Blind Channel Estimation Based on Constant Modulus for OFDM Systems

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***Abstract*—A blind channel estimation scheme is presented here for OFDM systems using PSK modulation. This scheme reduces the number of channels that is possible by using constant modulus property, and chooses the best channel among possible ones that fit by using finite alphabet property of information signals and achieves good performance with low computational complexity.**

***Index Terms*—Blind channel estimation, orthogonal frequency division multiplexing (OFDM)**

I. Introduction

CHANNEL estimation is a very important topic in orthogonal frequency division multiplexing (OFDM) when at the receiver, we need to know channel transfer function to correctly perform demodulation. Channel estimation techniques can be categorized in two sections, non-blind channel estimation and blind channel estimation. In non-blind channel estimation methods that exist, channel estimation is usually done using pilot tones or training sequences [3]. These methods often have a significant loss in channel utilization because of employing pilot tones or training sequences.

Recently blind channel estimation is more interested because they have better bandwidth efficiency. A finite alphabet blind channel estimator has been introduced in [1], but for large number of subcarriers and long channel impulse response, has high computational complexity and implementation problems.

The other method that is used for blind channel estimation is subspace based method that uses signal redundancy generated by cycle prefix or virtual carrier (VC). Also zero-padding (ZP-OFDM) was proposed in order to do symbol recovery even with channel zeros by replacing the CP and with cost of modifying the transmitter that is not good because, we supposed to uses CP (CP-OFDM) in standards.

Anyway subspace based methods need to collect data records that are almost long, to make data covariance matrix *full rank*, then they have slow convergence rate. But channel estimator based on PSK modulation (constant modulus modulation) just needs one OFDM symbol at high SNR to estimate channel that is also impossible for statistical methods and so good for estimating rapid time-varying channels.

Here we present a channel estimator based on constant modulus property and finite alphabet property of PSK signals.

II. System Model and Formulation of CP-OFDM

In this section, the OFDM system model and formulation that used in the rest of paper was explained.

Consider a OFDM system with N subcarriers. And consider the data in frequency domain is *Information Symbols*, $X(i) = [X_0(i),\cdots,X_k(i),\cdots,X_{N-1}(i)]^T$ where i, k, is for block index and subcarrier index. System uses constant envelope modulation method such as PSK that is named as constant modulus modulation. So the information sequence are constant modulus signals $|X_k(i)| = 1, \forall k \in [0, N-1]$.

Channel is modeled as a FIR filter with impulse response in time domain that has L tabs (CIR) $\mathbf{h} = [h_0,\cdots,h_L]^T$, (usually L<N/4) and frequency response $\mathbf{H} = [H_0,\cdots,H_{N-1}]^T = \mathbf{Wh},$ where $\mathbf{W}$ is N×L Fast Fourier Transform matrix.

At the receiver side, after FFT, we can write the equation below (equation (1)) for the signal of the k-th subcarrier in the i-th block (block = OFDM symbol).

$$Y_k(i) = X_k(i)H_k + W_k(i), 0 \le k \le N-1 \qquad (1)$$

where $W_k(i)$ is the white Gaussian noise with zero mean and variance σ_w^2. If we assume that the length of cyclic prefix (CP) is larger than the length of the channel (L), then we can cancel inter symbol interference (ISI) and in the rest of paper we neglect it.

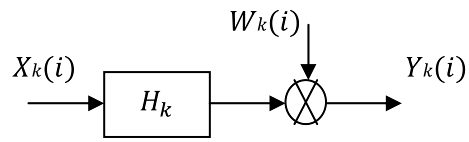

Figure 1: simplified system model

III. Blind Channel Estimation

With we neglect the noise effect on system and with assumption of constant modulus property, we can write the equation below (equation (2)).

$$|H_k| = |Y_k| \quad (2)$$

where $|Y_k|$ is the information received at the receiver in frequency domain. The algorithm that will be explained in continue is based on [1], and has two steps

A. Step one

When Frequency response of the real channel is obtained from (2), it is the time to find a channel that its frequency response magnitude is like real channel obtained from (2). Assume that the desired channel has the impulse response $\hat{\mathbf{h}} = [\hat{h}_0, \cdots, \hat{h}_L]^T$, and $\hat{\mathbf{H}} = [\hat{H}_0, \cdots, \hat{H}_{N-1}]^T$. For simplicity and without losing generality, we assume that $\sum_{l=0}^{L}|h_l| = \sum_{l=0}^{L}|\hat{h}_l| = 1$. Now the problem is to find the set of coefficients or coefficient vector $\hat{\mathbf{h}} = [\hat{h}_0, \cdots, \hat{h}_L]^T$ that minimizes the cost function defined by equation (3).

$$J = \frac{1}{4}\sum_{k=0}^{N-1}\left\{\left|\hat{H}\right|^2 - |H|^2\right\}^2 \quad (3)$$

J is minimum if and only if $\left|\hat{H}\right|^2 = |H|^2$ or $\left|\hat{H}\right| = |H|$, $\forall k \in [0, N-1]$. If we use gradient operator for iterative minimization, a factor 4 exist in equation, that why we use $\frac{1}{4}$ factor in equation (3). As shown in equation (4), a simple iterative algorithm is used to find the desired $\hat{\mathbf{h}}$.

$$\begin{aligned}\hat{\boldsymbol{h}}(m+1) &= \hat{\boldsymbol{h}}(m) - \mu(m)\nabla_{\hat{\mathrm{h}}}\mathrm{J}(\mathrm{m}) \\ &= \hat{\boldsymbol{h}}(m) - \mu(m)\sum_{k=0}^{N-1}\left\{\left|\hat{H}_k\right|^2 - |H_k|^2\right\}\hat{H}_k(m)W_k^H\end{aligned} \quad (4)$$

where m is the index for number of iteration and $\hat{\boldsymbol{h}}(m)$ is the channel response estimate. For initializing the iteration we can use $\hat{\mathbf{h}}(\mathrm{m}) = [1,0,\cdots,0]^T$, $\mu(m)$ is the step size of the algorithm and is positive. In relation to $\nabla_{\hat{\mathrm{h}}}$, it is obvious that gradient operation is done on element off the $\hat{\boldsymbol{h}}$. And finally W_k^H is a vector (1×(L+1) matrix) with (1, p) element, $e^{2jkp/N}$, $0 \le p \le L$.

At each iteration, it is better to normalized estimated vector like in equation (5)

$$\hat{\boldsymbol{h}}(m+1) = \frac{\hat{\boldsymbol{h}}(m+1)}{\left\|\hat{\boldsymbol{h}}(m+1)\right\|} \quad (5)$$

We can use steepest descent method for selecting $\mu(m)$, in this way $\mu(m)$ is the argument that minimizes the $\sum_{k=0}^{N-1}\left\{\left|W_k^H\left(\hat{\boldsymbol{h}}(m) - \mu(m)\nabla_{\hat{\mathrm{h}}}\mathrm{J}(\mathrm{m})\right)\right|^2 - |H_k|^2\right\}^2$.

If algorithm performs for enough number of iterations (m is sufficiently large), $\hat{\boldsymbol{h}}(m)$ will be coverage to a vector $\hat{\boldsymbol{h}}_{\boldsymbol{result}}$, that its magnitude equals to 1 due to equation (5). Condition for reaching to required number of iteration is that the gradient vector equals to zero. And $|H_k| = |\hat{H}_k|$, $0 \le k \le N-1$. So $\boldsymbol{h} = \hat{\boldsymbol{h}}$ (for proof see Appendix A).

B. Step two

If we assume $\hat{H}(z) = \sum_{l=0}^{L}\hat{h}_l z^{-l}$ *and* $\tilde{H}(z) = \sum_{l=0}^{L}\tilde{h}_l z^{-l}$ for z-transform of $\hat{\mathbf{h}} = [\hat{h}0, \cdots, \hat{h}L]^T$ and $\tilde{\mathbf{h}} = [\tilde{\mathrm{h}}_0, \cdots, \tilde{\mathrm{h}}_\mathrm{L}]^T$, with zeros $\{\tilde{z}_i\}_{i=1}^{L} = \{\hat{z}_i\}_{i=1}^{L}$, then we have the following proposition.

Proposition 1: (see Appendix B for proof)
If

$$|H(z)| = \left|\tilde{H}(z)\right| = \left|\hat{H}_k\right| \ , \forall k \in [0, N-1] \quad (7)$$

then the following equation comes true:

$$\tilde{H}(z) = \alpha\prod_{i=1}^{L}B_i = \sum_{l=0}^{L}\tilde{\mathrm{h}}_\mathrm{l}\mathrm{z}^{-\mathrm{l}} \quad (8)$$

where $B_i = 1 - \hat{z}_i z^{-1}$, or $z^{-1} - \hat{z}_i^{\,*}$, and α is a complex scaling factor that just only its phase is uncertain and it's magnitude is constant due to condition $\sum_{l=0}^{L}|h_l| = \sum_{l=0}^{L}\left|\hat{h}_l\right| = 1$.

It is obvious that if $\hat{\mathbf{h}}$ satisfied equation (6), for a scalar γ, $0 < \gamma \le 2\pi$, $\hat{\mathbf{h}} \times e^{j\gamma}$ also satisfied equation (6).

As a result of proposition that mentioned above is that 2^L channels (also including real channel) wit h their frequency responses satisfy equation (6) and can be identifiable with a complex scaling factor and $\hat{\mathbf{h}}$ is estimated in step one.

As explained in [1], we can use one pilot symbol to estimate and resolve scalar ambiguity of all possible channels. With use of finite alphabet property, it can be possible to find the argument that minimize and best fit over 2^L channels

$$\hat{H} = \arg_{\tilde{H}(p), p\in[1,2^L]}\min\sum_{k=0}^{N-1}\left|\tilde{H}^s{}_k(p) - H_k^s\right|^2 \quad (9)$$

Notice that with pilot symbol the scalar ambiguity can be forms and shows with $\tilde{h}(p)$. S is the constellation size defined in [1] and $H_k^S = \frac{1}{I}\sum_{i=0}^{I-1}y_k^s(i)$.

It can be easily shown that the complexity of the algorithm is of the order 2^L.

At the beginning of the paper, noise effect was omitted. If noise applies to algorithm, with equation below, estimation is done.

$$|H_k| = \frac{1}{I}\sum_{i=0}^{I-1}y_k^s(i) \quad (10)$$

And I is the number of blocks (symbols) that averaged.

IV. Conclusion

With use of constant modulus information symbols, an estimator present in this paper for OFDM system. Also be assumed that system uses finite alphabet. The best advantage of this method is its low computational complexity and non-use of high order statistics. Simplicity of this method makes it suitable for implementation on hardware such as FPGA or DSP. And with use of high order statistics of the signal, presented algorithm can reach better accuracy with a limit number of OFDM symbols.

APPENDIX

Appendixes A: Proof of Equation (6)

Form the condition

$$\nabla_{\hat{h}} J = \sum_{k=0}^{N-1} \left\{ \left|\hat{H}_k\right|^2 - |H_k|^2 \right\} \hat{H}_k W_k^H = [0, \cdots, 0]^T$$

The following equation can be obtained

$$IFFT \left\{ \left[\left|\hat{H}_k\right|^2 - |H_k|^2 \right] \hat{H}_k \right\}_{k=0}^{N-1} = [\underbrace{0, \cdots, 0}_{L+1}, a_1, \cdots, a_{N-L}]$$

where a_n represents a complex number. On the other hand, it can also be derived that

$$IFFT \left\{ \left[\left|\hat{H}_k\right|^2 - |H_k|^2 \right] \hat{H}_k \right\}_{k=0}^{N-1}$$
$$= IFFT \left\{ \left|\hat{H}_k\right|^2 - |H_k|^2 \right\}_{k=0}^{N-1} \otimes IFFT \{\hat{H}_k\}_{k=0}^{N-1}$$
$$= \{b_1, b_2, \cdots, b_{L+1}, 0, \cdots, 0, b^*{}_{L+2}, \cdots, b^*{}_2\} \otimes \{\hat{h}_0, \hat{h}_1, 0, \cdots, 0\}$$

where $\otimes$ represents circular convolution, b_n represents a complex number with $n \in [2, L+1]$ and $b_1 = \sum_{k=0}^{N-1} \left|\hat{H}_k\right|^2 - |H_k|^2 = 0$, because $\sum_{l=0}^{L} |h_l| = \sum_{l=0}^{L} \left|\hat{h}_l\right| = 1$. From the previous equations, it is not difficult to get that

$$b_n = 0, n \in [2, L+1]$$

Since $b_n = 0, n \in [2, L+1]$ can be uniquely determined by $\hat{\boldsymbol{h}}$, there is a unique solution and equation (6) has been proofed.

Appendixes B: Proof of Proposition 1

Given the condition $|\hat{H}_k|^2 = |\tilde{H}_k|^2, \forall k \in [0, N-1]$, it can be derived that $\hat{H}(z)/\tilde{H}(z)$ is an (L)-order allpass transfer function.

We can obtain that a (L)-order stable allpass transfer function can be written as

$$H_{ap}(z) = e^{j\varphi} \prod_{i=1}^{L} \{ \frac{z^{-1} - \gamma_i^*}{1 - \gamma_i z^{-1}} \}$$

where φ is a real number in the range [0, 2π]. If the stability is not considered, an allpass transfer function can be written as

$$H_{ap}(z) = e^{j\varphi} \prod_{i=1}^{L} A_i$$

where $A_i = \frac{z^{-1} - \gamma_i^*}{1 - \gamma_i z^{-1}}$ or $\frac{1 - \gamma_i z^{-1}}{1 - \gamma_i z^{-1}}$.

So it can be derived that $\tilde{z}_i = \hat{z}_i \; or \; \frac{1}{\hat{z}_i^*}$.

Therefore the proposition has been proofed.

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