

# Superposition Network Coding for wireless Cooperative Communication

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**Abstract**—Spatial diversity can be exploited by the use of cooperative communication technology. However, most of the well known cooperative protocols have some practical challenges. In this paper we introduce two disadvantages of several important cooperative diversity protocols including low spectral efficiency and requirement for perfect time and frequency synchronization of relay nodes. Then, a cooperative strategy is proposed that improves overall communication performance in comparison with non-cooperative protocols with high spectral efficiency and non synchronization problems. In this technique, we use a combination of superposition coding and network coding to achieve cooperative diversity in asymmetric wireless networks; hence the name superposition network coded cooperation (SNCC). The SNCC scheme is applied to a special but generalizable case of three users. In this case the receiver structure and error performance analysis of SNCC are presented. The results demonstrate performance improvement of wireless communication.

**Index Terms**—Cooperative diversity, Symbol Error Rate Selection Combining (SER-SC), Interference Ignorant Detector (IID), Network coding, Superposition coding.

## I. INTRODUCTION

WIRELESS transmitted signals are subject to large scale propagation effects including path loss, shadowing, and small scale multipath fading [1]. One of the most effective methods to combat the adverse effects of fading is spatial diversity. An approach to achieve spatial diversity is multiple input multiple output (MIMO) technique in which information symbols are transmitted over spatially independent antennas [2]. However, the implementation of MIMO encounters some substantial constraints such as size and complexity limitations [3]. Cooperative diversity is a technique to achieve the advantages of MIMO without the requirement for multiple antenna at each terminal [3]-[4].

Cooperative diversity makes the possibility of forming virtual antenna arrays to increase the reliability of communication. Relay nodes are the fundamental elements of every cooperative diversity protocol. These nodes cooperate with source nodes to transmit their information to destination.

An important application of cooperative communication is its capability to improve the performance of communication in asymmetric networks. In a wireless network, nodes are

distributed in different locations and experience different path-loss and shadowing effects, hence different channel conditions. This makes wireless networks asymmetric in nature. Direct transmission in asymmetric wireless networks results in high aggregate transmit power and uneven power distribution that reduces the network lifetime [6]. Cooperative diversity is a technique to improve the performance of communication in asymmetric networks [6].

Although cooperative communication is a practical solution to improve the performance of wireless systems, it has its own challenges. The performance analysis of multi-relay decode-and-forward (DF) and amplify-and-forward (AF) protocols using conventional repetition coding was studied in [7]-[10]. For these protocols, the relay nodes transmit in orthogonal time slots in the time division multiple access (TDMA) method. This is a low complexity approach. However, it leads to low spectral efficiency due to large time delay [5]. The spectral efficiency in TDMA-based protocols depends on the number of relay nodes. So, these protocols are not efficient for networks with large numbers of relays at all. To improve the system spectral efficiency, simultaneous-based protocols are proposed [11]-[14], in which relay nodes transmit in the same time slot. These strategies make a significant improvement of spectral efficiency at the cost of perfect time and frequency synchronization of the relay nodes [15]-[16]. This requirement for synchronization is very difficult to be satisfied due to the distributed nature of cooperative communication [17].

The above review clarifies the necessity of introducing new frameworks for cooperation in asymmetric networks with high spectral efficiency without synchronization problems. In [16] a new cooperation strategy has been proposed that eliminates synchronization problems with low transmission delay. However, this approach does not improve spectral efficiency at all.

In this paper a new cooperative diversity scheme for multipoint-to-point communication in asymmetric networks is proposed. We assume an asymmetric network that multiple source nodes transmit their information symbols to a common destination by the use of relay nodes. To overcome the synchronization problems, simultaneous transmission of terminals is avoided. Therefore, the TDMA approach is used

for transmission. In this case a technique called superposition coded cooperation (SCC) is used to mitigate the low spectral efficiency. The idea of SCC is based on merging source and relay transmission phases. In this technique a source node appears simultaneously as relay for other nodes in its transmission phase [18]-[22]. One approach for realization of SCC is that the source node assigns a fraction of its power to transmit its own information and the remainder for relaying [18]. This technique is named as superposition modulation. Clearly, a drawback of superposition modulation is its inefficiency when the amount of relay information increases.

In the proposed scheme, conventional superposition modulation is modified by network coding [23], so that it can be used for arbitrary amount of relay information. The idea of SNCC is based on mapping all the relay symbols to one symbol by bit-level XOR [24] and then using the SCC scheme for transmission. In this technique a source node transmits its own information symbol superposed with a relay symbol, by an efficient power assignment, in its transmission phase. The relay symbol may be achieved by bitwise XOR of the information symbols of all the source nodes whose information must be relayed. Reducing the set of relay symbols to one symbol makes superposition modulation efficient. This technique is called superposition network coded cooperation (SNCC). In this paper, SNCC is proposed for a simple but generalizable case. We analyze and simulate symbol error rate (SER) performance of SNCC for three users with M-PSK modulation. The simulation results show that SNCC may be used to achieve an even power distribution in the asymmetric network. Also, in comparison with non-cooperative transmission, SNCC can provide an improvement in the error performance of the first and the second terminals with an arbitrarily small degradation of the third terminal for sufficiently high signal-to-noise ratio (SNR). This issue proves the advantage of SNCC in the error performance improvement in comparison with non-cooperative transmission. Also, the SNCC may be used to achieve an even power distribution in the asymmetric network, that results in extended network life time. All of these advantages are provided by SNCC, while it does not require any extra time or frequency resources in comparison with the non-cooperative scheme.

The rest of this paper is organized as follows. Section II provides a system model. Section III elaborates the analysis behind the SNCC idea. Simulation results are presented in Section IV and finally a conclusion is drawn in Section V.

## II. SYSTEM MODEL

We consider an asymmetric network consisting of  $N$  source nodes denoted by  $U_1, U_2, \dots, U_N$  transmitting information to a common destination in a TDMA manner. Every source node can appear as a half-duplex relay node. Cooperation strategy for each relay is DF. The communication channel between every node and destination is modelled as narrowband Rayleigh fading with additive white Gaussian noise (AWGN)

[1]. The set of network terminals are partitioned into some groups with at most three members. Although we assume three nodes in this paper, our study reveals that the proposed method can be generalized to  $n > 3$  nodes. Grouping criterion is based on true detection. In this manner every terminal must be able to decode the information of other terminals in its group with negligible error.

Here, we consider a three member group. Without loss of generality, terminals are indexed according to their channel quality so that the terminal with lower channel quality has a lower index number. Priority of transmission is also determined according to channel quality, where the terminal with the worse channel quality transmits earlier. So, the terminal  $U_i, i = 1, 2, 3$ , transmits in the  $i$ th time slot.

By applying this transmission order, the transmitted signal from the terminal with worse channel quality may be transferred by the following terminals in its group. Therefore, it is possible for more terminals to detect the information of the terminal with worse channel quality and then transmit over their own channels to destination. Using this scheme, we may achieve higher diversity orders for terminals with poor channel quality, since the information of these terminals is transmitted over more spatially independent channels [6]. The transmitted signal from  $U_i$  is received by the terminal  $U_j, i < j \leq 4$ , where the index  $j = 4$  denotes the destination. The received signal at the terminal  $U_j$  from  $U_i$  can be written as

$$y_{i,j} = h_{i,j}x_i + v_{i,j}, \quad j = i + 1, \dots, 4, \quad (1)$$

where  $h_{i,j}$  is the channel coefficient between the terminals  $i$  and  $j$ , modelled as a circularly symmetric complex Gaussian random variable with mean 0 and variance  $\sigma^2$ ,  $h_{i,j} \sim \mathcal{CN}(0, \sigma^2)$ . According to the grouping criteria we expect a large value of  $\sigma$  that is equal to good channel condition. The coefficient  $h_{i,4} \sim \mathcal{CN}(0, \frac{1}{\lambda_i})$  represents the channel between the  $i$ th terminal and destination. Also,  $v_{i,j}$  is zero mean AWGN with variance  $N_0$ . The signal  $x_i$ , transmitted by the terminal  $U_i$ , is

$$x_i = c_{1,i}s_i + \sqrt{1 - c_{1,i}^2}s_{1,\dots,i-1}, \quad i = 1, 2, 3, \quad (2)$$

where the symbol  $s_i, i = 1, \dots, 3$ , is the source symbol of  $i$ th terminal. The source symbols are chosen independently from an equiprobable M-PSK constellation with average power  $E[|s_i^2|] = a^2$ . The superposition factor  $c_{1,i}$  is

$$c_{1,i} = \begin{cases} 1, & i = 1; \\ c_1, & i \neq 1, \end{cases} \quad (3)$$

where the coefficient  $c_1$  is close (but not equal) to 1. Also, the superposed symbol  $s_{1,\dots,i-1}$  is defined as

$$s_{1,\dots,i-1} = s_1 \oplus s_2 \oplus \dots \oplus s_{i-1}, \quad (4)$$

where  $\oplus$  denotes bit-level XOR between two symbols and then mapping the resulted bit sequence to a symbol from a constellation with the same average power (but not necessarily

TABLE I  
TRANSMISSION SCHEME IN SNCC

Transmission	$U_1$ Transmit $s_1$	$U_2$ Transmit $(s_2, s_1)$	$U_3$ Transmit $(s_3, s_1 \oplus s_2)$
Time slot	1	2	3

the same type) as source symbols. According to (2), the transmitted symbol from the  $i$ th terminal consists of the information of the symbols  $s_1, s_2, \dots, s_i$ .

To summarize, the transmission scheme is presented in Table I. According to the described model of SNCC, this technique requires no extra time and bandwidth resources compared to the TDMA-based non-cooperative scheme. This makes the spectral efficiency higher than other cooperative schemes that are based on TDMA. In addition to high spectral efficiency, SNCC has no synchronization challenges because of the fact that the terminals transmit in orthogonal time slots [16].

### III. ANALYSIS OF SNCC

In this section the receiver structure and SER analysis of SNCC for a three user group are presented. When the transmission phase of terminal  $U_i$  is finished, the next transmitting terminals  $U_j, i < j \leq n$  receive  $y_{i,j}$  and, in the role of relay nodes, detect the source symbol  $s_i$  by the use of an interference ignorant detector (IID) given as [25]

$$\hat{s}_i = \arg \min_{s_i} |y_{i,j} - h_{i,j} c_{1,i} s_i|. \quad (5)$$

According to (5), IID detects the desired symbol  $s_i$  by ignoring the presence of intentional interfering symbol  $\sqrt{1 - c_{1,i}^2} s_{1,\dots,i-1}$ . This issue provides low complexity for detection. However, the performance of IID is highly sensitive to interference. Exceeding the interfering power from a specific threshold makes an error floor in detection [25]. This error floor makes it impossible to achieve arbitrarily low error probability for detection, hence causing high degradation in detection performance [25]. Since the superposed symbol appears as interference, the interference is controllable in SNCC. This means that the superposed symbol can be designed so that IID has efficient performance in detecting the source symbol  $s_i$  from  $y_{i,j}$ .

To this end, we choose the superposed symbol  $s_{1,\dots,i-1}$  from an M-PAM constellation rotated by an angle equal to phase of the source symbol  $s_i$ . In this case, (1) can be written as

$$y_{i,j} = h_{i,j} (c_{1,i} s_i + \sqrt{1 - c_{1,i}^2} e^{j\varphi_{s_i}} s_{1,\dots,i-1}) + v_{i,j}, \quad j = i + 1, \dots, 4, \quad (6)$$

where  $s_i$  and  $s_{1,\dots,i-1}$  are the source symbols in M-PSK modulation and superposed symbol in M-PAM modulation respectively. Also,  $\varphi_{s_i}$  is the phase of the symbol  $s_i$ . In this manner, the transmitted symbols of all the terminals,

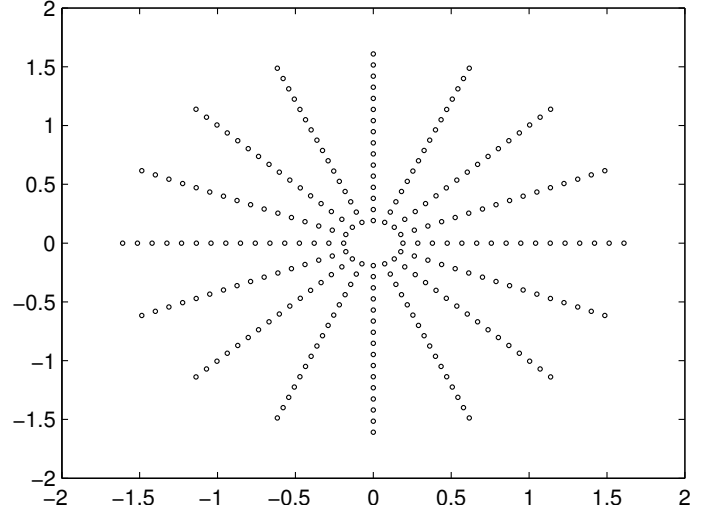


Fig. 1. The constellation points, resulted from superposition network coding for 16-PSK modulation of the source symbols and  $c_1 = 0.9$ .

$U_i, 1 \leq i \leq 3$ , are selected from the set  $\psi$  given as

$$\psi = \{(c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}}) a e^{j \frac{2\pi k}{M}}\}, \quad m, k = 1, 2, \dots, M \quad (7)$$

where  $A_m = 2m - 1 - M$ . As shown in appendix, if we set

$$c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}} > 0, \quad m = 1, 2, \dots, M, \quad (8)$$

which is satisfied by  $\sqrt{\frac{3}{4} \cdot \frac{M-1}{M-5}} < c_{1,i} \leq 1$ , the IID has the same performance as ML detector in detecting the source symbol  $s_i$  from  $y_{i,j}$ . Fig. 1 presents an example of signal points transmitted by the terminals  $U_j, j = 2, \dots, n$  with 16-PSK modulation for the source symbols and  $c_1 = 0.9$ .

The performance of communication in a wireless network depends on the detection method used at the destination. Implementation of optimum receiver for SNCC at the destination is not possible because of high complexity. So, it is necessary to propose a receiver structure with low complexity that exploits the benefits of SNCC. The proposed scheme for detection at destination is based on symbol error rate selection combining (SER-SC) [26] and successive interference cancellation (SIC) [25]. In this approach, the receiver waits for all the signals of a group to be received. According to (6), the received signals at the destination after three time slots are

$$\begin{cases} y_1 = h_1 s_1 + \nu_1, \\ y_2 = h_2 (c_1 s_2 + \sqrt{1 - c_1^2} e^{j\varphi_{s_2}} s_{1,1}) + \nu_2, \\ y_3 = h_3 (c_1 s_3 + \sqrt{1 - c_1^2} e^{j\varphi_{s_3}} s_{1,2}) + \nu_3, \end{cases} \quad (9)$$

where the index  $j = 4$  is removed for simplicity. After that, the destination begins the detection process from the last to the first received signal.

In the first phase, the receiver detects the source symbol  $s_3$  from  $y_3$  by IID. By the use of the nearest neighbor union

bound (NNUB) approximation [27], the instantaneous error probability for detecting the source symbol  $s_3$  from  $y_3$  by IID can be approximately given by [28]

$$P_{es_3} \approx \frac{2}{M} \sum_{m=1}^M Q \left[ u_m \sqrt{2|h_3|^2 \text{SNR}} \right], \quad (10)$$

where  $\text{SNR} = \frac{a^2}{N_0}$  and

$$u_m = \left( c_1 + A_m \cdot \sqrt{\frac{3(1-c_1^2)}{M^2-1}} \right) \sin\left(\frac{\pi}{M}\right). \quad (11)$$

By averaging (10) with respect to exponential random variable  $|h_3|^2 \sim \exp(\lambda_3)$ , the approximate average SER is given by

$$P_{es_3,avg} \approx \frac{2}{M} \sum_{m=1}^M \left( \frac{1}{2} - \frac{1}{2} \sqrt{\frac{u_m^2 \text{SNR}}{u_m^2 \text{SNR} + \lambda_3}} \right). \quad (12)$$

In high SNR, by the use of two first terms of the Taylor expansion, (12) can be approximated as

$$P_{es_3,avg} \approx \frac{1}{2M\text{SNR}} \sum_{m=1}^M \left( \frac{\lambda_3}{u_m^2} \right). \quad (13)$$

As we can see from (13), the SER is a function of  $\text{SNR}^{-1}$ , so the diversity order is equal to 1.

After detecting the symbol  $s_3$ , the receiver tries to remove the effect of source symbol  $s_3$  from  $y_3$  through the detected symbol  $\hat{s}_3$  as

$$\hat{y}_3 = e^{-j\varphi_{\hat{s}_3}} (y_3 - c_1 h_3 \hat{s}_3), \quad (14)$$

where  $\varphi_{\hat{s}_3}$  is the phase of detected symbol  $\hat{s}_3$ .

In the second phase, the receiver detects the source symbol  $s_2$ . The information of this symbol is placed in signals  $y_2$  and  $\hat{y}_3$ . Hence, it is possible for the receiver to gain the advantages of diversity in detection of  $s_2$ . However, the symbol  $s_2$  does not explicitly exist in  $\hat{y}_3$  due to bit level XOR. This issue makes it impossible to utilize a conventional maximal ratio combiner (MRC). Our approach to achieve spatial diversity is based on SER-SC. In this technique, the receiver selects a path among all received paths with minimum instantaneous SER and detects a desired symbol from it. If the selected signal is  $y_2$ , then the symbol  $s_2$  is directly detected by IID. Otherwise, the IID detects the superposed symbol  $s_{1,2}$  from  $\hat{y}_3$  and the symbol  $s_1$  from  $y_1$  and then calculates their bit level XOR. Clearly, in this case the resulted symbol will be  $s_2$  if the symbols  $s_{1,2}$  and  $s_1$  are detected correctly. The decision making in this phase is given as

$$\hat{s}_2 = \begin{cases} \hat{s}_2(y_1, \hat{y}_3), & P_{c_1} \cdot P_{c_3} > P_{c_2}, \\ \hat{s}_2(y_2), & \text{otherwise.} \end{cases} \quad (15)$$

where  $\hat{s}_2(y_2)$  and  $\hat{s}_2(y_1, \hat{y}_3)$  represent the detected symbol  $s_2$  from  $y_2$  and the pair  $(y_1, \hat{y}_3)$  respectively. Also  $P_{c_1}$ ,  $P_{c_2}$ , and  $P_{c_3}$  are the instantaneous probability of correct detection of the symbols  $s_1$  from  $y_1$ ,  $s_2$  from  $y_2$ , and  $s_{1,2}$  from  $\hat{y}_3$  respectively. According to (15), the instantaneous error probability in this

phase can be written as

$$P_{e_{s_2}} = \begin{cases} 1 - P_{c_1} \cdot P_{c_3}, & P_{c_1} \cdot P_{c_3} > P_{c_2}, \\ 1 - P_{c_2}, & \text{otherwise.} \end{cases} \quad (16)$$

By the use of Bayes rule we have

$$\begin{aligned} P_{c_3} &= \frac{1 - P_{e_{s_{1,2}|cs_3}} - P_{e_{s_3}} \times}{(P_{e_{s_{1,2}|es_3}} - P_{e_{s_{1,2}|cs_3}})} \\ &\approx \frac{1 - P_{e_{s_{1,2}|cs_3}}}{1 - 2Q\left(\sqrt{\frac{6(1-c_1^2)}{M^2-1}}|h_3|^2 \text{SNR}\right)}, \end{aligned} \quad (17)$$

where  $P_{\cdot|}$  denotes the conditioned instantaneous error probability and the notations  $cs_i$  and  $es_i$  represent correct and incorrect detection of  $s_i$  respectively. The first approximation in (17) is valid due to the fact that  $P_{e_{s_3}}$  is considerably less than  $P_{e_{s_{1,2}|cs_3}}$  for  $c_1$  near to 1. The second approximation is achieved by NNUB approximation. Also, by the use of NNUB we have

$$P_{c_1} \approx 1 - 2Q\left(\sin\left(\frac{\pi}{M}\right)\sqrt{2|h_1|^2 \text{SNR}}\right), \quad (18)$$

and

$$P_{c_2} \approx 1 - \frac{2}{M} \sum_{m=1}^M Q\left[u_m \sqrt{2|h_2|^2 \text{SNR}}\right], \quad (19)$$

respectively, where the term  $u_m$  is given by (11). To get the average error probability of detecting the symbol  $s_2$ , we have to average (16) over the joint probability density function (PDF) of  $P_{c_1}$ ,  $P_{c_2}$ , and  $P_{c_3}$ . To the best of our knowledge, there is not a known expression for the PDF of these random functions. This makes the analytical averaging of (16) impossible. Hence, we try to give an approximate term for average error probability of  $s_2$ , by approximating the random functions  $P_{c_1} \cdot P_{c_3}$  and  $P_{c_2}$  with the simpler random functions  $Y_1$  and  $Y_2$  respectively. To achieve sufficiently good approximations, two constraints on functions  $Y_1$  and  $Y_2$  are considered. Firstly is to assume approximately equal expectation values, so that for high SNR

$$\begin{aligned} E[Y_1] &\approx E[P_{c_1} \cdot P_{c_3}], \\ E[Y_2] &\approx E[P_{c_2}], \end{aligned} \quad (20)$$

where  $E[\cdot]$  denotes the expected value. Secondly  $Y_1$  and  $Y_2$  have similar properties to that of functions  $P_{c_1} \cdot P_{c_3}$  and  $P_{c_2}$  respectively. According to (17)-(19), these random functions are composed of the Gaussian Q-function of exponential random variables. So, they are both monotonically decreasing functions, varying in the range (0,1). In order to satisfy the second constraint, we apply above properties to choose  $Y_1$  and  $Y_2$ . This leads to the approximated functions as

$$\begin{aligned} Y_1 &= 1 - 2Q(\sqrt{|h|^2 \text{SNR}}), \\ Y_2 &= 1 - 2Q(\sqrt{2\alpha \sin^2\left(\frac{\pi}{M}\right)|h_2|^2 \text{SNR}}), \end{aligned} \quad (21)$$

where  $|h|^2$  is an exponential random variable with mean  $\frac{1}{\lambda}$  and the coefficient  $\alpha$  is a constant. With some simple computations

we find that (20) will be satisfied if

$$\lambda = \frac{(M^2 - 1)\lambda_3}{6(1 - c_1^2)} + \frac{\lambda_1}{2\sin^2(\frac{\pi}{M})} \quad (22)$$

and

$$\alpha = \frac{M}{\sum_{m=1}^M \frac{1}{\left(c_1 + A_m \sqrt{\frac{3(1-c_1^2)}{M^2-1}}\right)^2}} \quad (23)$$

By substituting (21) into (16), we have

$$P_{e_{s_2}} \approx \begin{cases} 2Q(\sqrt{|h|^2\text{SNR}}), & 2\alpha \sin^2(\frac{\pi}{M})|h_2|^2 < |h|^2, \\ 2Q(\sqrt{2\alpha \sin^2(\frac{\pi}{M})|h_2|^2\text{SNR}}), & \text{otherwise.} \end{cases} \quad (24)$$

where the intervals are simplified due to a monotonically decreasing manner of the Q-function. We can simplify (24) as

$$P_{e_{s_2}} \approx \begin{cases} 2Q(\sqrt{X_1}), & \text{if } X_1 > X_2, \\ 2Q(\sqrt{X_2}), & \text{otherwise.} \end{cases} \quad (25)$$

where  $X_1$  and  $X_2$  are exponential random variables defined as

$$X_1 \triangleq |h|^2\text{SNR} \sim \exp\left(\frac{\lambda}{\text{SNR}}\right),$$

$$X_2 \triangleq 2\alpha \sin^2\left(\frac{\pi}{M}\right)|h_2|^2\text{SNR} \sim \exp\left(\frac{\lambda_2}{2\alpha \sin^2\left(\frac{\pi}{M}\right)\text{SNR}}\right) \quad (26)$$

By averaging (25) over the joint PDF of independent random variables  $X_1$  and  $X_2$  given by (26), the average SER for detection of  $s_2$  is approximately equal to

$$P_{e_{s_2},avg} \approx 1 - \frac{1}{\sqrt{1+2\lambda_3}} + \frac{1}{\sqrt{1+2\lambda_3+2\lambda_4}} - \frac{1}{\sqrt{1+2\lambda_4}}, \quad (27)$$

where  $\lambda_3 = \frac{\lambda}{\text{SNR}}$  and  $\lambda_4 = \frac{\lambda_2}{2\alpha \sin^2(\frac{\pi}{M})\text{SNR}}$ . By the use of Taylor expansion, (27) at high SNR can be expressed approximately as

$$P_{e_{s_2},avg} \approx \frac{3\lambda\lambda_2}{2\alpha \sin^2\left(\frac{\pi}{M}\right)\text{SNR}^2} \quad (28)$$

varying  $P_{e_{s_2},avg}$  as a function of  $\text{SNR}^{-2}$  shows that the diversity gain is equal to 2 in detection of  $s_2$ .

After detecting  $s_2$ , its effect is removed from  $y_2$  with the same manner as (14), to create  $\hat{y}_2$ . Then the detection phase of  $s_1$  is started. In this phase, the receiver has three paths  $y_1$ ,  $\hat{y}_2$ , and  $(y_2, \hat{y}_3)$  for detection. Detection from  $y_1$  or  $\hat{y}_2$  is performed directly by IID. On the other hand, if the signal  $\hat{y}_3$  is selected, firstly the symbol  $s_{1,2}$  is detected by IID and secondly the symbol  $s_1$  is extracted by bit level XOR of the detected symbol  $\hat{s}_{1,2}$  and the symbol  $\hat{s}_2$ , detected in the second phase according to (15). So, the decision approach is given by

$$\hat{s}_1 = \begin{cases} \hat{s}_1(y_1), & \text{if } P_{c_1} > P_{c_4}, P_{c_2} \cdot P_{c_3}, \\ \hat{s}_1(\hat{y}_2), & \text{if } P_{c_4} > P_{c_1}, P_{c_2} \cdot P_{c_3}, \\ \hat{s}_1(y_2, \hat{y}_3), & \text{otherwise.}, \end{cases} \quad (29)$$

where  $\hat{s}_1(y_1)$ ,  $\hat{s}_1(\hat{y}_2)$ , and  $\hat{s}_1(y_2, \hat{y}_3)$  represent detected symbol  $s_1$  from  $y_1$ ,  $\hat{y}_2$ , and the pair  $(y_2, \hat{y}_3)$  respectively. Also,  $P_{c_4}$

is the instantaneous probability of correct detection of symbol  $s_{1,1}$  from  $\hat{y}_2$  and is approximated with the same manner as (17) by replacing  $h_3$  with  $h_2$ . In this phase the paths  $\hat{y}_2$  and  $(y_2, \hat{y}_3)$  are not independent. Therefore we expect a diversity gain of less than 3. In this case we do not provide analytical expression for average error probability due to its difficulty and suffice to simulation results.

#### IV. SIMULATION RESULTS

In this section computer simulations for SER performance of SNCC scheme in a three user group are presented. The SNCC scheme requires the same bandwidth and time slots as the non-cooperative scheme. Therefore, it is fair to compare the SER performance of SNCC with that of non-cooperative scheme.

Fig. 2 shows SER performance versus SNR for the terminals with  $\lambda_1 = 1$ ,  $\lambda_2 = .1$  and  $\lambda_3 = .05$  and 4-PSK modulation for the source symbols. We choose the superposition factor  $c_1$  equal to 0.93. The excellent agreement between simulation results and analytical results demonstrates the accuracy of our analysis for SNCC. In this figure, we also compare SNCC performance with repetition based cooperative scheme with the same overall power consumption as SNCC, as shown in table II. In this case, the required time slots for a group of three users is two times larger than that of SNCC. By comparing SER curves in SNCC scheme with the same curves in non-cooperative scheme we find that the SNCC scheme may improve the performance of the first and the second terminals at the cost of a performance degradation of the third terminal. On the other hand, by comparing the decaying slope of the curves we can infer that the first and the second terminals exploit a higher diversity order in the SNCC scheme than the non-cooperative scheme, while the diversity order of the third terminal is not changed. Therefore, the SNCC can provide significant improvement for the first and the second terminals at the cost of an arbitrarily small degradation of the third terminal for a sufficiently large SNR. This is the major advantage of SNCC compared to non-cooperative communication. We note that this important advantage is achievable without any extra resource requirement compared to the TDMA-based non-cooperative scheme and may be generalized to a group with an arbitrary size. The degradation in the SER performance of the third terminal can be made arbitrarily small by choosing the superposition factor  $c_1$  sufficiently close to one. However, as the coefficient  $c_1$  gets closer to one, the performance of two other terminals degrade for a specific SNR. This exhibits a trade-off between the performance degradation of the third terminal and the improvement of the others. Therefore, it is important to choose the factor  $c_1$  in an efficient manner to gain an acceptable performance improvement for the first and the second terminals without significant degradation in the performance of the third terminal for a specific SNR.

A criteria for choosing the coefficient  $c_1$  can be based on achieving even power distribution in the network, which

TABLE II  
 TRANSMISSION SCHEME IN REPETITION CODING WITH THE SAME POWER CONSUMPTION AS SNCC

Transmission	$U_1$ Transmit	$U_2$ Transmit	$U_3$ Transmit	$U_3$ Transmit	$U_3$ Transmit	$U_3$ Transmit
	$s_1$	$c_1 s_2$	$\sqrt{1-c_1^2} s_1$	$c_1 s_3$	$\sqrt{\frac{1-c_1^2}{2}} s_2$	$\sqrt{\frac{1-c_1^2}{2}} s_1$
Time slot	1	2	3	4	5	6

results in increasing the network lifetime. In this case we choose  $c_1$  to have approximately the same transmitting power for all the terminals in a specific SER. Fig. 3 shows SER performance for a three-user group with 16-PSK modulation of the source symbols and  $\lambda_1 = 0.5$ ,  $\lambda_2 = 0.1$  and  $\lambda_3 = 0.05$ . We can see from Fig. 3 that the SNR of the terminals in non-cooperative scheme is significantly different in order to achieve a SER about 0.005. However, by choosing  $c_1 = 0.95$  in SNCC scheme, all the terminals will have approximately the same SER performance (about 0.005), in the same SNR (22.5 dB).

Fig. 4 depicts SER performance of the terminals in the first example for various amounts of  $c_1$  in SNR=15 dB. As shown in this figure, SNCC scheme makes significant performance improvement for the first and the second terminals at the cost of a negligible degradation of the third terminal for the superposition factors close to one ( $.96 < c_1 < 1$ ). Therefore, these values can be selected if we need very small degradation of the third terminal.

A special case of SNCC occurs when all the terminals in a group have the same or close channel variances. This is the worst case for utilizing the benefits of SNCC, because the information of every terminal except the last one is repeated in all of the following transmitting terminals that do not have a better channel condition. In addition, due to the similar channel condition of all the terminals, there is no preference for the transmission order of the terminals. In this case, in order to achieve a more beneficial and fair scheme, we can change the transmission priority in consecutive cooperation periods. Fig. 5 and 6 present SER performance versus SNR and the coefficient  $c_1$  for the terminals in a three-user group with 4-PSK modulation of the source symbols,  $\lambda_1 = \lambda_2 = \lambda_3 = 1$  and  $c_1 = 0.94$ . These figures reveal the advantages of SNCC in comparison with direct transmission scheme, even for the worst case of channel conditions.

### V. CONCLUSION

In this paper, a novel framework for cooperation was proposed to improve the reliability of communication with high spectral efficiency and non synchronization problems. The new scheme was based on a combination of superposition coding and network coding. So, it was named as SNCC. The SNCC scheme was applied to a three-user group. Receiver structure, SER analysis, and simulation results for SNCC were presented. The results showed that in comparison with non-cooperative communication, the SNCC can provide significant

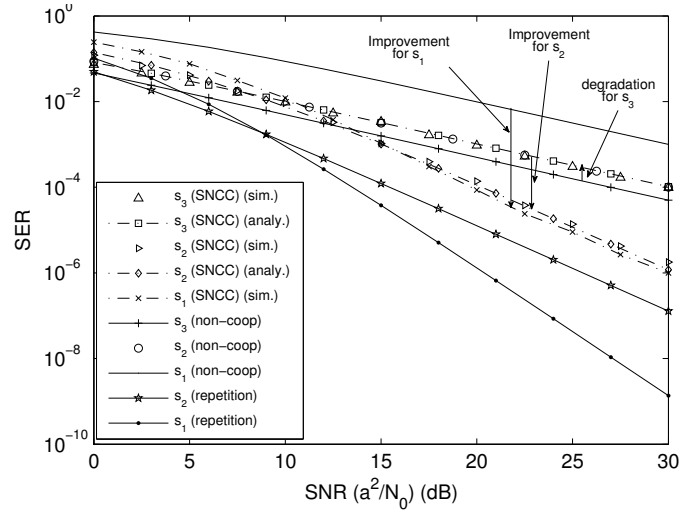


Fig. 2. SER performance of SNCC for 4-PSK modulation,  $c_1 = .93$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = .1$ ,  $\lambda_3 = .05$ .

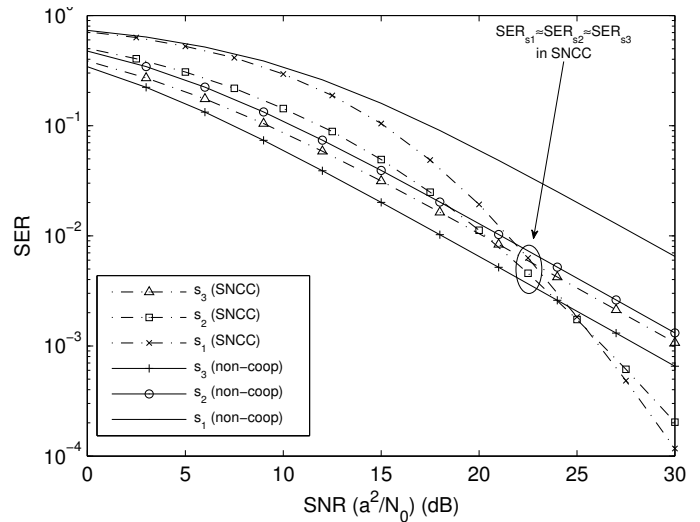


Fig. 3. SER performance of SNCC for 16-PSK modulation,  $c_1 = .95$ ,  $\lambda_1 = .5$ ,  $\lambda_2 = .1$ ,  $\lambda_3 = .05$ .

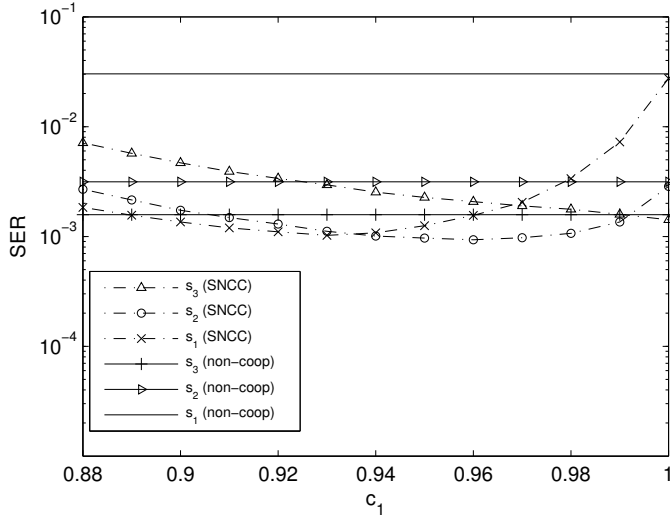


Fig. 4. SER performance versus superposition factor for 4-PSK modulation,  $\lambda_1 = 1, \lambda_2 = .1, \lambda_3 = .05, \text{SNR} = 15\text{dB}$ .

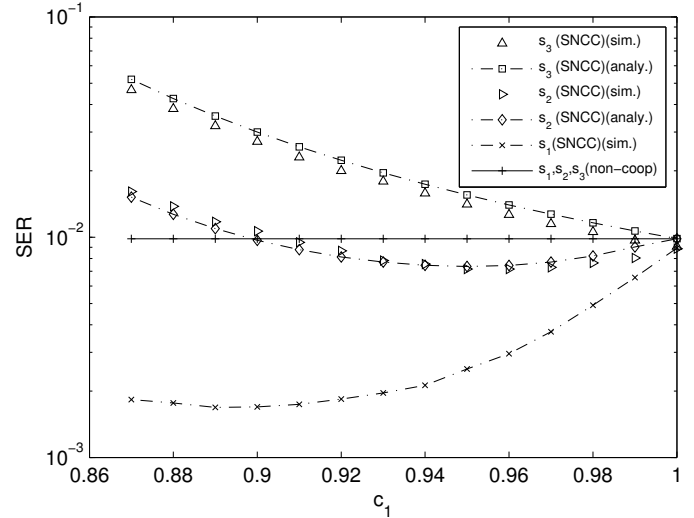


Fig. 6. SER performance versus superposition factor  $c_1$ , for 4-PSK modulation of the source symbols,  $\lambda_1 = \lambda_2 = \lambda_3 = 1$ , and  $\text{SNR} = 15\text{dB}$ .

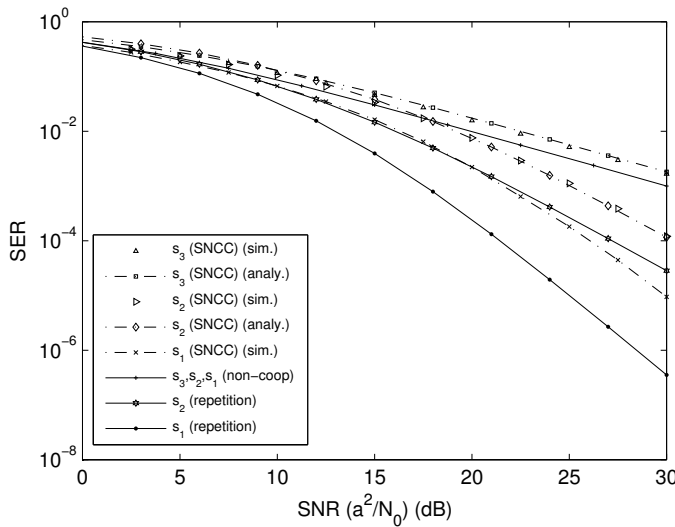


Fig. 5. SER performance of SNCC versus SNR, for 4-PSK modulation of the source symbols,  $c_1 = 0.94, \lambda_1 = \lambda_2 = \lambda_3 = 1$ , and  $\text{SNR} = 15\text{dB}$ .

improvement for the first and the second terminals at the cost of an arbitrarily small degradation of the third terminal for a sufficiently large SNR. This advantage is provided by SNCC without any extra transmission resources compared to the non-cooperative scheme.

## VI. APPENDIX I

In this part we show that if the condition in (8) is satisfied then IID in detecting the source symbol  $s_i$  from  $y_{i,j}$  in (6) has the same performance as optimal ML detector. According to (5), the decision area for IID to detect the  $k$ th constellation point of  $s_i$  is presented at the top of next page, where the second equality is valid if (8) is satisfied. Also the right hand

side expression in the third equality is the decision area for an optimal ML detector to detect  $k$ th constellation point of  $s_i$  from  $y_{i,j}$  [25].

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$$\begin{aligned}
 r_k &= \left\{ r : |r - h_i c_{1,i} a e^{j \frac{2\pi}{M} k}| < |r - h_i c_{1,i} a e^{j \frac{2\pi}{M} z}| \quad \forall z \neq k \right\} \\
 &= \left\{ r : |r - h_i a e^{j \frac{2\pi}{M} k} (c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}})| < |r - h_i a e^{j \frac{2\pi}{M} z} (c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}})| \quad \forall z \neq k, \forall A_m \right\} \\
 &= \left\{ r : \sum_m e^{-\frac{|r - h_i a e^{j \frac{2\pi}{M} k} (c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}})|^2}{N_0}} > \sum_m e^{-\frac{|r - h_i a e^{j \frac{2\pi}{M} z} (c_{1,i} + A_m \sqrt{\frac{3(1-c_{1,i}^2)}{M^2-1}})|^2}{N_0}} \quad \forall z \neq k \right\} \quad (30)
 \end{aligned}$$

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