Sensitivity Analysis of Interleaved OFDMA System Uplink to Carrier Frequency Offset

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***Abstract*—This paper investigates the sensitivity analysis of orthogonal frequency division multiple access (OFDMA) systems to carrier frequency offset (CFO) in the uplink. This analysis uses simple superposition principle approach, where the effects of different users are studied separately. We calculate a closed-form expression for signal-to-interference ratio (SIR) and derive very simple expressions for inter-carrier interference (ICI) and multiple access interference (MAI) in interleaved subcarrier allocation scheme. Finally theoretical results are verified using Monte Carlo simulation.**

***Index Terms*—OFDMA uplink, carrier frequency offset (CFO), interleaved subcarrier allocation, inter-carrier interference (ICI), multiple access interference (MAI).**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is widely used in many communication systems, e.g. digital audio broadcasting (DAB), digital video broadcasting (DVB), asymmetric digital subcarrier line (ADSL) and IEEE 802.11a/g. In OFDM, a set of equally spaced subcarriers are used for parallel data transmission. Furthermore, orthogonal frequency division multiple access (OFDMA) is a promising technology for broadband wireless communication and has been adopted by the IEEE 802.16e [1]. In OFDMA systems, the available subcarriers are divided into several mutually exclusive sets assigned to different users for simultaneous transmission. The orthogonality among subcarriers guarantees intrinsic protection against inter-carrier interference (ICI) and multiple access interference (MAI) [2].

Similarly to OFDM, OFDMA is sensitive to carrier frequency offset (CFO) due to oscillator instabilities and/or wireless channel effects introducing ICI and MAI consequently [2]. In downlink transmission, an OFDMA system is equivalent to an OFDM system with multiple users. The sensitivity analysis of OFDM systems to CFO has been studied extensively [3]-[6]. Some literature have studied the sensitivity of OFDMA to CFO for the uplink transmission [7]-[9]. One of the pioneering works on the uplink case, is presented in [7]. In [8], CFO is modelled as an independent and identically distributed (i.i.d) random variable and signal-to-interference-plus-noise ratio (SINR) is derived as a function of the variance of CFO. Reference [9] studies the combined effect of both CFO and timing offsets on signal-to-interference ratio (SIR). Generally, the derived expressions in the existing literature are not simple and their analysis are complex and incomplete in some cases. This paper studies the sensitivity analysis of OFDMA uplink to CFO more completely using the simple superposition principle and a simple closed-form expression for SIR is derived.

The sensitivity analysis of OFDMA uplink to CFO is closely related to the subcarrier allocation scheme [7]. There are two major allocation schemes, namely block and interleaved schemes [10]. In the former, disjoint blocks of contiguous subcarriers are allocated to distinct users. In the latter, subcarriers of each user are equally spaced over the whole transmission bandwidth. Due to the blockwise structure, the block allocation provides good robustness to CFO [10]. On the other hand, the block allocation is vulnerable to frequency selective channels, where the interleaved allocation provides maximum separation between subcarriers allocated to each user and then maximizes the frequency diversity for each user.

This paper studies the sensitivity analysis of OFDMA uplink to CFO using superposition principle, where the effects of different users are considered separately. We derive a simple closed-form expression for SIR in interleaved subcarrier allocation scheme. We will show that there is an excellent match between the theoretical and simulation results.

The rest of this paper is organized as follows. The system model is presented in section II. The SIR analysis for interleaved allocation scheme is derived in Section III. Section IV includes simulation results and finally the conclusion is drawn in Section V.

II. SYSTEM MODEL

Consider the uplink of an OFDMA system, where M active users are communicating with a base station. Assume that there are $N = M \cdot Q$ subcarriers, where Q is the number of subcarriers allocated to each user. The index set of Q subcarriers assigned to the mth user is denoted by $\mathcal{I}_m$. Clearly $\bigcup_{m=1}^{M} \mathcal{I}_m = \{0, 1, \cdots, N-1\}$ and $\mathcal{I}_m \cap \mathcal{I}_j = \emptyset, \forall\, m \neq j$. A baseband discrete time block diagram of an OFDMA uplink in the presence of CFO is depicted in Fig. 1. In this figure subscript m denotes the mth user and subscript i denotes the ith uplink transmitted block. Also N_g is the number of cyclic prefix (CP) samples and $N_T = N_g + N$ is the total number of samples for an OFDMA uplink block. The data stream of each user is divided into blocks of Q symbols and the ith block of the mth user (i.e., $\widetilde{\boldsymbol{X}}_{m,i}$) is assigned to its own subcarriers by

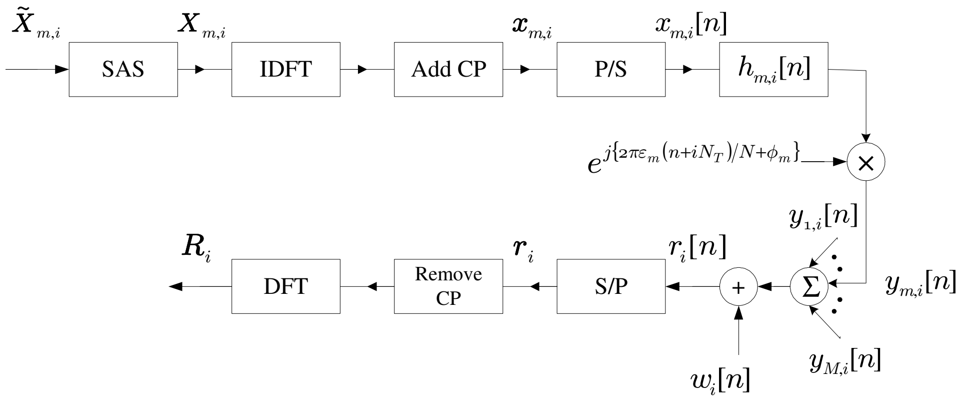

Fig. 1. The baseband equivalent of the uplink of an OFDMA system for the ith transmitted block in the presence of CFO.

the subcarrier allocation scheme (SAS) unit. Thus $X_{m,i}[k]$, $k \in \mathcal{I}_m$ are information symbols of the mth user allocating to the corresponding subcarriers. We focus on the ith uplink block and for notational simplicity, the i subscript will not be used throughout this paper. The received signal at the receiver is

$$r[n] = \sum_{m=1}^{M} y_m[n] + w[n], \tag{1}$$

where $w[n]$ is the additive white Gaussian noise (AWGN) and $y_m[n]$ is the received signal from the mth user in the presence of CFO as

$$y_m[n] = e^{j\left\{\frac{2\pi\varepsilon_m}{N}(n+iN_T)+\phi_m\right\}}\left(x_m[n] * h_m[n]\right), \tag{2}$$

where ϕ_m is the phase offset and ε_m is the normalized CFO (to the subcarrier spacing) considered within interval $[-0.5, 0.5]$. Also $*$ denotes the linear convolution and $x_m[n]$ is the transmitted signal by the mth user

$$\begin{aligned} x_m[n] &= \frac{1}{N}\sum_{k\in\mathcal{I}_m} X_m[k]e^{j2\pi k(n-N_g)/N}; \\ &\qquad n = 0, 1, \cdots, N_T - 1. \end{aligned} \tag{3}$$

The signal $x_m[n]$ is transmitted through a multipath fading channel assumed static over an OFDMA uplink block and $h_m[n]$ is the overall channel impulse response (CIR) between the mth user and base station. The channel taps are assumed to be complex and statistically independent circular Gaussian random variables with zero mean (Rayleigh fading) and power delay profile $\mathrm{E}\left\{|h_m[n]|^2\right\} = \beta_m e^{-n/L_m}$; $n = 0, 1, \cdots, L_m - 1$, where L_m is the CIR order and β_m is a scaling factor for the average energy of CIR as $\sum_{n=0}^{L_m-1}\mathrm{E}\left\{|h_m[n]|^2\right\} = \overline{\gamma}_m$.

Furthermore, we assume that the system is time-synchronized OFDMA. It can be shown that if $N_g > \max_m\{L_m + \theta_m\}$, where $\theta_m = \mathrm{int}(\triangle t_m/T_s)$ is the normalized timing error to the sampling period, the system will be quasi-synchronous and then the timing errors can be compensated for by the channel equalizer [2].

III. SIR ANALYSIS

In this section, the sensitivity of OFDMA uplink to CFO is studied. The symbols of different users are independent and also are independent of AWGN. Thus we can apply the superposition principle to different users due to linearity of the system. At this point, we assume that the symbols of all users, except for the mth user, are zero. In other words, we calculate the effect of desired user's CFO on all subcarriers. For simplicity AWGN is not considered in SIR analysis. By substituting (2) in (1), we have

$$r[n] = y_m[n] = Nc_m[n]e^{j\psi_m}\left(x_m[n] * h_m[n]\right), \tag{4}$$

where $c_m[n] \triangleq \frac{1}{N}e^{j2\pi\varepsilon_m n/N}$ and $\psi_m \triangleq 2\pi\varepsilon_m iN_T/N + \phi_m$. After removing CP from sequence $r[n]$ and taking DFT, we have

$$\begin{aligned} R[k] &= e^{j\psi_m}\left\{C_m[k] \circledast \left(H_m[k]X_m[k]\right)\right\}; \\ &\qquad k = 0, 1, \cdots, N-1, \end{aligned} \tag{5}$$

where $C_m[k]$, $H_m[k]$ and $X_m[k]$ are the DFT of sequences $c_m[n]$, $h_m[n]$ and $x_m[n]$ respectively and $\circledast$ denotes the circular convolution. It can be shown that

$$C_m[k] = \frac{\sin\left(\pi\left(\varepsilon_m - k\right)\right)}{N\sin\left(\frac{\pi}{N}\left(\varepsilon_m - k\right)\right)}e^{j2\pi(\varepsilon_m-k)(N-1)/N}. \tag{6}$$

Relation (5) may be rewritten as

$$\begin{aligned} R[k] = {} & \underbrace{e^{j\psi_m}C_m[0]H_m[k]X_m[k]}_{\text{desired signal term}} \\ & \underbrace{+\, e^{j\psi_m}\sum_{r\in\mathcal{I}_m, r\neq k} C_m[k-r]H_m[r]X_m[r]; \quad k \in \mathcal{I}_m}_{\text{ICI term}} \end{aligned} \tag{7a}$$

$$R[k] = \underbrace{e^{j\psi_m}\sum_{r\in\mathcal{I}_m} C_m[k-r]H_m[r]X_m[r]; \quad k \notin \mathcal{I}_m}_{\text{MAI term}}. \tag{7b}$$

In the absence of CFO, $R[k] = e^{j\phi_m}H_m[k]X_m[k]$, $k \in \mathcal{I}_m$ denote received symbols on the subcarriers of the mth user affected by the channel. Also, we do not have any interference over subcarriers of other users, i.e., $R[k]=0,\ k\notin\mathcal{I}_m$. So the first term for $k\in\mathcal{I}_m$ in (7) is the desired signal term which is attenuated, and the second term is the ICI term caused by the mth user's CFO. The term for $k\notin\mathcal{I}_m$ in (7) is MAI caused by the mth user for the kth subcarrier belonging to other users. We denote the average powers of these terms by $\mathrm{P}_{\mathrm{S}}^{k}$, $\mathrm{P}_{\mathrm{ICI}}^{k}$ and $\mathrm{P}_{\mathrm{MAI}}^{m,k}$ respectively. The symbols assigned to the subcarriers of each user are zero mean and uncorrelated $E\left\{X_m[r]X_m^*[s]\right\}=\sigma_m^2\delta[r-s]$. They are also independent of the channel taps. In addition, it can be shown $E\left\{\left|H_m[k]\right|^2\right\}=\overline{\gamma}_m$. In the following, the calculation of $\mathrm{P}_{\mathrm{S}}^{k}$, $\mathrm{P}_{\mathrm{ICI}}^{k}$ and $\mathrm{P}_{\mathrm{MAI}}^{m,k}$ are explained. For the desired signal power $\mathrm{P}_{\mathrm{S}}^{k}$ we have

$$\begin{aligned}\mathrm{P}_{\mathrm{S}}^{k} &= \left|C_m[0]\right|^2\mathrm{E}\left\{\left|H_m[k]X_m[k]\right|^2\right\}\\ &= \left|C_m[0]\right|^2\sigma_m^2\overline{\gamma}_m\\ &\triangleq f_N^2\left(\varepsilon_m\right)\sigma_m^2\overline{\gamma}_m;\quad k\in\mathcal{I}_m,\end{aligned}\tag{8}$$

where $f_N(x)\triangleq\left|\frac{\sin(\pi x)}{N\sin(\pi x/N)}\right|$. The average ICI power $\mathrm{P}_{\mathrm{ICI}}^{k}$ is equal to

$$\begin{aligned}\mathrm{P}_{\mathrm{ICI}}^{k} &= \mathrm{E}\left\{\left|\sum_{r\in\mathcal{I}_m,r\neq k}C_m[k-r]H_m[r]X_m[r]\right|^2\right\}\\ &= \sum_{r\in\mathcal{I}_m,r\neq k}\left|C_m[k-r]\right|^2\sigma_m^2\overline{\gamma}_m\\ &\triangleq \mathrm{IP}(k)\sigma_m^2\overline{\gamma}_m;\quad k\in\mathcal{I}_m,\end{aligned}\tag{9}$$

where $\mathrm{IP}(k)$ is defined as normalized ICI power to $\sigma_m^2\overline{\gamma}_m$. The name IP stands for ICI power. For $k\in\mathcal{I}_m$, $\mathrm{IP}(k)$ is the normalized ICI power caused by the mth user on the kth subcarrier. In general, the normalized ICI power for $k\in\mathcal{I}_m$ depends on system parameters, the mth user's CFO and the subcarrier index. Similarly, MAI power caused by the mth user on the subcarriers of other users is

$$\begin{aligned}\mathrm{P}_{\mathrm{MAI}}^{m,k} &= \mathrm{E}\left\{\left|\sum_{r\in\mathcal{I}_m}C_m[k-r]H_m[r]X_m[r]\right|^2\right\}\\ &= \sum_{r\in\mathcal{I}_m}\left|C_m[k-r]\right|^2\sigma_m^2\overline{\gamma}_m\\ &\triangleq \mathrm{MP}(m,k)\sigma_m^2\overline{\gamma}_m;\quad k\notin\mathcal{I}_m,\end{aligned}\tag{10}$$

where $\mathrm{MP}(m,k)$ is defined as normalized MAI power to $\sigma_m^2\overline{\gamma}_m$. The name MP means MAI power. For $k\notin\mathcal{I}_m$, $\mathrm{MP}(m,k)$ is the normalized MAI power caused by the mth user on the kth subcarrier which does not belong to this user. Generally, the normalized MAI power caused by the mth user over $k\notin\mathcal{I}_m$, depends on system parameters, the mth user's CFO and the subcarrier index.

Now we use superposition principle to calculate instantaneous SIR. The signal term is calculated according to (8) and the interference term consists of ICI power caused by the desired user plus MAI power caused by other users on the given subcarrier. An exact SIR may be calculated by averaging instantaneous SIR over the distribution of the channel gains [6], as shown in (11). From (11) it can be seen that exact calculation of SIR is too complex. Thus the expectation in (11) is approximated by the expectation of the nominator to the expectation of denominator as [6]

$$\mathrm{SIR}(k)\approx\frac{f_N^2\left(\varepsilon_m\right)\sigma_m^2\overline{\gamma}_m}{\mathrm{IP}(k)\sigma_m^2\overline{\gamma}_m+\sum\limits_{\substack{m'=1\\ m'\neq m}}^{M}\mathrm{MP}(m',k)\sigma_{m'}^2\overline{\gamma}_{m'}};\quad k\in\mathcal{I}_m.\tag{12}$$

If the average energy of the CIR is assumed to be unity (i.e., $\overline{\gamma}_m=1,\forall m$) and power control is performed (i.e., $\sigma_m^2=\sigma_X^2,\forall m$), then (12) is simplified to

$$\mathrm{SIR}(k)\approx\frac{f_N^2\left(\varepsilon_m\right)}{\mathrm{IP}(k)+\sum\limits_{\substack{m'=1\\ m'\neq m}}^{M}\mathrm{MP}(m',k)};\quad k\in\mathcal{I}_m.\tag{13}$$

According to (13), we need to calculate $\mathrm{IP}(k)$ and $\mathrm{MP}(m,k)$. In the following, we calculate a closed-form expressions for $\mathrm{IP}(k)$ and $\mathrm{MP}(m,k)$ in interleaved allocation scheme. In this scheme, $\mathcal{I}_m=\{i_m+qM;q=0,1,\cdots,Q-1\}$, where i_m is an integer number within interval $[0,M-1]$. Under this condition, it can be proved that the normalized ICI and MAI powers are (see Appendix)

$$\mathrm{IP}(k)=f_M^2(\varepsilon_m)-f_N^2(\varepsilon_m);\quad k\in\mathcal{I}_m\tag{14}$$

$$\mathrm{MP}(m,k)=f_M^2\left(i_m-i_{m'}+\varepsilon_m\right);\quad k\in\mathcal{I}_{m'},m'\neq m.\tag{15}$$

Equations (14) and (15) are important results that define the normalized ICI and MAI powers in very simple forms. According to (14) the normalized ICI power on all subcarriers of the mth user is the same and it depends on ε_m. Equation (15) shows that the normalized MAI power over all subcarriers of the m'th ($m'\neq m$) user, caused by the mth user, is also the same and depends on ε_m and on the spacing between their subcarriers. Fig. 2 and Fig. 3 show these equations as a function of CFO for $N=64$ subcarriers with $M=4$ and $M=8$ number of users, respectively. In these figures $s\triangleq i_m-i_{m'}$ denotes spacing between subcarriers of distinct users. As seen from these figures, the normalized ICI and MAI powers are increasing functions of the absolute value of CFO within interval $[-0.5,0.5]$ and MAI is the dominant interference in interleaved allocation especially in the spacing of $s=\pm1$. Normalized MAI power for negative values of s can be obtained by mirroring the corresponding curves in positive values vertically. When the number of subcarriers allocated to each user is decreased, the normalized ICI and MAI powers are decreased but not significantly especially for dominant interference terms.

The analysis of SINR can be performed with similar approach from (13), when AWGN is present. The result is

$$\mathrm{SINR}(k)\approx\frac{f_N^2\left(\varepsilon_m\right)\mathrm{SNR}_0}{\left(\mathrm{IP}(k)+\sum\limits_{\substack{m'=1\\ m'\neq m}}^{M}\mathrm{MP}(m',k)\right)\mathrm{SNR}_0+1};\quad k\in\mathcal{I}_m,\tag{16}$$

$$\mathrm{SIR}(k) = E\left\{ \frac{f_N^2\left(\varepsilon_m\right)\sigma_m^2|H_m[k]|^2}{\sum_{r\in\mathcal{I}_m, r\neq k}|C_m[k-r]H_m[r]|^2\sigma_m^2 + \sum_{m'=1,m'\neq m}^{M}\sum_{r\in\mathcal{I}_{m'}}|C_{m'}[k-r]H_{m'}[r]|^2\sigma_{m'}^2} \right\}; k\in\mathcal{I}_m. \tag{11}$$

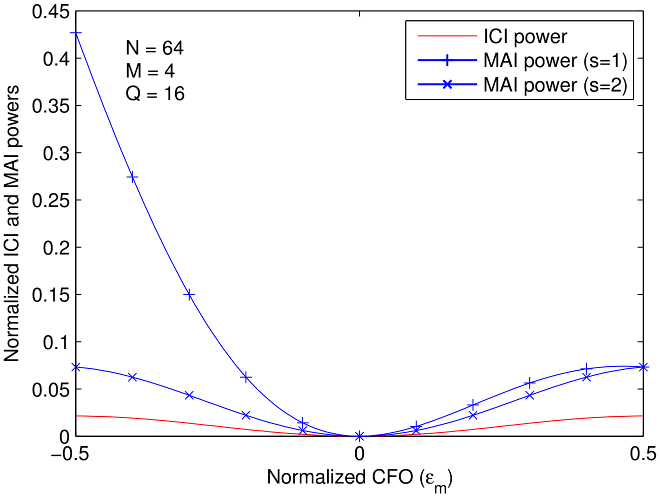

Fig. 2. Normalized ICI and MAI powers as a function of CFO ($N = 64, M = 4$).

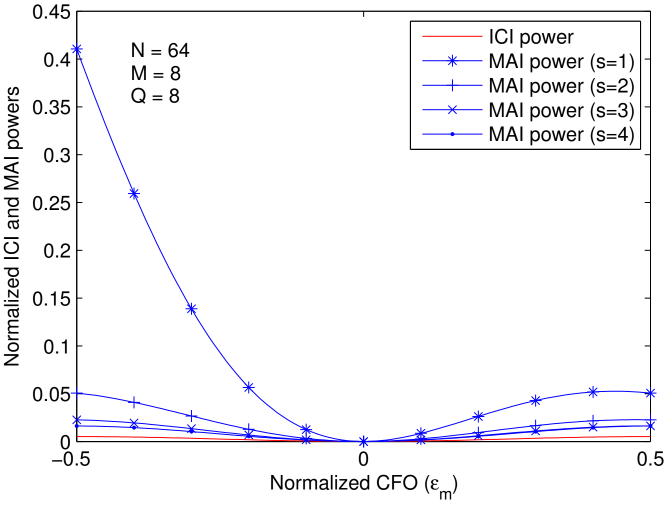

Fig. 3. Normalized ICI and MAI powers as a function of CFO ($N = 64, M = 8$).

where $\mathrm{SNR}_0 = \frac{\sigma_X^2}{\sigma_W^2}$ is received SNR in the absence of CFO and $\sigma_W^2 = \mathrm{E}\left\{|W[k]|^2\right\} = N\mathrm{E}\left\{|w[n]|^2\right\}$ is AWGN average power on the given subcarrier at the receiver. In the special cases, where $M = 1$ or $\varepsilon_m = \varepsilon, \forall m$, it can be shown that (16) is simplified to

$$\mathrm{SINR}(k) \approx \frac{f_N^2\left(\varepsilon\right)\mathrm{SNR}_0}{\left(1-f_N^2(\varepsilon)\right)\mathrm{SNR}_0+1}, \tag{17}$$

which is the derived expression in [6], for SINR of an OFDM system in the presence of CFO.

The above analysis is also applicable when some users are absent and the number of active users is less than M. In the proposed analysis the channel equalizer is not considered. It is straight forward to show that the channel equalization using a bank of one-tap multipliers in the frequency domain, dose not change the derived expression for SIR. Therefore, these results can be used for sensitivity analysis to uncompensated CFO after synchronization.

IV. SIMULATION RESULTS

In this section the theoretical results are verified using Monte Carlo simulation. We consider $M = 4$ active users communicating with the base station. We let the number of subcarriers $N = 64$, where $Q = 16$ subcarriers are assigned to each user. Also, the CP length is $N_g = 16$ and the normalized CFO vector is assumed as $[\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4] = [0.25, -0.15, 0.20, -0.10]$. The channels for all users are considered to have the same order $L_m = 4, \forall m$. In addition, the average energy of the CIR is assumed to be unity (i.e., $\overline{\gamma}_m = 1, \forall m$) and power control is performed (i.e., $\sigma_m^2 = \sigma_X^2, \forall m$). The simulation and theoretical results for SIR in interleaved allocation are shown in Fig. 4. As shown in this figure, the simulation result is in exact match with the theoretical result. As we expect, SIR is constant on the subcarriers of each user. Fig. 5 shows the simulation and theoretical results for SINR as a function of received SNR in the absence of CFO. We can also derive from this figure that there is an excellent match between the two sets of results in this case.

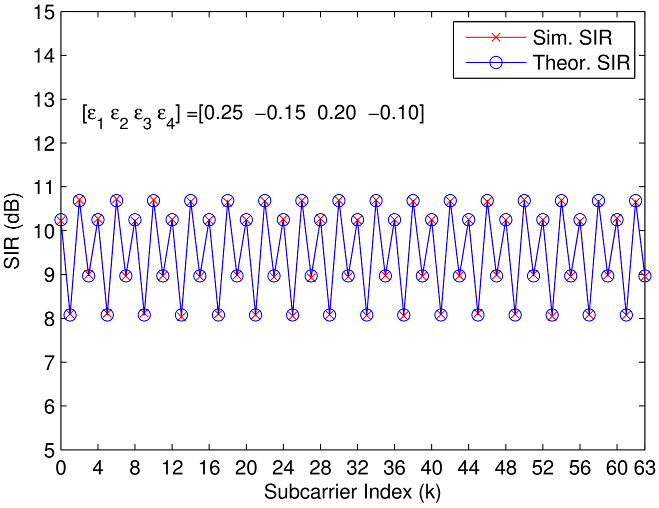

Fig. 4. SIR as a function of subcarrier index.

V. CONCLUSION

We studied the sensitivity analysis of OFDMA systems to CFO in the uplink using superposition principle, where the

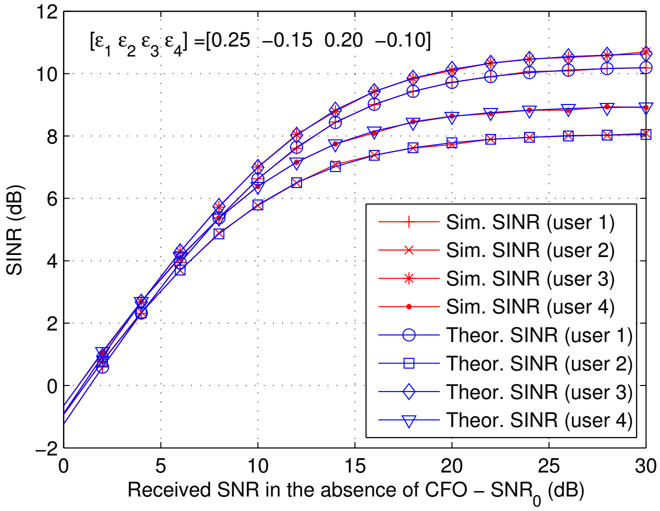

Fig. 5. SINR as a function of received SNR.

effects of different users are studied separately. We derived a closed-form expression for SIR in interleaved allocation scheme. Finally theoretical results are validated using Monte Carlo simulation.

APPENDIX A
NORMALIZED ICI AND MAI POWERS CALCULATION

Using Parseval's theorem, we have the following identity

$$\begin{aligned}\sum_{k=0}^{N-1} f_N^2\left(\varepsilon_m - k\right) &= \sum_{k=0}^{N-1} \left|C_m[k]\right|^2 \\ &= N\sum_{n=0}^{N-1} \left|c_m[n]\right|^2 \\ &= 1. \end{aligned} \tag{A.1}$$

For interleaved allocation, the normalized ICI power for $k \in \mathcal{I}_m$, is equal to

$$\begin{aligned}\mathrm{IP}(k) &= \sum_{r\in\mathcal{I}_m, r\neq k} \left|C_m[k-r]\right|^2 \\ &= \sum_{q'=0,q'\neq q}^{Q-1} \left|C_m[i_m+qM-i_m-q'M]\right|^2 \\ &= \sum_{q'=0,q'\neq q}^{Q-1} \frac{\sin^2\left(\pi\varepsilon_m\right)}{N^2\sin^2\left(\frac{\pi}{N}\left(\varepsilon_m+(q'-q)M\right)\right)} \\ &= \frac{\sin^2\left(\pi\varepsilon_m\right)}{M^2\sin^2\left(\frac{\pi\varepsilon_m}{M}\right)}\sum_{q'=0,q'\neq q}^{Q-1}\frac{\sin^2\left(\pi\varepsilon_m/M\right)}{Q^2\sin^2\left(\frac{\pi}{Q}\left(\varepsilon_m/M+q'-q\right)\right)} \\ &= f_M^2(\varepsilon_m)\sum_{q'=0,q'\neq q}^{Q-1} f_Q^2\left(\frac{\varepsilon_m}{M}+q'-q\right) \\ &= f_M^2(\varepsilon_m)\left(1-f_Q^2\left(\frac{\varepsilon_m}{M}\right)\right) \quad (\text{ by (A.1)}) \\ &= f_M^2(\varepsilon_m)-f_N^2(\varepsilon_m). \end{aligned} \tag{A.2}$$

Similarly normalized MAI power caused by the mth user over the kth subcarrier for $k \in \mathcal{I}_{m'}, m' \neq m$, is given by

$$\begin{aligned}\mathrm{MP}(m,k) &= \sum_{r\in\mathcal{I}_m} \left|C_m[k-r]\right|^2 \\ &= \sum_{q'=0}^{Q-1} \left|C_m[i_{m'}+qM-i_m-q'M]\right|^2 \\ &= f_M^2(i_m-i_{m'}+\varepsilon_m)\sum_{q'=0}^{Q-1} f_Q^2\left(\frac{i_m-i_{m'}+\varepsilon_m}{M}+q'-q\right) \\ &= f_M^2(i_m-i_{m'}+\varepsilon_m), \end{aligned} \tag{A.3}$$

where in the last step we have used (A.1).

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