieved with convergence in the result (Table II, 2).

In the next section, the convergence rate and the evaluation of our approach for the energy efficiency and throughput performance will be depicted through the simulation results.

4.3. Computational complexity

The proposed solution in the previous section consists of an iterative algorithm with the joint inner-layer and outer-layer optimization. The overall computational complexity of this solution depends on the product of the number of iterations in the inner, N_{In_Layer}, and outer, N_{Out_Layer}, layer optimization. According to [27], if we consider δ optimality definition as $\eta_{EE}(P_S) - \eta_{EE}^*(P_S^*) < \delta$, the number of water-filling and subcarrier selection required for each inner-layer optimization would be at least $O(\alpha\beta N(1/\delta^2))$. The value of $\beta \in \{1, 2\}$ shows that the water-filling process is only used for source ($\beta = 1$) or for the source and relay separately ($\beta = 2$). Parameter α shows the number of iterations required for the convergence of the subcarrier selection process. The total computational complexity of the proposed solution is equal to $O(N_{Out\_Layer}\alpha\beta N(1/\delta^2))$.

5. SIMULATION RESULTS

We evaluate the proposed EE subcarrier relay selection and power allocation method using different source transmit power budgets for OFDM nonregenerative relay links by means of Monte Carlo simulations. In our simulations, we assume that all three nodes, source, relay and destination, are located on a line. We assume that relay is located in the middle of the source–destination link. Source–relay, relay–destination and source–destination distance is denoted by d_0, d_1 and d_2, respectively. We consider an OFDM system with $N = 64$ subcarriers each with 20-kHz bandwidth and that is frequency selective with a complex normal distribution where the complex amplitude of the ℓth channel path between two nodes with distance d metres is defined as [15]

$$h_\ell \sim CN\left(0, \frac{1}{L(1+d)^\alpha}\right) \tag{27}$$

and the power delay profile is defined as

$$PDP(t) = \sum_{\ell=0}^{L-1} |h_\ell|^2 \, \delta(t - \tau_\ell) \tag{28}$$

where α is the path loss exponent, L is the number of channel taps and τ_ℓ is the channel propagation delay of path ℓ. The frequency domain channel power gain is given by Fourier transformation of the power delay profile with N subcarriers. According to [28], we assume $P_{C,S} = 0.5P_S, P_{C,R} = 0.5P_R$ and $\zeta_S = \zeta_R = 2.5$. Table III shows our simulation parameters.

5.1. Convergence

Figure 4 illustrates the convergence of the EE subcarrier selection algorithm, described in Section 4.1. This

Table III. Simulation parameters.

$N = 64$	$2d_0 = 2d_1 = d_2 = 10\,\text{km}$	$P_S = [0.2\ 4]$ for low and [2 40] for high relay power regimes	$P_{C,S} = 0.5P_S$	$\zeta_S = 2.5$
$L = 4,\ \alpha = 4$	$\sigma_d^2 = \sigma_r^2 = 8 \times 10^{-17}$	$P_R = 1\,\text{W}$ for low and 10 W for high relay power regimes	$P_{C,R} = 0.5P_R$	$\zeta_R = 2.5$

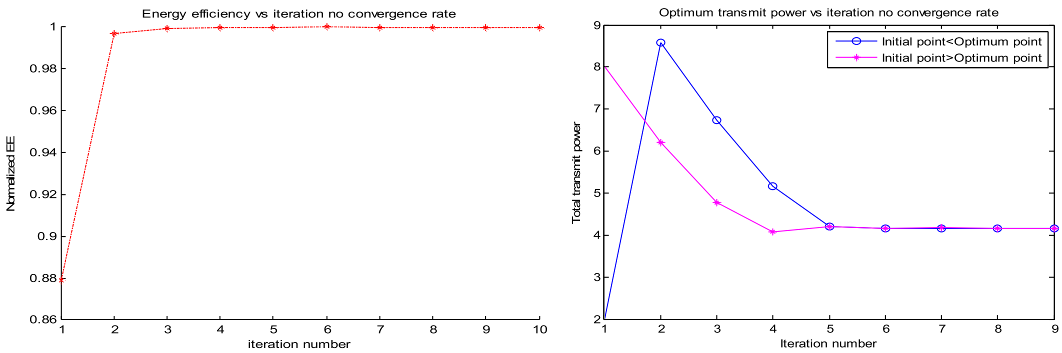

Figure 4. Convergence of heuristic described in Table I (left side) and Table II (right side) algorithms.

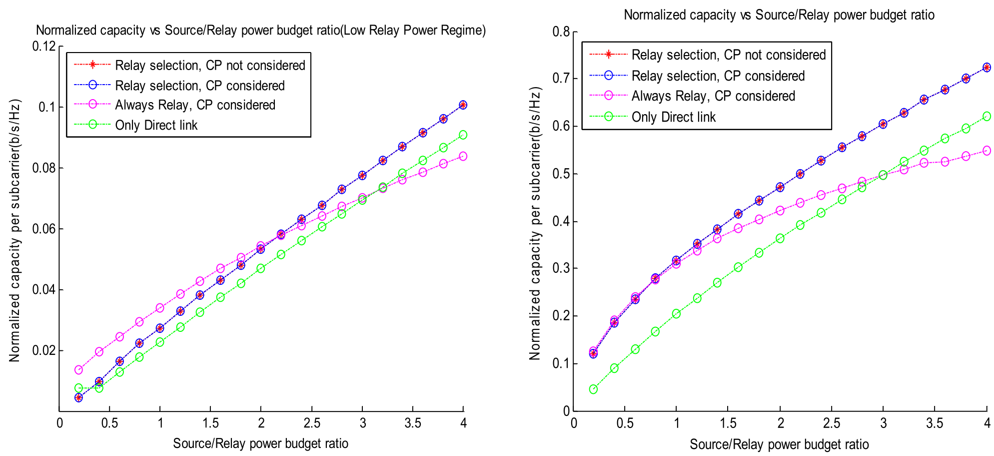

Figure 5. Throughput versus source/relay power budget ratio variation in low (left side) and high (right side) relay power regimes.

algorithm converges to maximum energy efficiency after three iterations.

Figure 4 depicts the convergence of EE source power selection algorithm for a given relay power, described in Section 4.2. This algorithm converges to EE source transmit power (P_S) that maximizes energy efficiency after four iterations independent of the initial point selection.

5.2. Throughput

The impact of source transmit power variation (for a given relay power) on the total system throughput performance in four scenarios [without relay (direct link) [29], always relayed [15] and relay selection with and without circuit power considerations] for low and high relay power regimes is plotted in Figure 5. In all scenarios, throughput increased with an increase in the source/relay power budget. Moreover, the difference between our heuristic relay selection (with and without circuit power consideration) and the other scenarios is increased with increasing the source/relay power budgets in both low and high relay power regimes.

5.3. Energy efficiency

The impact of source/relay power budget ratio variation (for a given relay power) on the energy efficiency performance in four scenarios [without relay (direct link) [29], always relayed [15] and relay selection with and without circuit power consideration] is compared in Figure 6. The energy efficiency concavity with P_S variation in both low (left side) and high relay transmit power regimes is depicted in Figure 6. The EE subcarrier relay selection algorithm described in Section 4.1 shows better performance compared with other methods for $(P_S/P_R) > 2$ and $(P_S/P_R) > 1$ in both low and high relay transmit power regimes, respectively. Without considering circuit power in the high relay transmit power regime, the peak value for energy efficiency is higher compared with that in other methods; however, this peak value is not valid in practical scenarios.

5.4. Energy efficiency versus spectral efficiency

According to [30], energy efficiency versus SE is one of the four fundamental trade-offs. The impact of SE variations

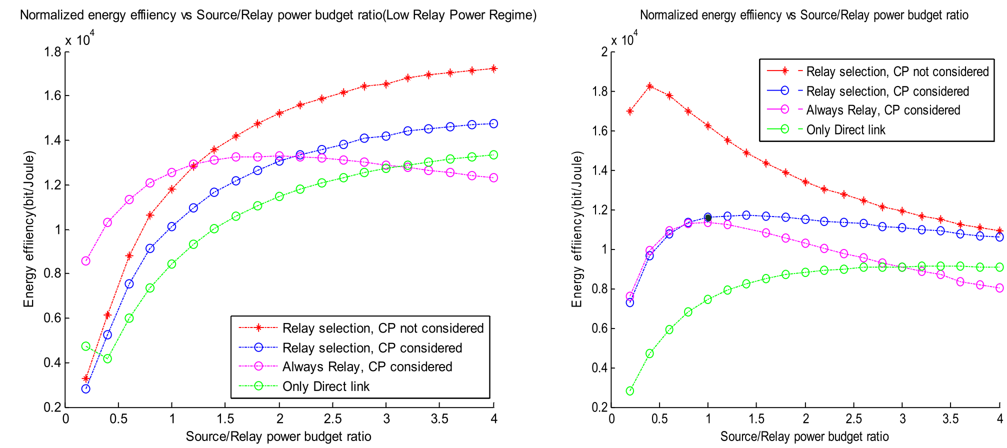

Figure 6. Energy efficiency versus source/relay power budget ratio variation in low (left side) and high (right side) relay power regimes.

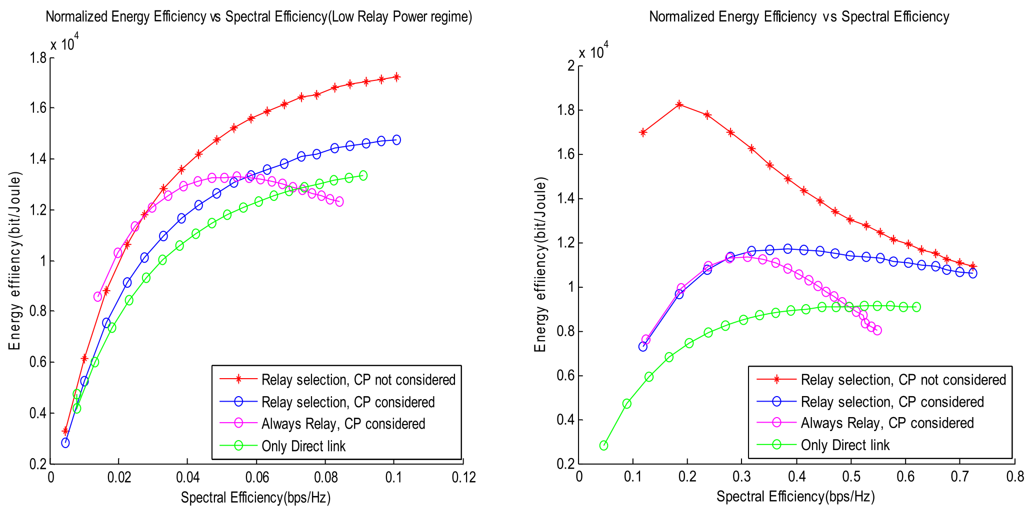

Figure 7. Energy efficiency variation versus spectral efficiency variation in low (left side) and high (right side) relay power regimes.

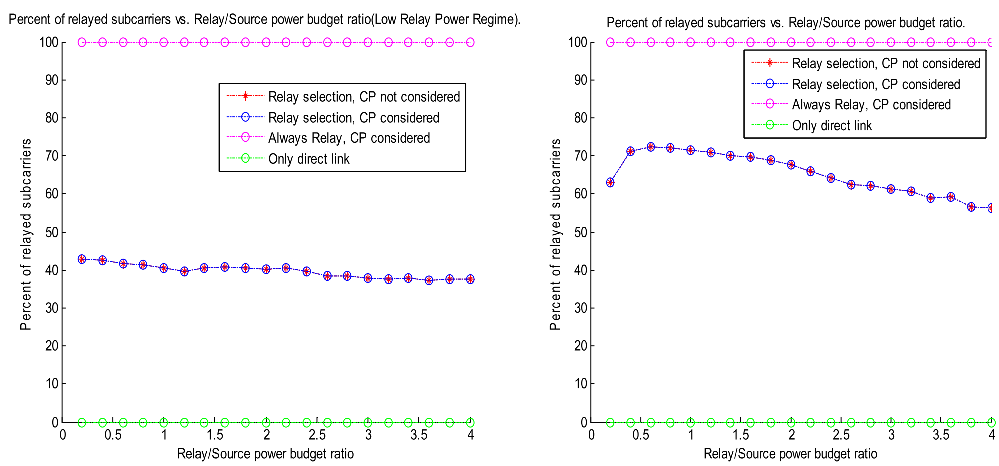

Figure 8. Energy efficiency variation versus spectral efficiency variation in low (left side) and high (right side) relay power regimes.

on the energy efficiency performance in four scenarios [without relay (direct link) [29], always relayed [15] and with relay selection with and without circuit power consideration] is compared in Figure 7. It can be seen that the EE subcarrier selection algorithm has better performance compared with other methods for $SE > 0.05$ and $SE > 0.25$ in low and high relay transmit power regimes, respectively.

5.5. Relaying subcarrier ratio

The impact of source/relay power budget ratio variation (for a given relay power) on the ratio of relayed subcarriers in four scenarios for low (left side) and high (right side) relay transmit power regimes is compared in Figure 8. The ratio of relayed subcarriers with variation on source/relay power budget ratio is almost constant in the low relay transmit power regime. However, in the high relay transmit power regime, this parameter is higher in comparison with the low relay power regime and varies in a concave fashion.

6. CONCLUSION

In this paper, we studied the end-to-end energy efficiency of an OFDM relay-based system under power constraints for the base station and relay nodes with system circuit power considerations. Using a two-phase amplify-and-forward relaying protocol, we assumed that relay has a fixed transmit power constraint and the source transmit power can vary in transmission power interval where the source employs a selective relaying mechanism for each individual subcarrier.

We first formulated the problem as an EE mixed binary integer programming problem and then proposed a heuristic algorithm that is composed of several subproblems to answer the following three questions. What is the EE source transmit power? Which subcarriers are the EE set for relaying? How can one distribute source and relay transmit power between subcarriers in the first and second time slots to maximize the end-to-end energy efficiency?

To answer these questions, we proposed a joint inner-layer and outer-layer optimization scheme consisting of two iterative algorithms where each algorithm uses the results of the other iteratively. The first question is answered in the outer layer, and the other questions are answered in the inner layer. We observed in our simulations that no more than five iterations are required for these two algorithms to converge into the suboptimal solutions.

To evaluate the efficiency of the proposed method, we compared its efficiency through simulations with the cases without using selective relaying and without circuit power consideration. The simulation results indicate that using our proposed method results in significant improvement in energy efficiency for low and high transmit power regimes.

APPENDIX A: APPLYING KKT CONDITIONS

This optimization problem is quasi-convex [objective function is quasi-convex (Appendix B), unequal constraint ($R - C_I \leqslant 0$) is convex (Appendix B) and other constraints are affine], so we can apply convex optimization techniques to calculate and $P_{S,n}$. This optimization problem is formulated as follows:

$$\begin{aligned} &\min -\frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)} \\ &\text{s.t} \\ &R - C_I \leqslant 0, \\ &P_S - \sum_n P_{S,n} = 0, \\ &P_R - \sum_n P_{R,n} = 0 \end{aligned} \tag{A1}$$

We start with the Lagrangian function as

$$\begin{aligned} L(\mathbf{P_R}, \boldsymbol{\lambda}, \upsilon_1) = {} & \frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)} \\ & + \boldsymbol{\lambda}^T (R - C_I) - \upsilon_r (\mathbf{1}^T \mathbf{P_R} - P_R) \end{aligned} \tag{A2}$$

The derivative of the Lagrangian with respect to $P_{R,k}$ is given by

$$\begin{aligned} \frac{\partial L(\mathbf{P_R}, \boldsymbol{\lambda}, \upsilon_1)}{\partial P_{R,k}} = {} & \frac{\frac{\partial C_I}{\partial P_{R,k}}}{\left(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)\right)} \\ & + \lambda_k \frac{\partial C}{\partial P_{R,k}} - \upsilon_r = 0 \end{aligned} \tag{A3}$$

$$\lambda_k = \frac{\upsilon_r - \frac{\frac{\partial C_I}{\partial P_{R,k}}}{(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R))}}{\frac{\partial C}{\partial P_{R,k}}} \tag{A4}$$

from the KKT conditions [23], $\lambda_k \geqslant 0$. Thus, we obtain

$$\upsilon_r \geqslant \frac{\frac{\partial C_I}{\partial P_{R,k}}}{\left(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)\right)} \tag{A5}$$

Another KKT condition is that $\lambda_k P_{R,k} = 0$; that is,

$$\upsilon_r = \frac{\frac{\partial C_I}{\partial P_{R,k}}}{\left(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)\right)} \tag{A6}$$

After some mathematical manipulations, we obtain

$$P_{R,n} = \frac{-(2 + P_{S,n}a_n) + \sqrt{(2 + P_{S,n}a_n)^2 - 4\left(P_{S,n}a_n + 1 - \frac{P_{S,n}b_n a_n}{\nu_r(P_{C,S}+P_{C,R}+\frac{1}{2}(\zeta_S P_S + \zeta_R P_R))\ln(2)}\right)}}{2b_n} \tag{A7}$$

APPENDIX B: ENERGY EFFICIENCY QUASI-CONCAVITY WITH P_S VARIATION

Using the energy efficiency definition according to Equation (6), we have

$$\eta_{EE} = \frac{C_I}{P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)} \tag{B1}$$

to prove η_{EE} concavity with respect to P_S; first, we prove that C_I is concave with respect to P_S. According to Equation (2), we have

$$P_S = P_{S,1} + P_{S,2} + \cdots + P_{S,n},$$

$$\begin{aligned} P_{S,1} &= \beta_1 P_S, \\ P_{S,2} &= \beta_2 P_S, \\ &\ \ \vdots \\ P_{S,n} &= \beta_n P_S \end{aligned} \tag{B2}$$

Using the derivation chain rule, we have

$$\begin{aligned} \frac{\partial C_I}{\partial P_S} &= \frac{\partial C_I}{\partial P_{S,1}}.\frac{\partial P_{S,1}}{\partial P_S} + \frac{\partial C_I}{\partial P_{S,2}}.\frac{\partial P_{S,2}}{\partial P_S} + \cdots + \frac{\partial C_I}{\partial P_{S,n}}.\frac{\partial P_{S,n}}{\partial P_S} \\ &= \frac{\partial C_I}{\partial P_{S,1}}\beta_1 + \frac{\partial C_I}{\partial P_{S,2}}\beta_2 + \cdots + \frac{\partial C_I}{\partial P_{S,n}}\beta_n \end{aligned} \tag{B3}$$

or

$$\frac{\partial C_I}{\partial P_S} = \sum_{n=1}^{N} \frac{\partial C_I}{\partial P_{S,n}}\beta_n$$

similarly,

$$\frac{\partial^2 C_I}{\partial P_S^2} = \sum_{n=1}^{N} \frac{\partial^2 C_I}{\partial P_{S,n}^2}\beta_n$$

Using the theorem proved in [23] that says the sum of the concave function is also concave and the relations available in Equation (B4), we prove concavity with respect to P_S. Concavity with respect to $P_{S,n}$ is proved as follows:

$$\begin{aligned} \frac{\partial C_I}{\partial P_{S,n}} = &\frac{P_{R,n}a_n b_n \mu_n}{2(1 + P_{S,n}a_n)(1 + P_{S,n}a_n + P_{R,n}b_n)\ln(2)} \\ &+ \frac{(1-\mu_n)c_n}{(1 + P_{S,n}c_n)\ln(2)} \end{aligned} \tag{B4}$$

according to Equation (B4), by increasing $P_{S,n}$, $\frac{\partial C_I}{\partial P_{S,n}}$ is decreased. With a second-order derivation, we have

$$\begin{aligned} \frac{\partial^2 C_I}{\partial P_{S,n}^2} = &-\frac{P_{R,n}a_n b_n \mu_n (2a_n(1 + P_{S,n}a_n) + a_n b_n P_{S,n})}{2(1 + P_{S,n}a_n)(1 + P_{S,n}a_n + P_{R,n}b_n)^2\ln(2)} \\ &- \frac{(1-\mu_n)c_n^2}{(1 + P_{S,n}c_n)^2\ln(2)} \end{aligned} \tag{B5}$$

therefore,

$$\frac{\partial^2 C_I}{\partial P_{S,n}^2} < 0 \tag{B6}$$

Therefore, C_I concavity with respect to P_S is proved. Using the C_I concavity, we prove that η_{EE} is quasi-concave with respect to P_S in the sequence as follows:

$$\eta_{EE} = \frac{C_I}{P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_R P_R + \zeta_S P_S)} \tag{B7}$$

denote the super level set of $\eta_{EE}(P_S)$ as

$$S_\alpha = \left\{P_{S,\min} \leqslant P_S \leqslant P_{S,\max} \middle| \eta_{EE}(P_S) \geqslant \alpha\right\} \tag{B8}$$

According to [23], $\eta_{EE}(P_S)$ is quasi-concave in P_S if S_α is convex for any real value of α. Substituting Equation (B7) in Equation (B8), we have

$$\begin{aligned} S_\alpha &= \left\{(P_{S,\min} \leqslant P_S \leqslant P_{S,\max}) \middle| \frac{C_I(P_S)}{P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)} \geqslant \alpha\right\} \\ &= \left\{(P_{S,\min} \leqslant P_S \leqslant P_{S,\max}) \middle| \left(P_{C,S} + P_{C,R} + \frac{1}{2}(\zeta_S P_S + \zeta_R P_R)\right)\alpha - C_I(P_S) \leqslant 0\right\} \end{aligned} \tag{B9}$$

Because $-C_I(P_S)$ is convex, S_α is convex in P_S; therefore, is quasi-concave in P_S.

APPENDIX C: ENERGY EFFICIENCY GRADIENT CALCULATION

Energy-efficient quasi-convexity in P_S was proved (Appendix B). Another question is how we can obtain $\frac{\partial \eta_{EE}}{\partial P_S}$ while we have $\frac{\partial \eta_{EE}}{\partial P_{S,n}}$ for each subcarrier. We start with the energy efficiency definition as follows:

$$\eta_{EE} = \frac{\sum_{n=1}^{N} C_{I,n}}{P_{C,S} + P_{C,R} + \frac{1}{2}\left(\zeta_S P_S + \zeta_R P_R\right)} \tag{C1}$$

where

$$\begin{aligned} C_{I,n} &= \mu_n \frac{1}{2}\log_2(1+\rho_n) + (1-\mu_n)\log_2(1+P_{S,n}c_n) \\ &= \frac{1}{2}\log_2\left(\frac{1+\rho_n}{(1+P_{S,n}c_n)^2}\right)^{\mu_n} + \log_2(1+P_{S,n}c_n) \end{aligned}$$

We know

$$P_S = P_{S,1} + P_{S,2} + \cdots + P_{S,n},$$

$$\begin{aligned} P_{S,1} &= \beta_1 P_S, \\ P_{S,2} &= \beta_2 P_S, \\ &\ \ .\ \ .\ \ . \\ &\ \ .\ \ .\ \ . \\ &\ \ .\ \ .\ \ . \\ P_{S,n} &= \beta_n P_S \end{aligned} \tag{C2}$$

Using the derivation chain rule, we have

$$\begin{aligned} \frac{\partial \eta_{EE}}{\partial P_S} &= \frac{\partial \eta_{EE}}{\partial P_{S,1}}.\frac{\partial P_{S,1}}{\partial P_S} + \frac{\partial \eta_{EE}}{\partial P_{S,2}}.\frac{\partial P_{S,2}}{\partial P_S} + \cdots + \frac{\partial \eta_{EE}}{\partial P_{S,n}}.\frac{\partial P_{S,n}}{\partial P_S} \\ &= \frac{\partial \eta_{EE}}{\partial P_{S,1}}\beta_1 + \frac{\partial \eta_{EE}}{\partial P_{S,2}}\beta_2 + \cdots + \frac{\partial \eta_{EE}}{\partial P_{S,n}}\beta_n \end{aligned} \tag{C3}$$

or

$$\frac{\partial \eta_{EE}}{\partial P_S} = \sum_{n=1}^{N} \frac{\partial \eta_{EE}}{\partial P_{S,n}}\beta_n$$

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