PAPR REDUCTION IN OFDM SYSTEMS USING POLYNOMIAL-BASED COMPRESSING AND ITERATIVE EXPANDING

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ABSTRACT

In this paper we propose a novel algorithm for PAPR reduction of an OFDM system, based on a companding scheme. In this method a compressing polynomial is appended to the IFFT block at the transmitter and at the receiver the FFT block is combined with a reverse expanding function where the iterative Jacobi's method is used for solving equations. The proposed method entails less complexity at the transmitter in comparison with other PAPR reduction algorithms. It also requires less increase in SNR for the same BER compared to other companding methods. A trade off between complexity and performance can set the order of compressing polynomial and the number of iterations for the proposed algorithm at the receiver.

1. INTRODUCTION

Recently, Orthogonal Frequency Division Multiplexing (OFDM) signaling has gained considerable interest for high data rate transmission applications, because of its high spectral efficiency and the immunity to frequency selective channels [1]. One major drawback of OFDM is the high peak-to-average power ratio (PAPR) of the output signal. Transmitting a signal with high PAPR requires highly linear power amplifiers with a large back-off to avoid adjacent channel interference due to nonlinear effects [2]. Also high values of PAPR result in low efficient usage of the ADC and DAC word length at the Analog Front Ends (AFE) of the transceiver. With a limited number of ADC/DAC bits the designer has to decide about clipping the peaks, which has a deteriorating effect on OFDM signals, or burying the small variations of the signal in the quantization noise. Therefore, dynamic range reduction plays an important role for the application of OFDM signals in both power and band-limited communication systems.

Many PAPR reduction techniques have been proposed in the literature, each with certain advantages and drawbacks. The simplest one is to clip the peak amplitude of the OFDM signal to some desired maximum level but this technique will cause an unacceptable level of noise and out of band distortion in the OFDM signal [3][4].

Other methods focus on the frequency domain, by shaping the signal constellation. Two recently introduced methods are the Partial Transmit Sequence (PTS) [5] and Selected Mapping (SLM) [6]. A drawback of these methods is high computational cost at the transmitter and extra information sent to the receiver. Another effective method is companding that reduces the PAPR with low complexity at the cost of a loss in SNR [7] [8]. Since all the companding techniques are sensitive to channel noise, due to the nonlinear processing, more PAPR reduction could lead to lower performance.

In this paper, the proposed companding method tries to reduce the PAPR with a polynomial-based compressing function in the transmitter. Also using an iterative technique in the receiver, the increase in SNR of the system for a given BER is lowered in comparison with other companding methods. In section 2, the signal model and the PAPR problem is explained. Section 3 introduces a novel method for companding the time domain signal. The simulation results and conclusion are given in section 4.

2. OFDM SIGNAL MODEL AND PAPR PROBLEM

Fig. 1 shows the system model considered in this paper. In an OFDM system, the input bit stream is mapped to a QAM constellation space and then the stream of weighted carriers $X[k]$ is fed to the IFFT block as explained bellow.

Assume N is the length of $X[k]$ in the frequency domain. The output of the IFFT $x[n]$ in the time domain can be written as:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{\,j2\pi k n/N}, \quad 0 \le n < N \qquad (1)$$

The resulting time domain signal that possesses a high PAPR can be compressed by a compression block. In our proposed method, this block is appended to the IFFT block.

The PAPR is a figure of merit that describes the dynamic range of the OFDM time signal. The conventional definition of the PAPR for the OFDM symbol in the time domain $x[n]$ may be expressed as:

$$PAPR(x[n]) = \frac{\max|x[n]|^2}{E[|x[n]|^2]} \qquad (2)$$

where E denotes the expectation operator.

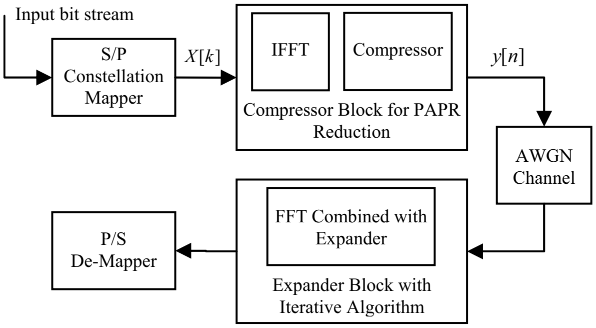

Fig. 1. The considered system block diagram.

3. THE PROPOSED METHOD

In this section a new companding method is proposed based on appending a compressor block to the IFFT at the transmitter and combining an expander with the FFT block in the receiver to come up with new extended blocks. At the transmitter the signal is compressed by a compressor block that scales the signal based on a nonlinear function that amplifies the signal average and keeps the peak constant. At the receiver, the signal expansion is performed through an iterative technique following the FFT operation. In the following, we will explain the new transmitter and receiver units respectively.

3.1. Appending a compressor to the IFFT block

The applied method at the transmitter is based on using a special family of compressing polynomials. It will be shown that this operation will effectively reduce the PAPR, and the order of the polynomial function can be chosen according to a performance-complexity trade off.

The compression is performed on a normalized signal using a nonlinear polynomial function at the transmitter, which reduces the PAPR by increasing the average signal energy while keeping the peak constant. In order to employ the iterative algorithm successfully at the receiver, the function must be selected carefully. Based on the required characteristics for the curve, there are some restrictions in choosing the coefficients of the polynomial. These requirements are as follows:

1. In order to have a one-to-one mapping applied to the signal, the function must be invertible in the range of [-1,1]. This allows for the inverse operation in the above range at the receiver. Obviously, it has to be an increasing function and its extreme points (where the sign of derivative changes) must be out of this range.
2. The function must be odd; therefore the terms with even degrees should take zero coefficients.
3. The function must take its minimum value at the point (-1,-1) and its maximum value at the point (1,1) in the mentioned range and pass through the origin.
4. For the best efficiency in PAPR reduction, it is necessary to have the steepest possible slope at the origin and the lowest slope (ideally zero) at the extreme points. It is shown in Fig. 2 that higher order polynomials results in sharper slopes at point (0,0).
5. Based on simulation results, the existence of a turning point (where the concavity changes) within the range of the function is an important limiting factor. Presence of a turning point increases the required SNR for a given BER performance. Therefore, the function within the range of [-1,1] must not have any turning point except for the origin.

The above requirements can be satisfied by any polynomial function with an odd order. Let's define:

$$f(x) = a_p x^p + a_{p-2} x^{p-2} + \ldots + a_5 x^5 + a_3 x^3 + a_1 x \quad (3)$$

where p is an odd number. Coefficients for polynomials of order 3, 5 and 7, are obtained as follows.

$$\begin{aligned} p = 3 &: a_3 = -1/2,\ a_1 = 3/2 \\ p = 5 &: a_5 = 3/8,\ a_3 = -5/4,\ a_1 = 15/8 \\ p = 7 &: a_7 = -5/16,\ a_5 = 21/16,\ a_3 = -35/16,\ a_1 = 35/16 \end{aligned} \quad (4)$$

These functions are depicted in Fig. 2. One should note that for polynomials of higher orders the coefficients cannot be directly obtained in the same manner, due to the lack of known parameters compared to the unknown ones.

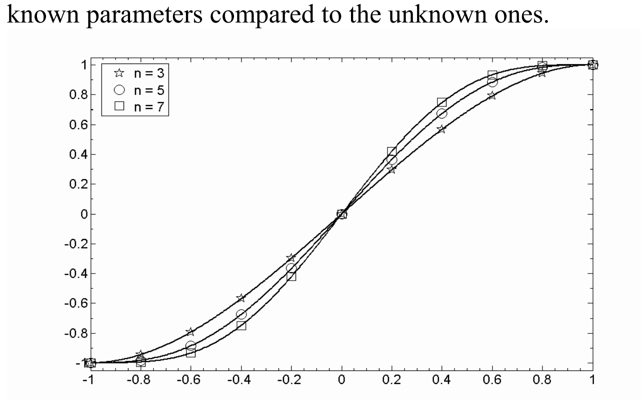

Fig. 2. Calculated polynomials of order 3, 5, 7.

To append the polynomial-based compressor to the IFFT, $x[n]$ should be computed as a real and imaginary part. So:

$$\mathrm{Re}\{x[n]\} = \frac{1}{N}\sum_{k=0}^{N-1}\left(A_r[n,k]\right) \quad (5a)$$

$$\mathrm{Im}\{x[n]\} = \frac{1}{N}\sum_{k=0}^{N-1}\left(A_i[n,k]\right) \quad (5b)$$

where

$$A_r[n,k] = \mathrm{Re}\{X[k]\}Cos(\frac{2\pi kn}{N}) - \mathrm{Im}\{X[k]\}Sin(\frac{2\pi kn}{N}) \quad (6a)$$

$$A_i[n,k] = \mathrm{Im}\{X[k]\}Cos(\frac{2\pi kn}{N}) + \mathrm{Re}\{X[k]\}Sin(\frac{2\pi kn}{N}) \quad (6b)$$

After applying the pth order compressing function on $\mathrm{Re}\{x[n]\}$ and $\mathrm{Im}\{x[n]\}$, the results can be shown respectively as:

$$\mathrm{Re}\{y[n]\} = a_p\left(\frac{1}{N}\sum_{k=0}^{N-1} A_r[n,k]\right)^p + \cdots + a_1\left(\frac{1}{N}\sum_{k=0}^{N-1} A_r[n,k]\right) \quad (7a)$$

$$\mathrm{Im}\{y[n]\} = a_p\left(\frac{1}{N}\sum_{k=0}^{N-1} A_i[n,k]\right)^p + \cdots + a_1\left(\frac{1}{N}\sum_{k=0}^{N-1} A_i[n,k]\right) \quad (7b)$$

where $y[n]$ represents the compressed signal. Fig. 3 shows the constellation shape of the original and the compressed signal.

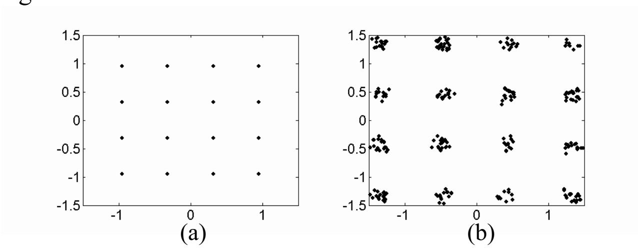

Fig. 3. The signal constellation for (a) the original signal and (b) its compressed version in the frequency domain.

The following subsection, introduces an iterative method for the combined Expansion/FFT blocks at the receiver over a noiseless channel and then the effect of AWGN channel on the signal will be studied.

3.2. Expansion via an iterative algorithm

The expander block, has to invert what has been applied to the signal at the transmitter. The problem is to find the data sequence $X[k]$ with length N in the frequency domain using the compressed time signal $y[n]$ with the same length. As explained before, the appended block at the transmitter relates $y[n]$ to $X[k]$ using a set of nonlinear equations, through which the unknown values can be computed. There are N known values of $\mathrm{Re}\{y[n]\}$ and $\mathrm{Im}\{y[n]\}$ and N unknown values of $\mathrm{Re}\{X[k]\}$ and $\mathrm{Im}\{X[k]\}$ with $n = 0,\ldots, N-1$ and $k = 0,\ldots, N-1$.

To solve the set of N equations, an iterative method is proposed to obtain the X values. The Jacobi Method is employed here for this purpose [9].

In Jacobi's method, initial values for $X[k]$ are needed for the first iteration. Referring to Fig. 3, it can be seen heuristically, that the constellation shape of the compressed signal has a good resemblance to that of the original signal. So in the first place use of the FFT block, can yield a good approximation of the original signal in the frequency domain. Thus the FFT of $y[n]$ can provide proper initial values for the iterative algorithm.

We start with computing $A_r[n,k]$ and $A_i[n,k]$ by using equation 6. Let's define:

$$\mathrm{Re}\{\hat{x}[n]\} = a_p\left(\frac{1}{N}\sum_{k=0}^{N-1} A_r[n,k]\right)^p + \cdots + a_1\left(\frac{1}{N}\sum_{k=0}^{N-1} A_r[n,k]\right) \quad (8a)$$

$$\mathrm{Im}\{\hat{x}[n]\} = a_p\left(\frac{1}{N}\sum_{k=0}^{N-1} A_i[n,k]\right)^p + \cdots + a_1\left(\frac{1}{N}\sum_{k=0}^{N-1} A_i[n,k]\right) \quad (8b)$$

and

$$B_r[n,k] = \frac{N}{a_1}\left(\mathrm{Re}\{\, y[n]\} - \left(\mathrm{Re}\{\,\hat{x}[n]\} - \frac{a_1}{N}A_r[n,k]\right)\right) \quad (9a)$$

$$B_i[n,k] = \frac{N}{a_1}\left(\mathrm{Im}\{\, y[n]\} - \left(\mathrm{Im}\{\,\hat{x}[n]\} - \frac{a_1}{N}A_i[n,k]\right)\right) \quad (9b)$$

Next, we compute $\hat{X}[n,k]$ as follows:

$$\mathrm{Re}\{\hat{X}[n,k]\} = B_r[n,k]Cos(\frac{2\pi kn}{N}) + B_i[n,k]Sin(\frac{2\pi kn}{N}) \quad (10a)$$

$$\mathrm{Im}\{\hat{X}[n,k]\} = B_i[n,k]Cos(\frac{2\pi kn}{N}) - B_r[n,k]Sin(\frac{2\pi kn}{N}) \quad (10b)$$

We can observe from equation 10 that for any value of k, there will be N results for $\mathrm{Re}\{\hat{X}[n,k]\}$ and $\mathrm{Im}\{\hat{X}[n,k]\}$. From the results obtained through simulations it is deduced that the average of the calculated values is the best candidate to be used as $X[k]$ for the next iteration.

Generally, for a noiseless channel, after a few iterations the results will converge to the desired values. However, in the presence of noise, for low SNR conditions, the values of $(1/N)\sum A_r[n,k]$ and $(1/N)\sum A_i[n,k]$ obtained through the algorithm, lie outside of the range [-1,1]. As explained before, our polynomials have their extreme points at the beginning and end of this range. Moreover, polynomials used in this algorithm have a high derivative out of the mentioned range. In this case, the obtained values beyond the [-1,1] range will be exposed to the high derivative part of the function, and during the following iterations, will move further from the values of the original signal. Consequently, the values of $\mathrm{Re}\{X[k]\}$ and $\mathrm{Im}\{X[k]\}$ will drastically diverge. As a result the signal points will be spread in the constellation map, approximately, in the form of circular shape with an enormous radius.

In order to prevent the algorithm from diverging in the presence of channel noise, one can replace the out-of-range values of the $(1/N)\sum A_r[n,k]$ and $(1/N)\sum A_i[n,k]$ with the values of the extreme points related to the sign of computed terms. It can prevent values from moving out of the range and make the system converge. Another suggestion for avoiding divergence is to use a linear function such as a ramp with unit slope outside of the [-1,1] range. The simulation results show that this ramp is a good choice for any SNR value to avoid divergence. However, using the unit ramp function in a noisy condition will result in more iterations before convergence in comparison with the noiseless situation.

4. SIMULATION RESULTS AND CONCLUSION

Reduction of PAPR in an OFDM system using a companding method is obtained at the cost of increase in SNR for a given BER. In order to analyze the relationship between BER performance and PAPR reduction of the applied algorithm, randomly generated data modulated by 16-QAM and 64-subcarriers is used.

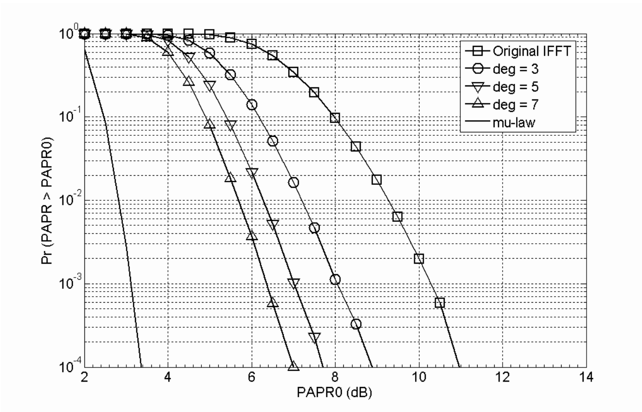

Fig. 4. The CCDF of PAPR for original and compressed signals.

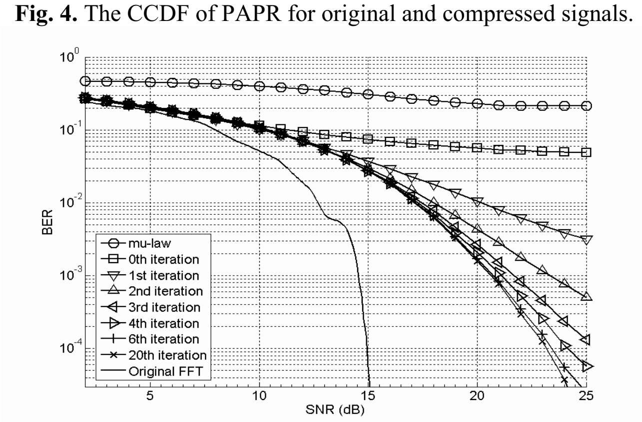

Fig. 5. Effect of iteration on improving performance.

Fig. 4 shows the results for a number of 10^5 random OFDM symbols where the original signal and different companding methods are considered. The Complementary Cumulative Distribution Function (CCDF) of PAPR for original signals, μ-law compressed signals with $\mu = 255$ and compressed signals with polynomials of different orders are shown. As it is observed, the algorithm reduces the PAPR of the signal effectively compared to the original signal. For the 3rd, 5th, 7th order polynomials, and for CCDF = 10^{-4}, the PAPR is reduced 2.1, 3.3 and 4 dB respectively. Therefore, with higher order polynomials, more PAPR reduction can be achieved.

The advantage of using more iterations on system performance is illustrated in Fig. 5 where the 5th order polynomial is employed. It can be seen that for low SNR values, i.e. less than 10dB, there is no difference between different curves, hence, FFT with no iteration can be used. Also, the negligible difference between 6th and 20th iteration curves is noticeable. It is obvious that after a certain number of iterations, the performance cannot be improved further.

Fig. 6 shows the effect of polynomials with different orders used, where the number of iterations at the receiver is the minimum required for the best performance. Curves in this figure are obtained by 3rd, 5th and 7th order polynomials through 3, 7 and 9 iterations respectively.

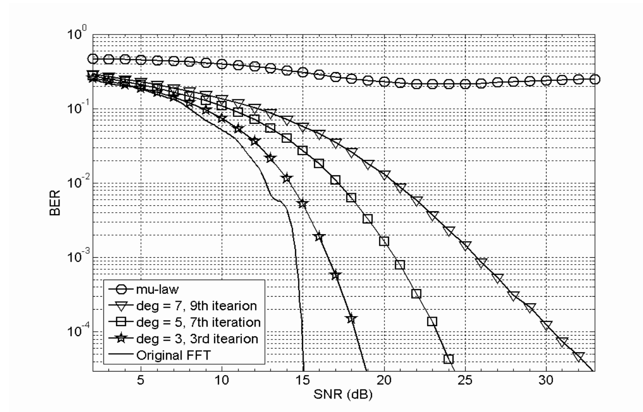

Fig. 6. Effect of polynomial order on system performance.

Although μ-law reduces the PAPR more than our proposed algorithm, Fig. 6 states that its required SNR is much higher for the same BER.

As a result of the simulations, increasing the polynomial order reduces the PAPR while lowering the BER performance, and running more iterations at the receiver improves BER performance in exchange for more computation. In designating an OFDM system the polynomial order and the effective number of iterations can be selected based on the above results.

5. REFERENCES

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